

# Analysis of Natural Frequency and Mode Shape of All Edge Fixed Condition Plate with Uncertain Parameters

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**Abstract:** Vibration analysis of plates has been an active research subject of engineering field. The analytical solutions have been found for plates with specified forms of mass and stiffness modifications, but for the plates with uncertain mass and stiffness modifications have not been addressed confidently. The concept of uncertainty plays an important role in the design of practical mechanical system. So it becomes important to study its effects on mechanical system for different frequency domain i.e. low, medium and high frequency. Here in this paper structural element square plate with all edge fixed boundary condition is selected on which mass, stiffness and combination of it, uncertainty is taking in account. By using Finite Element Method technique finding out, how plate is dynamically behaves in vibration. During modal analysis natural frequency and mode shapes are find and compared the response of bare plate and the plate with mass, stiffness ,mass-stiffness combination uncertainty.

**Keywords:** Vibration analysis, FEM, Modal Analysis, Uncertain parameters and Ansys software.

## I. INTRODUCTION

Plates and plate-like components are widely used in engineering structures such as several machine structures, civil engineering structure, boards in electronic equipment. Vibration analysis of plates and plate-like components has been an active research subject and numerous technical papers have been published. For to calculating the natural frequencies and mode shapes of a structure Modal analysis method is used. This method determined the dynamic response of complicated structural dynamic problems. In general, applications of modal analysis today cover a broad range of objectives identification and evaluation of vibration phenomena, validation, structural integrity assessment, structural modification, and damage detection. In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of given set of design parameters. The study of mathematical models which involve physical and geometric parameters such as mass density  $\rho$ , elastic modulus  $E$ , Poisson's ratio  $\nu$ , lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes. In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an important role in investigation of various engineering problems.

## II. LITERATURE SURVEY

Natural frequencies and mode shapes of the modified plate are identified by analysing the receptance data based on the concept of modal analysis [1]. A more recent and complete review of vibration of plates can be found in Ref. [2]. However, for most cases where plates are used in practical engineering structures, mass and/or stiffness modifications of these plates become necessary, while the natural frequencies and mode shapes of the modified plates are of interest. Such problems of determining the natural frequencies and mode shapes of a plate with mass and/or

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stiffness modifications has been treated by several investigators. By assuming the fundamental shape function and using the Rayleigh-Ritz method, fundamental natural frequency of a rectangular plate with a rigid mass of finite width under certain boundary conditions was calculated by Cohen and Handelman [3]. The fundamental frequency of a square plate clamped all around and having a mass at the center of twice its own mass was solved by Solecki [4]. Stokey and Zorowski [5] developed a general method for determining approximately the natural frequencies of a rectangular plate with arbitrarily located masses.

The study of mathematical models which involve physical and geometric parameters such as mass density  $\rho$ , elastic modulus  $E$ , Poisson's ratio  $\nu$ , lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes, structural loading and environmental conditions [6]. In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an important role in investigation of various engineering problems [7].

Here in this paper square plate with all edge fixed boundary condition and two major uncertain parameters are consider i.e. mass and stiffness. Hence the analysis is done on bare plate and plate with uncertainty due to mass, stiffness and combination of mass and stiffness. For all these combination modal analysis is done and different natural frequency and mode shapes were found. Again for these combinations harmonic analysis is done and FRF were plotted and compared it for all the conditions.

### III. MATHEMATICAL MODELING

All the Vehicles, aircraft and home appliances structures are made up of Plate or combination of Plates so it becomes necessary to study Plate vibration. Using the Lagrange- Rayleigh -Ritz technique [8], the equations of motion of a dynamic system in modal space can be derived.

#### 3.1: Bare Square Plate:

Considering the All Edge Fixed bare square plate (with no structural uncertainty). For this plate eigenfunction is described by sinusoidal mode shapes in the  $x$  and  $y$  directions, respectively.

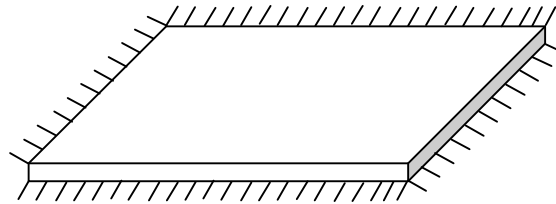


Fig. 1 All Edge Fixed bare square plate

Lagrange's equation results in the equation of motion of the bare plate as given below

$$\ddot{q} + \omega^2 q = 0 \tag{1}$$

The natural frequencies can then be obtained by eigenvalue analysis [9]

#### 3.2: Uncertain Mass Loaded Plate:

Now consider the uncertain mass loaded plate as shown in Fig 2 for this the equation of motion is

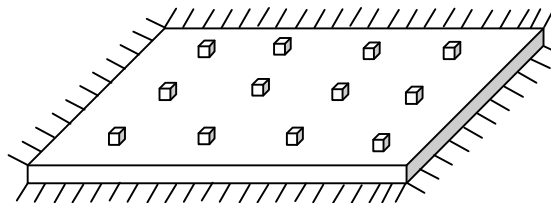


Fig. 2 All Edge Fixed bare square plate with mass uncertainty

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$$\ddot{q}_{pq} + \sum_{N_m} \sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m) \psi_{pq}(x_m) + \omega_{pq}^2 q_{pq} = 0 \tag{2}$$

Where;

$N_m$  is number of point masses,  $\psi_{mn}(x)$  are the mass-normalized eigenfunctions and  $x_m$  correspond to the random locations of the added masses [10]

### 3.3: Uncertain Spring (Stiffness) Loaded Plate:

Now consider the uncertain spring loaded plate as shown in Fig 3 for this the equation of motion is

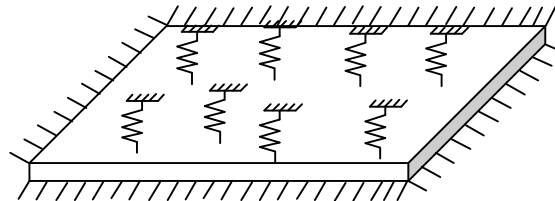


Fig. 3 All Edge Fixed bare square plate with stiffness uncertainty

$$\ddot{q}_{pq} + \sum_{N_k} \sum_{mn} k q_{mn} \psi_{mn}(x_k) \psi_{pq}(x_k) + \omega_{pq}^2 q_{pq} = 0 \tag{3}$$

Where;  $N_k$  springs to ground (of stiffness  $k$ ),  $\psi_{mn}(x)$  are the mass-normalized eigenfunctions and  $X_k$  correspond to the random locations of the added springs

### 3.4: Uncertain Mass-and-Spring-Loaded Plate:

Now consider a mass-and-spring-loaded plate as shown in Fig 4 for this the equation of motion is [10]

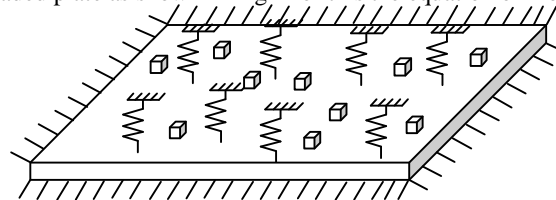


Fig. 4 All Edge Fixed bare square plate with mass and stiffness uncertainty

$$\ddot{q}_{pq} + \sum_{N_m} \sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m) \psi_{pq}(x_m) + \sum_{N_k} \sum_{mn} k q_{mn} \psi_{mn}(x_k) \psi_{pq}(x_k) + \omega_{pq}^2 q_{pq} = 0 \tag{4}$$

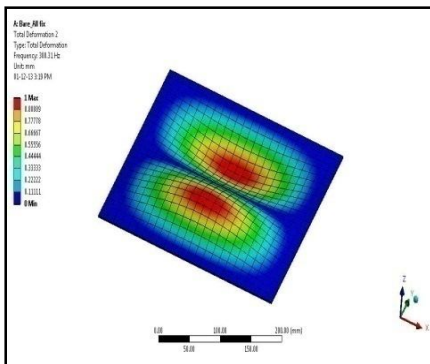
Where;  $N_m$  number of point masses and  $N_k$  springs to ground (of stiffness  $k$ ),  $\psi_{mn}(x)$  are the mass-normalized eigenfunctions, and  $x_m$ ,  $x_k$  respectively correspond to the random locations of the added masses and springs.

## IV. MODAL ANALYSIS OF SQUARE PLATE WITH AND WITHOUT UNCERTAINTY

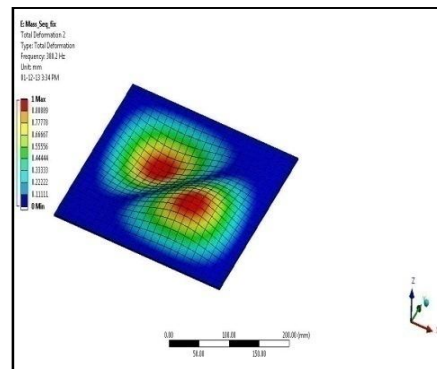
A square plate of dimension 304.8mm×304.8mm×2mm of steel material with properties of  $\rho=7.86 \times 10^{-9}$ ,  $\nu=0.3$ ,  $Y=2e5$  MPa was created. Applied all material properties and boundary condition of all edges fixed. Selecting the element type quadratic shell element 181. SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node translations in the x, y, and z directions, and rotations about the x, y, and z-axes. The degenerate triangular option should only be used as filler elements in mesh generation. Shell 181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses [11].

Total mass of plate is  $1.5 \times 10^{-3}$  tones. Mass uncertainty is taken 2% of total mass [12]. Number of masses taken is 13. They are sequentially placed on a plate.

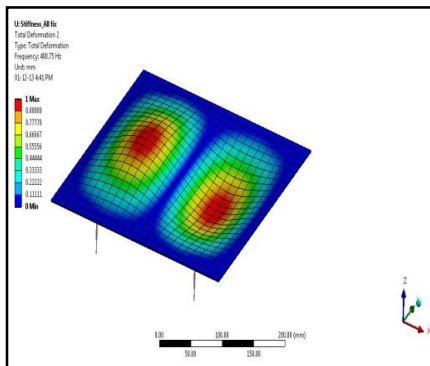




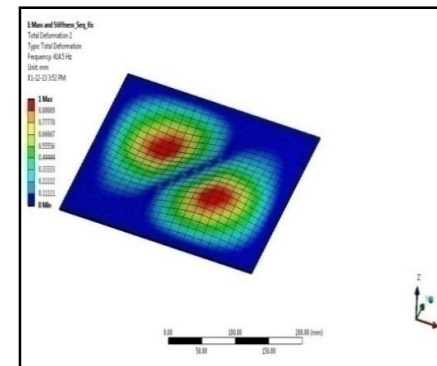
(a)



(b)



(c)



(d)

Fig. 6. Second mode shape of plate: (a) bare plate, (b) mass uncertainty, (c) stiffness uncertainty, (d) mass and stiffness uncertainty

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TABLE I  
NATURAL FREQUENCY VALUE FOR 30 MODE SHAPES

Mode No.	Bare Plate (Hz)	Mass Uncertainty (Hz)	% Error	Stiffness Uncertainty (Hz)	% Error	Mass And Stiffness Uncertainty (Hz)	% Error
1	189.08	183.08	-3.17	207.15	9.56	217.19	14.86
2	388.31	380.20	-2.09	400.75	3.20	414.50	6.74
3	388.31	380.20	-2.09	409.61	5.49	414.50	6.74
4	572.59	562.57	-1.75	624.15	9.00	866.18	51.27
5	705.04	687.24	-2.52	746.37	5.86	937.53	32.98
6	708.39	691.14	-2.44	758.15	7.02	948.77	33.93
7	880.45	861.27	-2.18	910.01	3.36	1047.00	18.92
8	880.45	861.27	-2.18	916.02	4.04	1047.00	18.92
9	1147.70	1131.80	-1.39	1174.70	2.35	1168.30	1.79
10	1147.70	1131.80	-1.39	1184.90	3.24	1168.30	1.79
11	1176.60	1145.90	-2.61	1239.40	5.34	1336.10	13.56
12	1311.90	1302.70	-0.70	1396.50	6.45	1366.20	4.14
13	1317.40	1311.90	-0.42	1477.70	12.17	1765.20	33.99
14	1600.70	1590.20	-0.66	1691.60	5.68	1785.60	11.55
15	1600.70	1590.20	-0.66	1712.60	6.99	1785.60	11.55
16	1721.00	1706.80	-0.83	1844.10	7.15	2009.70	16.78
17	1722.40	1708.70	-0.80	1881.60	9.24	2219.00	28.83
18	1884.00	1868.20	-0.84	2009.20	6.65	2237.00	18.74
19	1884.00	1868.20	-0.84	2073.90	10.08	2237.00	18.74
20	2014.00	1995.60	-0.91	2178.40	8.16	2788.50	38.46
21	2158.60	2124.40	-1.58	2321.90	7.57	2806.30	30.01
22	2165.30	2156.60	-0.40	2423.70	11.93	2809.20	29.74
23	2444.50	2393.50	-2.09	2490.20	1.87	2809.20	14.92
24	2444.50	2393.50	-2.09	2548.30	4.25	3114.30	27.40
25	2564.70	2536.50	-1.10	2689.60	4.87	3219.00	25.51
26	2564.70	2563.20	-0.06	2709.70	5.65	3243.80	26.48
27	2601.30	2563.20	-1.46	2736.00	5.18	3289.50	26.46
28	2604.70	2601.30	-0.13	3001.90	15.25	3289.50	26.29
29	2874.10	2827.50	-1.62	3038.70	5.73	3563.70	23.99
30	2874.10	2827.50	-1.62	3213.90	11.82	3563.70	23.99

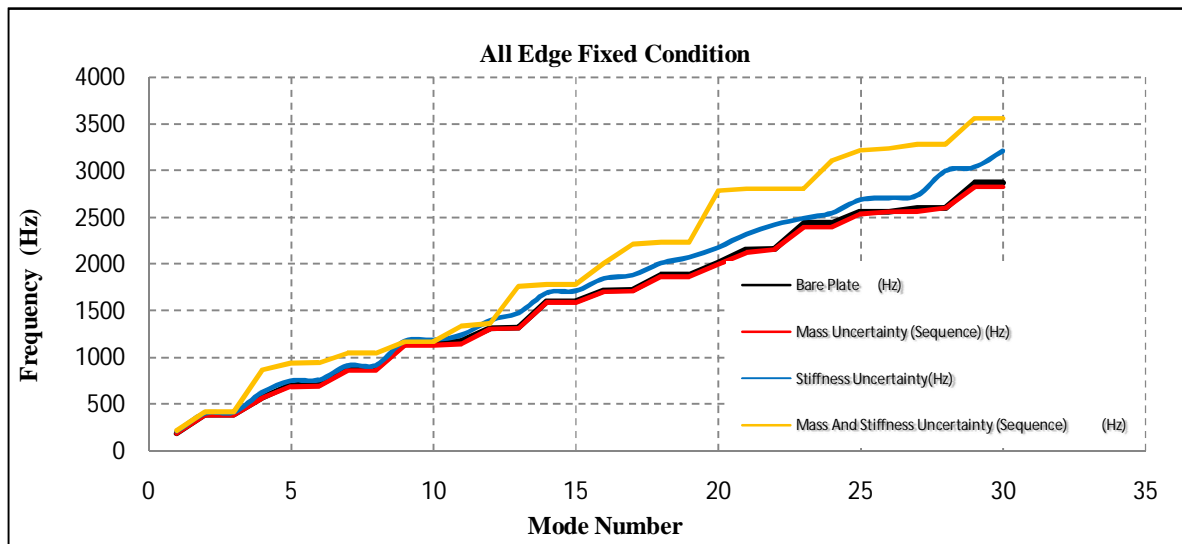


Fig. 7 Graph showing frequency deviations of bare plate with uncertain parameter contain plate.

### VI. CONCLUSIONS

By adding the uncertain mass, we are getting the different dynamic characteristic of plate for the same boundary condition. Though, the amount of uncertain mass is very low still it has influence on the structure. First frequency of the plate has been shift from 189.08 Hz to 183.08 Hz in case of all edge fixed boundary condition. It is clearly showing the effect of addition of uncertain mass. This trend of decreasing the plate frequencies are continuing for all the modes.

If we compare the characteristics of bare plate with that of plate with uncertain for the first 30 modes, there is clear deviation in frequency shift which may affect the working condition of entire structure. In case of uncertain mass in sequence pattern there is a deviation of -3.17% in fundamental mode which may not be acceptable in case precise instrumentation or precision manufacturing systems.

Addition of uncertain mass to the bare plate not only shift its frequencies but it also affects the mode shapes of plate. By adding an uncertain mass to the plate structure, apart from fundamental mode, most of the modes are losing their pattern. Irrespective of the frequency shift, the changing vibrating behavior of plate itself is a critical and unavoidable issue.

Addition of the uncertain stiffness dominantly changes dynamic characteristic of plate for the same boundary condition. First frequency of the plate has been shift from 189.08 Hz to 207.15 Hz in this boundary condition having the deviation of 9.55% in fundamental mode which may not be acceptable.

By adding the uncertain mass and stiffness, we are getting the drastically different dynamic characteristic of plate for the same boundary condition. First frequency of the plate has been shift from 189.08 Hz to 217.19 Hz in case and deviation of 14.86 % in first mode and in forth mode it is 51.27% of fundamental mode which may not be acceptable in any case.

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