

# The Trapezoid Fuzzy Prioritized Operators And Their Applications In Group Decision Making

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**Abstract:** In this paper we present a new class of operators based on Trapezoid Fuzzy numbers such as Trapezoid Fuzzy Prioritized Weighted Mean (TFPWM) operator, Trapezoid Fuzzy Prioritized Weighted Geometric Average (TFPWG), Trapezoid Fuzzy Prioritized Weighted Harmonic Average (TFPWH) operators to solve multiple attribute group decision making (MAGDM) problems. Unlike other (MAGDM) techniques here the attributes and experts are at different priority levels. Some desirable properties of TFPWM, TFPWG and TFPWH operator are investigated. Finally, we give a numerical example to illustrate the application of these operators to group decision making problems with trapezoidal fuzzy information.

**Keywords:** Aggregation Operators, Trapezoidal Fuzzy numbers, Prioritized aggregation operators, Decision making.

## I. INTRODUCTION

Multiple attribute decision making methods are widely used to rank many real world alternatives with respect to several competing attributes. Since the decision making process is governed by human perception and judgment which is ambiguous in nature, using fuzzy sets instead of exact numerical values will be a suitable selection in these cases. Due to the increase in complexity of real world decision making environments, most decision making processes take place in group. The group decision making under fuzzy environment is an interesting research topic and has produced good results [1,2,3]. In many group decision making situations under uncertainty, because of the ambiguity in human thinking, the information given by the decision makers (DMs) often takes the form of fuzzy numbers. The key step in the process of group decision making is to aggregate these information by using a proper aggregation operator [4,5]. Aggregation operators (AOs) are mathematical functions that are used to combine information [6]. AOs have the ability to fuse the fuzzy information. Different types of AOs are found in the literature [7].

A very common aggregation method is the ordered weighted averaging operator (OWA) [8]. The OWA operators were used for aggregating information expressed as exact numbers. Mostly the information available was imprecise and uncertain which can be expressed as fuzzy numbers. In this context fuzzy OWA (FOWA) operator was introduced to aggregate information represented by fuzzy numbers. The FOWA operator has been in the literature [9,10]. Yager [11] introduced the prioritized aggregation operators which allowed inclusion of priority between the attributes. Zhao [12] investigated the triangular fuzzy multiple attribute group decision making problem using fuzzy prioritized operators. The trapezoidal fuzzy numbers are a more general case encompassing the triangular ones with support in  $[0,1]$  [13]. Thus the Trapezoid fuzzy numbers are more suitable for use in depicting imprecise or vague fuzzy information.

In all MAGDM methods so far developed under trapezoidal fuzzy environment the attributes and DMs are at the same priority level. But in many types of applications we prioritize among the attribute or criteria and also among the DM's. Yager [11,14] first investigated this kind of issue.

In this work we propose some fuzzy prioritized aggregation operators such as TFPWM, TFPWG, TFPWH which allows prioritization among the attribute and DM's. We have also studied some properties of these operators. In section 2, we briefly review some basic concepts. In sections 3-5 three trapezoid fuzzy prioritized operators TFPWM, TFPWG, TFPWH and their properties are studied. Section 6 demonstrates a procedure for multiple attribute group decision making problems using these operators. Section 7 provides a numerical example to demonstrate the validity and effectiveness of the proposed operators.

## II. BASIC CONCEPTS AND OPERATIONS

In this section we briefly recall some basic concepts related to trapezoid fuzzy number and fuzzy OWA operators.

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**A. Definition**

A trapezoid fuzzy number  $\tilde{a}$  is defined by a quadruplet  $\tilde{a} = [a_1, a_2, a_3, a_4]$  with membership function given by

$$\mu_{\tilde{a}}(x) = \begin{cases} x, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_2 \geq x \geq a_1 \\ 1, & a_2 \geq x \geq a_3 \\ \frac{x - a_3}{a_3 - a_4}, & a_3 \geq x \geq a_4 \\ 0, & x > a_4 \end{cases}$$

Let  $\tilde{a} = [a_1, a_2, a_3, a_4]$  and  $\tilde{b} = [b_1, b_2, b_3, b_4]$  be two trapezoidal fuzzy numbers. Then the operations with these fuzzy numbers are defined as follows:

(1)  $\tilde{a} \oplus \tilde{b} = [a_1, a_2, a_3, a_4] \oplus [b_1, b_2, b_3, b_4] = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4]$

(2)  $\tilde{a} \otimes \tilde{b} = [a_1, a_2, a_3, a_4] \otimes [b_1, b_2, b_3, b_4] = [a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4]$

(3)  $\tilde{a} / \tilde{b} = [a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1]$

(4)  $\lambda \tilde{a} = \lambda [a_1, a_2, a_3, a_4] = [\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4]$

(5)  $\frac{1}{\tilde{a}} = \left[ \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right]$ .

**B. Definition**

The expected value of a fuzzy set A [14] is equal to  $E(A) = \frac{\int_{-\infty}^{\infty} x \mu_A(x)}{\int_{-\infty}^{\infty} \mu_A(x)}$  where the integral converges absolutely. (ie)  $\int_{-\infty}^{\infty} |x \mu_A(x)| dx < \infty$ . Otherwise A has no finite expected value. In other words for a trapezoidal fuzzy number

$$\tilde{a} = [a_1, a_2, a_3, a_4], \quad E(\tilde{a}) = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

**C. Aggregation Operators (AO)**

Aggregation operators [6] are mathematical functions used to combine different pieces of information. Although there is no standard definition of AOs, an AO of dimension n is a mapping  $\Omega : D^n \rightarrow D$  where  $D = [0, 1]$ . Some examples of AOs are maximum, minimum, weighted maximum, mean, median etc.

**D. OWA Operators**

Another class of AO's are the Ordered Weighted Averaging (OWA) operators [8] that provide a parameterized class of mean type AOs.

An OWA operator of dimension n is a mapping  $f : R^n \rightarrow R$  such that  $f [a_1, a_2, \dots, a_n] = \sum_{j=1}^n \omega_j b_j$  where  $b_j$  is

the  $j^{\text{th}}$  largest of the  $a_i$  and  $\omega_j$  is the weight of  $b_j$  which satisfies  $\omega \in [0, 1]$  and  $\sum_{j=1}^n \omega_j = 1$

The OWA operator is symmetrical, monotonic, bounded and satisfies idempotency [8]

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### E. Prioritized Average (PA)

This operator, originally introduced by Yager [11] was defined as follows

Let  $C = \{C_1, C_2, \dots, C_n\}$  be a collection of criteria and there is prioritization between the criteria expressed by linear ordering  $C_1 > C_2 > C_3 > \dots > C_n$  which indicate that criteria  $C_j$  has higher priority than  $C_k$  if  $j < k$ . The value of  $C_j(x)$  is the performance of any alternative  $x$  under criteria  $C_j$  and satisfies  $C_j(x) \in [0, 1]$ . If

$$PA(C_j(x)) = \sum_{j=1}^n \omega_j C_j(x) \text{ where } \omega_j = \frac{T_j}{\sum_{j=1}^n T_j}, T_j = \sum_{k=1}^{j-1} C_k(x), (j = 2, \dots, n) \text{ and } T_1 = 1 \text{ then PA is called the}$$

prioritized average operator.

### III. THE TRAPEZOID FUZZY PRIORITIZED AND WEIGHED MEAN OPERATOR (TFPWM)

#### A. Definition

Let  $a_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a collection of the trapezoidal fuzzy numbers. Then we define the trapezoidal fuzzy prioritized weighted mean (TFPWM) operator as follows

$$TFPWM(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \frac{T_1}{\sum_{j=1}^n T_j} \bar{a}_1 \oplus \frac{T_2}{\sum_{j=1}^n T_j} \bar{a}_2 \oplus \dots \oplus \frac{T_n}{\sum_{j=1}^n T_j} \bar{a}_n \quad (1)$$

where  $T_j = T \sum_{k=1}^{j-1} E(\bar{a}_k) (j = 2, 3, \dots, n), T_1 = 1$  and  $E(\bar{a}_j)$  is the expected value of  $\bar{a}_j$ .

#### B. Theorem 3.1

Let  $a_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a collection of trapezoidal fuzzy numbers. Then the aggregated result by equation (1) is also a trapezoidal fuzzy number and

$$TFPWM(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \left[ \sum_{j=1}^n \left( \frac{T_j s_{\alpha_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\beta_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\gamma_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\eta_j}}{\sum_{j=1}^n T_j} \right) \right] \quad (2)$$

where  $T_j = T \sum_{k=1}^{j-1} E(\bar{a}_k) (j = 2, 3, \dots, n), T_1 = 1$  and  $E(\bar{a}_j)$  is the expected value of  $\bar{a}_j$

Proof: First let us prove that

$$T_1 \bar{a}_1 \oplus T_2 \bar{a}_2 \oplus \dots \oplus T_n \bar{a}_n = \left[ \sum_{j=1}^n T_j s_{\alpha_j}, \sum_{j=1}^n T_j s_{\beta_j}, \sum_{j=1}^n T_j s_{\gamma_j}, \sum_{j=1}^n T_j s_{\eta_j} \right] \quad (3)$$

We use mathematical induction to prove the theorem.

Since the case of  $n=1$  is trivial let us prove the result for  $n=2$

$$\begin{aligned} T_1 \bar{a}_1 \oplus T_2 \bar{a}_2 &= T_1 [s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}] \oplus T_2 [s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}] \\ &= (T_1 s_{\alpha_1}, T_1 s_{\beta_1}, T_1 s_{\gamma_1}, T_1 s_{\eta_1}) \oplus (T_2 s_{\alpha_2}, T_2 s_{\beta_2}, T_2 s_{\gamma_2}, T_2 s_{\eta_2}) \\ &= [T_1 s_{\alpha_1} + T_2 s_{\alpha_2}, T_1 s_{\beta_1} + T_2 s_{\beta_2}, T_1 s_{\gamma_1} + T_2 s_{\gamma_2}, T_1 s_{\eta_1} + T_2 s_{\eta_2}] \end{aligned}$$

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$$= \left[ \sum_{j=1}^2 T_j s_{\alpha_j}, \sum_{j=1}^2 T_j s_{\beta_j}, \sum_{j=1}^2 T_j s_{\gamma_j}, \sum_{j=1}^2 T_j s_{\eta_j} \right]$$

Thus proving the case when n=2. Now assume that equation (2) is true when n=k

$$T_1 \bar{a}_1 \oplus T_2 \bar{a}_2 \oplus \dots \oplus T_k \bar{a}_k = \left[ \sum_{j=1}^k T_j s_{\alpha_j}, \sum_{j=1}^k T_j s_{\beta_j}, \sum_{j=1}^k T_j s_{\gamma_j}, \sum_{j=1}^k T_j s_{\eta_j} \right]$$

Consider the case when n=k+1,

$$T_1 \bar{a}_1 \oplus T_2 \bar{a}_2 \oplus \dots \oplus T_{k+1} \bar{a}_{k+1}$$

$$= \left[ \sum_{j=1}^k T_j s_{\alpha_j}, \sum_{j=1}^k T_j s_{\beta_j}, \sum_{j=1}^k T_j s_{\gamma_j}, \sum_{j=1}^k T_j s_{\eta_j} \right] + \left[ T_{k+1} s_{\alpha_{k+1}}, T_{k+1} s_{\beta_{k+1}}, T_{k+1} s_{\gamma_{k+1}}, T_{k+1} s_{\eta_{k+1}} \right]$$

$$= \left[ \sum_{j=1}^k T_j s_{\alpha_j} + T_{k+1} s_{\alpha_{k+1}}, \sum_{j=1}^k T_j s_{\beta_j} + T_{k+1} s_{\beta_{k+1}}, \sum_{j=1}^k T_j s_{\gamma_j} + T_{k+1} s_{\gamma_{k+1}}, \sum_{j=1}^k T_j s_{\eta_j} + T_{k+1} s_{\eta_{k+1}} \right]$$

$$= \left[ \sum_{j=1}^{k+1} T_j s_{\alpha_j}, \sum_{j=1}^{k+1} T_j s_{\beta_j}, \sum_{j=1}^{k+1} T_j s_{\gamma_j}, \sum_{j=1}^{k+1} T_j s_{\eta_j} \right] \quad (4)$$

Thus proving equation (3).

By equation (1)

$$TFPWM(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \frac{T_1}{\sum_{j=1}^n T_j} \bar{a}_1 \oplus \frac{T_2}{\sum_{j=1}^n T_j} \bar{a}_2 \dots \oplus \frac{T_n}{\sum_{j=1}^n T_j} \bar{a}_n$$

$$= \frac{1}{\sum_{j=1}^n T_j} (T_1 \bar{a}_1 \oplus T_2 \bar{a}_2 \oplus \dots \oplus T_n \bar{a}_n) \text{ using eqn (4)}$$

$$= \frac{1}{\sum_{j=1}^n T_j} \left[ \sum_{j=1}^{k+1} T_j s_{\alpha_j}, \sum_{j=1}^{k+1} T_j s_{\beta_j}, \sum_{j=1}^{k+1} T_j s_{\gamma_j}, \sum_{j=1}^{k+1} T_j s_{\eta_j} \right] = \left[ \sum_{j=1}^n \left( \frac{T_j s_{\alpha_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\beta_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\gamma_j}}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_{\eta_j}}{\sum_{j=1}^n T_j} \right) \right]$$

Hence the proof

### C.Theorem 3.2 (Idempotency)

Let  $\bar{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$  where  $j = 1, 2, \dots, n$  be a set of trapezoidal fuzzy numbers where

$$T_j = \prod_{k=1}^{j-1} E(\bar{a}_k) \quad (j = 2, \dots, n), T_1 = 1 \text{ and } E(\bar{a}_j) \text{ is the expected value of } \bar{a}_j. \text{ If all } \bar{a}_j \quad (j = 1, 2, \dots, n) = \tilde{a}$$

$$\text{then } TFPWM(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \tilde{a}$$

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Proof: Let  $\tilde{a} = [s_\alpha, s_\beta, s_\gamma, s_\eta]$  then by equation (2) we have

$$\begin{aligned}
 TFPWM(\tilde{a}, \tilde{a}, \dots, \tilde{a}) &= \left[ \sum_{j=1}^n \left( \frac{T_j s_\alpha}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_\beta}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_\gamma}{\sum_{j=1}^n T_j} \right), \sum_{j=1}^n \left( \frac{T_j s_\eta}{\sum_{j=1}^n T_j} \right) \right] \\
 &= \left[ s_\alpha \frac{\sum_{j=1}^n T_j}{\sum_{j=1}^n T_j}, s_\beta \frac{\sum_{j=1}^n T_j}{\sum_{j=1}^n T_j}, s_\gamma \frac{\sum_{j=1}^n T_j}{\sum_{j=1}^n T_j}, s_\eta \frac{\sum_{j=1}^n T_j}{\sum_{j=1}^n T_j} \right] \\
 &= [s_\alpha, s_\beta, s_\gamma, s_\eta] \quad (5) \\
 &= \tilde{a}
 \end{aligned}$$

*D.Theorem 3.3*

Let  $\tilde{a}'_i = (\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$  be any permutation of  $\tilde{a}_i = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$  then

$$TFPWM(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n) = TFPWM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$$

Proof: Let  $\tilde{a}'_j = [s'_{\alpha_j}, s'_{\beta_j}, s'_{\gamma_j}, s'_{\eta_j}]$  then

$$TFPWM(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n) = \left[ \sum_{j=1}^n \frac{T_j s'_{\alpha_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\beta_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\gamma_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\eta_j}}{\sum_{j=1}^n T_j} \right]$$

Since  $(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$  is any permutation of  $(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$  by equation (5) and theorem 3.1

$$\left[ \sum_{j=1}^n \frac{T_j s'_{\alpha_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\beta_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\gamma_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s'_{\eta_j}}{\sum_{j=1}^n T_j} \right] = \left[ \sum_{j=1}^n \frac{T_j s_{\alpha_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s_{\beta_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s_{\gamma_j}}{\sum_{j=1}^n T_j}, \sum_{j=1}^n \frac{T_j s_{\eta_j}}{\sum_{j=1}^n T_j} \right]$$

i.e.,  $TFPWM(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n) = TFPWM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$

*E.THEOREM 3.4 (Monotonicity)*

Let  $\tilde{a}'_i = [s'_{\alpha_i}, s'_{\beta_i}, s'_{\gamma_i}, s'_{\eta_i}]$  and  $\tilde{a}_i = [s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}, s_{\eta_i}]$ ,  $i = 1, 2, \dots, n$  be a collection of trapezoidal fuzzy numbers. If  $\alpha'_i \geq \alpha_i, \beta'_i \geq \beta_i, \gamma'_i \geq \gamma_i, \eta'_i \geq \eta_i$  for all  $i$  then

$$TFPWM(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n) \geq TFPWM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$$

Proof:  $\alpha'_i \geq \alpha_i, \beta'_i \geq \beta_i, \gamma'_i \geq \gamma_i, \eta'_i \geq \eta_i$  for all  $i$

Hence  $s'_{\alpha_j} \geq s_{\alpha_j}, s'_{\beta_j} \geq s_{\beta_j}, s'_{\gamma_j} \geq s_{\gamma_j}$ , and  $s'_{\eta_j} \geq s_{\eta_j}$

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$$\left[ \frac{T_j s'_{\alpha_j}}{\sum T_j}, \frac{T_j s'_{\beta_j}}{\sum T_j}, \frac{T_j s'_{\gamma_j}}{\sum T_j}, \frac{T_j s'_{\eta_j}}{\sum T_j} \right] \geq \left[ \frac{\sum T_j s_{\alpha_j}}{\sum T_j}, \frac{\sum T_j s_{\beta_j}}{\sum T_j}, \frac{\sum T_j s_{\gamma_j}}{\sum T_j}, \frac{\sum T_j s_{\eta_j}}{\sum T_j} \right]$$

Therefore  $TFPWM(\underline{a}'_1, \underline{a}'_2, \dots, \underline{a}'_n) \geq TFPWM(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n)$

*F.THEOREM 3.5 (Boundedness)*

Let  $\left[ s_{\min_i(\alpha_i)}, s_{\min_i(\beta_i)}, s_{\min_i(\gamma_i)}, s_{\min_i(\eta_i)} \right]$  and  $\left[ s_{\max_i(\alpha_i)}, s_{\max_i(\beta_i)}, s_{\max_i(\gamma_i)}, s_{\max_i(\eta_i)} \right]$  be the min and max operators.

Then the TFPWM operator is bounded by the min and max operators

$$\left[ s_{\min_i(\alpha_i)}, s_{\min_i(\beta_i)}, s_{\min_i(\gamma_i)}, s_{\min_i(\eta_i)} \right] \leq TFPWM(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n) \leq \left[ s_{\max_i(\alpha_i)}, s_{\max_i(\beta_i)}, s_{\max_i(\gamma_i)}, s_{\max_i(\eta_i)} \right]$$

**Proof:** By theorem 3.1, i.e. by equation (2)  $TFPWM(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n) = \left[ \frac{\sum_{i=1}^n T_j s_{\alpha_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_j s_{\beta_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_j s_{\gamma_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_j s_{\eta_i}}{\sum_{i=1}^n T_i} \right]$

$$\leq \left[ \frac{\sum_{i=1}^n T_i s_{\max \alpha_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_i s_{\max \beta_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_i s_{\max \gamma_i}}{\sum_{i=1}^n T_i}, \frac{\sum_{i=1}^n T_i s_{\max \eta_i}}{\sum_{i=1}^n T_i} \right] = \left[ s_{\max(\alpha_i)}, s_{\max(\beta_i)}, s_{\max(\gamma_i)}, s_{\max(\eta_i)} \right]$$

Similarly  $TFPWM(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n) \geq \left[ s_{\min(\alpha_i)}, s_{\min(\beta_i)}, s_{\min(\gamma_i)}, s_{\min(\eta_i)} \right]$

### IV. THE TRAPEZOID FUZZY PRIORITIZED WEIGHTED GEOMETRIC OPERATOR (TFPWG)

*A. Definition 4.1*

Let  $\underline{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be set of trapezoid fuzzy numbers, then TFPWG

is defined as  $TFPWG(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n) = \underline{a}_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes \underline{a}_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \dots \otimes \underline{a}_n^{\frac{T_n}{\sum_{j=1}^n T_j}}$

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k) (j = 2, 3, \dots, n)$   $T_1 = 1$  and  $E(\underline{a}_j)$  is the expected value of  $\underline{a}_j$

Based on the operations of trapezoidal fuzzy numbers we state the following theorem.

*B. Theorem 4.1*

Let  $\underline{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a set of trapezoidal fuzzy numbers then their aggregated value by using the TFPWG operator is also a trapezoidal fuzzy number and

$$TFPWG(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n) = \underline{a}_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes \underline{a}_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \dots \otimes \underline{a}_n^{\frac{T_n}{\sum_{j=1}^n T_j}}$$

$$= \left[ \prod_{j=1}^n (s_{\alpha_j})^{\frac{T_j}{\sum_{j=1}^n T_j}}, \prod_{j=1}^n (s_{\beta_j})^{\frac{T_j}{\sum_{j=1}^n T_j}}, \prod_{j=1}^n (s_{\gamma_j})^{\frac{T_j}{\sum_{j=1}^n T_j}}, \prod_{j=1}^n (s_{\eta_j})^{\frac{T_j}{\sum_{j=1}^n T_j}} \right] \text{ where } T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k) (j = 2, 3, \dots, n) \text{ and } T_1 = 1$$

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*C.Theorem 4.2(Idempotency)*

Let  $\bar{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$  ( $j = 1, 2, \dots, n$ ) be a set of trapezoidal fuzzy numbers

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k)$  ( $j = 2, 3, \dots, n$ ) and  $T_1 = 1$ . If all  $\bar{a}_j = \tilde{a}$  for all  $j$  then  $TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \tilde{a}$

i.e.  $TFPWG(\bar{a}, \bar{a}, \dots, \bar{a}) = \tilde{a}$

Proof: Let  $\bar{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] = [s_{\alpha}, s_{\beta}, s_{\gamma}, s_{\eta}]$

$$TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \left[ \prod_{j=1}^n (s_{\alpha_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\beta_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\gamma_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\eta_j})^{\frac{T_j}{\sum T_j}} \right], (j = 1, 2, \dots, n)$$

$$= \left[ \prod_{j=1}^n (s_{\alpha})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\beta})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\gamma})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\eta})^{\frac{T_j}{\sum T_j}} \right]$$

$$= \left[ (s_{\alpha})^{\frac{\sum T_j}{\sum T_j}}, (s_{\beta})^{\frac{\sum T_j}{\sum T_j}}, (s_{\gamma})^{\frac{\sum T_j}{\sum T_j}}, (s_{\eta})^{\frac{\sum T_j}{\sum T_j}} \right] = [s_{\alpha}, s_{\beta}, s_{\gamma}, s_{\eta}] = \bar{a}$$

*D. Theorem 4.4 (Boundedness)*

Let  $\bar{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$  ( $j = 1, 2, \dots, n$ ) be a set of trapezoidal fuzzy numbers

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k)$  ( $j = 2, 3, \dots, n$ ) and  $T_1 = 1$ . Let  $\tilde{a}^+ = [\max s_{\alpha_j}, \max s_{\beta_j}, \max s_{\gamma_j}, \max s_{\eta_j}]$  and

$\tilde{a}^- = [\min s_{\alpha_j}, \min s_{\beta_j}, \min s_{\gamma_j}, \min s_{\eta_j}]$ . Then  $\bar{a}^- \leq TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) \leq \bar{a}^+$

Proof:

$$TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \left[ \prod_{j=1}^n (s_{\alpha_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\beta_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\gamma_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\eta_j})^{\frac{T_j}{\sum T_j}} \right]$$

$$\geq [\min s_{\alpha_j}, \min s_{\beta_j}, \min s_{\gamma_j}, \min s_{\eta_j}] = \tilde{a}^-$$

$$TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) = \left[ \prod_{j=1}^n (s_{\alpha_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\beta_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\gamma_j})^{\frac{T_j}{\sum T_j}}, \prod_{j=1}^n (s_{\eta_j})^{\frac{T_j}{\sum T_j}} \right]$$

$\leq (\max s_{\alpha_j}, \max s_{\beta_j}, \max s_{\gamma_j}, \max s_{\eta_j}) = \tilde{a}^+$ . Hence the TFPWG operator is bounded by the max and min operators.

*E. Theorem 4.5(Monotonicity)*

Let  $\bar{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$  and  $\bar{a}'_j = [s_{\alpha'_j}, s_{\beta'_j}, s_{\gamma'_j}, s_{\eta'_j}]$  ( $j = 1, 2, 3, \dots, n$ ) be two set of trapezoidal fuzzy

numbers. If  $\bar{a}_j \leq \bar{a}'_j$  for all  $j$ , then  $TFPWG(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) \leq TFPWG(\bar{a}'_1, \bar{a}'_2, \dots, \bar{a}'_n)$

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### V. THE TRAPEZOID FUZZY PRIORITIZED WEIGHTED HARMONIC OPERATOR (TFPWH)

A. Definition

Let  $\tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a set of trapezoidal fuzzy numbers and let

$$H = \left( \frac{T_1}{\sum_{j=1}^n T_j} \frac{1}{\tilde{a}_1} \oplus \frac{T_2}{\sum_{j=1}^n T_j} \oplus \dots \oplus \frac{T_n}{\sum_{j=1}^n T_j} \frac{1}{\tilde{a}_n} \right)$$

then we define the trapezoid fuzzy prioritized weighted harmonic

operator (TFPWH) as  $TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = 1/H$  where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k)$ ,  $(j = 1, 2, \dots, n)$   $T_1 = 1$  and

$$E(\tilde{a}_j) \text{ is the expected value of } \tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$$

Based on the operations of trapezoidal fuzzy numbers we can prove the following theorems

B. Theorem 5.1

Let  $\tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a set of trapezoidal fuzzy numbers

$$\text{Let } H_1 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\alpha_j}} \right), H_2 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\beta_j}} \right), H_3 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\gamma_j}} \right) \text{ and } H_4 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\eta_j}} \right)$$

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k) (j = 2, 3, \dots, n)$   $T_1 = 1$  and  $E(\tilde{a}_j)$  is the expected value of  $\tilde{a}_j$ . Then

$$TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left[ \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4} \right]$$

Proof:

$$TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = 1/H$$

$$= \frac{1}{\sum_{j=1}^n T_j \left[ \frac{1}{s_{\alpha_1}}, \frac{1}{s_{\beta_1}}, \frac{1}{s_{\gamma_1}}, \frac{1}{s_{\eta_1}} \right] + \sum_{j=1}^n T_j \left[ \frac{1}{s_{\alpha_2}}, \frac{1}{s_{\beta_2}}, \frac{1}{s_{\gamma_2}}, \frac{1}{s_{\eta_2}} \right] + \dots + \sum_{j=1}^n T_j \left[ \frac{1}{s_{\alpha_n}}, \frac{1}{s_{\beta_n}}, \frac{1}{s_{\gamma_n}}, \frac{1}{s_{\eta_n}} \right]}$$

$$= \left[ \frac{1}{\sum_{j=1}^n T_j \left( \frac{T_1 + T_1 + \dots + T_n}{s_{\alpha_1}} \right)}, \frac{1}{\sum_{j=1}^n T_j \left( \frac{T_1 + T_2 + \dots + T_n}{s_{\beta_1}} \right)}, \frac{1}{\sum_{j=1}^n T_j \left( \frac{T_1 + T_2 + \dots + T_n}{s_{\gamma_1}} \right)}, \frac{1}{\sum_{j=1}^n T_j \left( \frac{T_1 + T_2 + \dots + T_n}{s_{\eta_1}} \right)} \right]$$

$$= \left[ \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4} \right]$$

Hence, the proof

C. Theorem 5.2 (Idempotency)



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Let  $\tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a set of trapezoidal fuzzy numbers

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k) (j = 2, 3, \dots, n), T_1 = 1$  and  $E(\tilde{a}_j)$  is the expected value of  $\tilde{a}_j$ . If all  $\tilde{a}_j (j = 1, 2, \dots, n)$  are

equal to  $\tilde{a}$  then  $TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \tilde{a}$

Proof:

$$TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = TFPWH(\tilde{a}, \tilde{a}, \dots, \tilde{a})$$

$$= \frac{1}{\sum_{j=1}^n T_j \frac{1}{\tilde{a}} \oplus \sum_{j=1}^n T_j \frac{1}{\tilde{a}} \oplus \dots \oplus \sum_{j=1}^n T_j \frac{1}{\tilde{a}}} = \frac{1}{\frac{1}{\tilde{a}} \left( \frac{T_1 + T_2 + \dots + T_n}{\sum_{j=1}^n T_j} \right)} = \frac{1}{\tilde{a}} = \tilde{a}$$

The following properties can easily be proved.

*D.Theorem 5.3 (Boundedness)*

Let  $\tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}] (j = 1, 2, \dots, n)$  be a set of trapezoidal fuzzy numbers  $TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$

$$= \left[ \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4} \right] \text{ where } H_1 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\alpha_j}} \right), H_2 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\beta_j}} \right), H_3 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\gamma_j}} \right) \text{ and}$$

$$H_4 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\eta_j}} \right) \text{ where } T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k), (j = 2, 3, \dots, n) \text{ and } T_1 = 1$$

If  $\tilde{a} = [\min s_{\alpha_j}, \min s_{\beta_j}, \min s_{\gamma_j}, \min s_{\eta_j}]$  and  $\tilde{a}^+ = [\max s_{\alpha_j}, \max s_{\beta_j}, \max s_{\gamma_j}, \max s_{\eta_j}]$  then,

$$\tilde{a} \leq TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \leq \tilde{a}^+$$

*E. Theorem 5.4 (Monotonicity)*

Let  $\tilde{a}_j = [s_{\alpha_j}, s_{\beta_j}, s_{\gamma_j}, s_{\eta_j}]$  and  $\tilde{a}'_j = [s'_{\alpha_j}, s'_{\beta_j}, s'_{\gamma_j}, s'_{\eta_j}] (j = 1, 2, 3, \dots, n)$  be two set of trapezoidal fuzzy

numbers. If  $\tilde{a}_j \leq \tilde{a}'_j$  for all j, then  $TFPWH(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \leq TFPWH(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$

**VI. A GROUP DECISION MAKING MODEL BASED ON THE TRAPEZOID FUZZY PRIORITY OPERATORS**

A multiple attribute group decision making problem under the trapezoid fuzzy environment is represented as follows:

Let  $X = \{X_1, X_2, \dots, X_m\}$  be the alternatives and  $U = \{G_1, G_2, \dots, G_n\}$  be the set of attributes. The prioritisation between the attribute is expressed by the linear ordering  $G_1 > G_2 > \dots > G_n$ . The attribute  $G_i$  has higher priority

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than  $G_j$  if  $i < j$ . Let  $D = \{D_1, D_2, \dots, D_t\}$  be the set of decision makers (DMs) having prioritization between the DM's expressed as a linear ordering  $D_1 > D_2 > \dots > D_t$ . Suppose that  $A_k = \left( \tilde{a}_{ij}^{(k)} \right)$  where

$\tilde{a}_{ij}^{(k)} = \left[ \left( \tilde{a}_{ij}^{(\alpha)} \right)^{(k)}, \left( \tilde{a}_{ij}^{(\beta)} \right)^{(k)}, \left( \tilde{a}_{ij}^{(\gamma)} \right)^{(k)}, \left( \tilde{a}_{ij}^{(\eta)} \right)^{(k)} \right]$  is the fuzzy decision matrix of trapezoidal fuzzy numbers. Here

$\tilde{a}_{ij}^{(k)}$  is an attribute expressed as trapezoidal fuzzy number given by the decision maker  $D_k \in D$  for the alternative  $X_i \in X$  with respect to the attribute  $G_j \in U$ .

In the following, we consider three methods to solve the multiple attribute decision making problems.

**(i) First method (using TFPWM operator)**

Step 1 Normalize each attribute value  $\tilde{a}_{ij}^{(k)} = \left[ \left( a_{ij}^{\alpha} \right)^k, \left( a_{ij}^{\beta} \right)^k, \left( a_{ij}^{\gamma} \right)^k, \left( a_{ij}^{\eta} \right)^k \right]$  in the matrix  $A_k$  into a corresponding

element in the matrix  $\tilde{r}_{ij}^{(k)} = \left[ \left( r_{ij}^{(\alpha)} \right)^{(k)}, \left( r_{ij}^{(\beta)} \right)^{(k)}, \left( r_{ij}^{(\gamma)} \right)^{(k)}, \left( r_{ij}^{(\eta)} \right)^{(k)} \right]$

$(i = 1, 2, \dots, m), (j = 1, 2, \dots, n), k = (1, 2, \dots, t)$  using the relation

$$\tilde{r}_{ij}^{(k)} = \left[ \frac{\left( a_{ij}^{(\alpha)} \right)^{(k)}}{l_{ik}^+}, \frac{\left( a_{ij}^{(\beta)} \right)^{(k)}}{l_{ik}^+}, \frac{\left( a_{ij}^{(\gamma)} \right)^{(k)}}{l_{ik}^+}, \frac{\left( a_{ij}^{(\eta)} \right)^{(k)}}{l_{ik}^+} \right] \quad (i = 1, 2, \dots, m), (j = 1, 2, \dots, n), k = (1, 2, \dots, t)$$

for the benefit attribute  $G_j$

$$= \left[ \frac{l_{ik}^-}{\left( a_{ij}^{(\alpha)} \right)^{(k)}, \frac{l_{ik}^-}{\left( a_{ij}^{(\beta)} \right)^{(k)}, \frac{l_{ik}^-}{\left( a_{ij}^{(\gamma)} \right)^{(k)}, \frac{l_{ik}^-}{\left( a_{ij}^{(\eta)} \right)^{(k)}} \right] \quad (i = 1, 2, \dots, m), (j = 1, 2, \dots, n), k = (1, 2, \dots, t)$$

for cost attribute  $G_j$  where  $l_{ik}^+ = \max \left[ \left( a_{ij}^{(n)} \right)^k \right], l_{ik}^- = \min \left[ \left( a_{ij}^{\alpha} \right)^k \right]$  for  $j = 1, 2, \dots, n$

Step 2 Calculate the values of  $T_{ij}^{(k)}, (k = 1, 2, \dots, t)$  based on the formula

$$T_{ij}^{(k)} = \prod_{\lambda=1}^{k-1} E \left( \tilde{r}_{ij}^{(\lambda)} \right), k = (1, 2, \dots, t) \quad (6) \quad \text{and} \quad T_{ij}^{(1)} = 1 \quad (7)$$

Step 3 Utilize the TFPWM operator to get the decision information matrix  $\bar{R}_k$

$$\bar{r}_{ij} = \left[ \tilde{r}_{ij}^{(\alpha)}, \tilde{r}_{ij}^{(\beta)}, \tilde{r}_{ij}^{(\gamma)}, \tilde{r}_{ij}^{(\eta)} \right] = TFPWM \left( \tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \dots, \tilde{r}_{ij}^{(t)} \right)$$

$$= \left[ \sum_{k=1}^t \left( \frac{T_{ij}^{(k)} \left( \tilde{r}_{ij}^{\alpha} \right)^{(k)}}{\sum_{k=1}^t T_{ij}^{(k)}} \right), \sum_{k=1}^t \left( \frac{T_{ij}^{(k)} \left( \tilde{r}_{ij}^{\beta} \right)^{(k)}}{\sum_{k=1}^t T_{ij}^{(k)}} \right), \sum_{k=1}^t \left( \frac{T_{ij}^{(k)} \left( \tilde{r}_{ij}^{\gamma} \right)^{(k)}}{\sum_{k=1}^t T_{ij}^{(k)}} \right), \sum_{k=1}^t \left( \frac{T_{ij}^{(k)} \left( \tilde{r}_{ij}^{\eta} \right)^{(k)}}{\sum_{k=1}^t T_{ij}^{(k)}} \right) \right] \quad (i = 1, 2, \dots, m), (j = 1, 2, \dots, n)$$

Step 4 Calculate the values of  $T_{ij}$  based on the following equation

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$$T_{ij} = \prod_{\lambda=1}^{j-1} E(\tilde{r}_{i\lambda}) \quad (i=1,2,\dots,m)(j=1,2,\dots,n) \text{ and } T_{i1}=1, (i=1,2,\dots,m) \tag{8}$$

Step 5 Aggregate all the trapezoidal fuzzy preference values  $\tilde{r}_j (j = 1, 2, \dots, m)$  for each alternative  $X_i$  by using TFPWM operator to derive the overall preference value

$$\tilde{r}_i = [r_i^\alpha, r_i^\beta, r_i^\gamma, r_i^\eta] \quad (i = 1, 2, \dots, m)$$

$$= TFPWM(\tilde{r}_{i_1}, \tilde{r}_{i_2}, \dots, \tilde{r}_{i_n}) = \left[ \sum_{j=1}^n \left( \frac{T_{ij} r_i^\alpha}{\sum_{j=1}^n T_{ij}} \right), \sum_{j=1}^n \left( \frac{T_{ij} r_i^\beta}{\sum_{j=1}^n T_{ij}} \right), \sum_{j=1}^n \left( \frac{T_{ij} r_i^\gamma}{\sum_{j=1}^n T_{ij}} \right), \sum_{j=1}^n \left( \frac{T_{ij} r_i^\eta}{\sum_{j=1}^n T_{ij}} \right) \right]$$

Step 6 Calculate the expected values as  $E(\tilde{r}_i) = \frac{r_i^\alpha + r_i^\beta + r_i^\gamma + r_i^\eta}{4}$

Step 7 Rank all the alternatives  $X_i (i = 1, 2, \dots, m)$  based on their expected values. The bigger value of  $E(\tilde{r}_i)$  (the larger the overall  $\tilde{r}_i$ ) and thus the alternative  $X_i$  is selected as the best

**(ii) Second method using TFPWG operator:**

Step 1 and Step 2 Same as outlined in the first method.

Step 3 Utilize the TFPWG operator and the  $\tilde{R}_k$  matrix.

$$\tilde{r}_{ij} = \left[ \prod_{k=1}^t \left( (r_{ij}^\alpha)^{(k)} \right)^{\frac{\frac{r_{ij}^\alpha}{\sum_{k=1}^t r_{ij}^\alpha}}{\sum_{k=1}^t r_{ij}^\alpha}}, \prod_{k=1}^t \left( (r_{ij}^\beta)^{(k)} \right)^{\frac{\frac{r_{ij}^\beta}{\sum_{k=1}^t r_{ij}^\beta}}{\sum_{k=1}^t r_{ij}^\beta}}, \prod_{k=1}^t \left( (r_{ij}^\gamma)^{(k)} \right)^{\frac{\frac{r_{ij}^\gamma}{\sum_{k=1}^t r_{ij}^\gamma}}{\sum_{k=1}^t r_{ij}^\gamma}}, \prod_{k=1}^t \left( (r_{ij}^\eta)^{(k)} \right)^{\frac{\frac{r_{ij}^\eta}{\sum_{k=1}^t r_{ij}^\eta}}{\sum_{k=1}^t r_{ij}^\eta}} \right]$$

$$= TFPWG(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \dots, \tilde{r}_{ij}^{(t)})$$

Step 4 Same as in first method

Step 5: Aggregate the trapezoid fuzzy preference value  $\tilde{r}_{ij} (j = 1, 2, \dots, n)$  for each alternative  $X_i$  based on the

TFPWG operator and derive the overall preference value  $\tilde{r}_i (i = 1, 2, \dots, m)$ ,  $\tilde{r}_i = [r_i^\alpha, r_i^\beta, r_i^\gamma, r_i^\eta] (i = 1, 2, \dots, m)$

$$= TFPWG(\tilde{r}_{i_1}^{(1)}, \tilde{r}_{i_2}^{(2)}, \dots, \tilde{r}_{i_n}^{(n)}) = \left[ \prod_{j=1}^n \left( (r_{ij}^\alpha) \right)^{\frac{\frac{r_{ij}^\alpha}{\sum_{j=1}^n T_{ij}}}{\sum_{j=1}^n T_{ij}}}, \prod_{j=1}^n \left( (r_{ij}^\beta) \right)^{\frac{\frac{r_{ij}^\beta}{\sum_{j=1}^n T_{ij}}}{\sum_{j=1}^n T_{ij}}}, \prod_{j=1}^n \left( (r_{ij}^\gamma) \right)^{\frac{\frac{r_{ij}^\gamma}{\sum_{j=1}^n T_{ij}}}{\sum_{j=1}^n T_{ij}}}, \prod_{j=1}^n \left( (r_{ij}^\eta) \right)^{\frac{\frac{r_{ij}^\eta}{\sum_{j=1}^n T_{ij}}}{\sum_{j=1}^n T_{ij}}} \right]$$

Step 6 and Step 7 same as in first method

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(iii) **Third Method based on TFPWH operator:**

Step 1 and Step 2 Same as in first method

Step 3 Utilize the TFPWH operator and the  $\bar{R}_k$  matrix

$$r_{ij}^{\square} = TFPWH \left( \tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \dots, \tilde{r}_{ij}^{(t)} \right) = TFPWH \left( \bar{a}_1, \bar{a}_2, \dots, \bar{a}_n \right) = \left[ \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4} \right]$$

$$\text{where } H_1 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\alpha_j}} \right), H_2 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\beta_j}} \right), H_3 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\gamma_j}} \right) \text{ and } H_4 = \sum_{j=1}^n \left( \frac{T_j}{\sum_{j=1}^n T_j} \frac{1}{s_{\eta_j}} \right)$$

where  $T_j = \prod_{k=1}^{j-1} E(\tilde{a}_k)$  ( $j = 2, 3, \dots, n$ )  $T_1 = 1$  and  $E(\bar{a}_j)$  is the expected value of  $\bar{a}_j$ .

Step 4 Same as in first method

Step 5 Aggregate the trapezoid fuzzy preferences value  $r_{ij}^{\square}$  ( $j = 1, 2, \dots, n$ ) for each alternative  $x_i$  based on the TFPWH

operator and derive the overall preference value  $\tilde{r}_i = [r_i^{\alpha}, r_i^{\beta}, r_i^{\gamma}, r_i^{\eta}]$  ( $i = 1, 2, \dots, m$ )

$$= TFPWH \left( \bar{a}_1, \bar{a}_2, \dots, \bar{a}_n \right) = \left[ \frac{1}{\sum_{j=1}^n \frac{T}{r_{ij}^{\alpha}}}, \frac{1}{\sum_{j=1}^n \frac{T}{r_{ij}^{\beta}}}, \frac{1}{\sum_{j=1}^n \frac{T}{r_{ij}^{\gamma}}}, \frac{1}{\sum_{j=1}^n \frac{T}{r_{ij}^{\eta}}} \right] \text{ where } T = \frac{T_{ij}}{\sum_{j=1}^n T_{ij}}$$

Step 6 & Step 7 Same as in first method.

### VII. ILLUSTRATIVE EXAMPLE

Let us suppose that a university wants to sponsor candidates for overseas research education. Suppose there are four candidates  $X_i$  ( $i=1, 2, 3, 4$ ) and they are evaluated in 4 attributes namely (1)  $G_1$  the research aptitude; (2)  $G_2$  the subject knowledge; (3)  $G_3$  morality, (4)  $G_4$  the academic consistency. The decision makers in the order of priority are  $D_1$  the Director,  $D_2$  the Dean and  $D_3$  the Academic Head of the concerned school. The prioritization relationship for the criteria is  $G_1 > G_2 > G_3 > G_4$

Step 1 Four possible candidates are evaluated using trapezoid fuzzy numbers by the three decision makers under the above four attributes and the trapezoid fuzzy decision matrices are shown in Table 1-3.

**Table 1 Decision Matrix A<sub>1</sub>**

|                | G <sub>1</sub>           | G <sub>2</sub>           | G <sub>3</sub>           | G <sub>4</sub>           |
|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| X <sub>1</sub> | (0.89, 0.92, 0.94, 0.97) | (0.79, 0.84, 0.87, 0.9)  | (0.61, 0.63, 0.65, 0.69) | (0.70, 0.73, 0.77, 0.78) |
| X <sub>2</sub> | (0.61, 0.66, 0.69, 0.7)  | (0.86, 0.88, 0.9, 0.92)  | (0.67, 0.71, 0.73, 0.75) | (0.66, 0.76, 0.80, 0.83) |
| X <sub>3</sub> | (0.76, 0.78, 0.8, 0.82)  | (0.94, 0.96, 0.97, 0.99) | (0.92, 0.94, 0.96, 0.97) | (0.89, 0.93, 0.95, 0.96) |
| X <sub>4</sub> | (0.96, 0.97, 0.98, 0.99) | (0.81, 0.82, 0.86, 0.89) | (0.96, 0.98, 0.99, 1.00) | (0.96, 0.97, 0.99, 1.00) |

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Table 2 Decision Matrix A2

|                | G <sub>1</sub>           | G <sub>2</sub>           | G <sub>3</sub>           | G <sub>4</sub>           |
|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| X <sub>1</sub> | (0.62, 0.66, 0.69, 0.74) | (0.58, 0.60, 0.62, 0.66) | (0.69, 0.77, 0.82, 0.85) | (0.51, 0.53, 0.55, 0.57) |
| X <sub>2</sub> | (0.69, 0.74, 0.76, 0.81) | (0.61, 0.63, 0.65, 0.66) | (0.51, 0.53, 0.54, 0.55) | (0.53, 0.55, 0.59, 0.62) |
| X <sub>3</sub> | (0.81, 0.82, 0.83, 0.84) | (0.76, 0.79, 0.81, 0.82) | (0.61, 0.62, 0.64, 0.68) | (0.77, 0.79, 0.81, 0.83) |
| X <sub>4</sub> | (0.66, 0.69, 0.71, 0.74) | (0.81, 0.83, 0.84, 0.85) | (0.81, 0.83, 0.85, 0.86) | (0.82, 0.83, 0.84, 0.86) |

Table-3 Decision Matrix A3

|                | G <sub>1</sub>           | G <sub>2</sub>           | G <sub>3</sub>           | G <sub>4</sub>           |
|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| X <sub>1</sub> | (0.67, 0.69, 0.72, 0.76) | (0.55, 0.59, 0.64, 0.66) | (0.74, 0.77, 0.79, 0.88) | (0.47, 0.49, 0.52, 0.55) |
| X <sub>2</sub> | (0.71, 0.74, 0.77, 0.80) | (0.50, 0.61, 0.67, 0.69) | (0.44, 0.50, 0.52, 0.53) | (0.52, 0.55, 0.57, 0.60) |
| X <sub>3</sub> | (0.81, 0.82, 0.84, 0.85) | (0.73, 0.76, 0.79, 0.80) | (0.60, 0.62, 0.64, 0.68) | (0.75, 0.79, 0.81, 0.83) |
| X <sub>4</sub> | (0.65, 0.68, 0.71, 0.74) | (0.80, 0.82, 0.84, 0.85) | (0.82, 0.84, 0.85, 0.87) | (0.80, 0.83, 0.85, 0.87) |

Step 2 The attributes  $G_j (j=1,2,3,4)$  are normalized and the normalized decision table  $R^{(k)} = \left( r_{ij}^{(k)} \right)_{4 \times 4}$  are shown in Tables 4-6.

Table-4 Decision Matrix  $R^{(1)}$

|                | G <sub>1</sub>               | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.256, 0.270, 0.282, 0.301) | (0.214, 0.233, 0.249, 0.265) | (0.179, 0.189, 0.199, 0.218) | (0.196, 0.208, 0.227, 0.243) |
| X <sub>2</sub> | (0.175, 0.194, 0.207, 0.217) | (0.232, 0.244, 0.257, 0.271) | (0.196, 0.213, 0.294, 0.237) | (0.185, 0.217, 0.236, 0.259) |
| X <sub>3</sub> | (0.218, 0.229, 0.240, 0.255) | (0.254, 0.267, 0.277, 0.291) | (0.270, 0.282, 0.294, 0.307) | (0.249, 0.265, 0.280, 0.299) |
| X <sub>4</sub> | (0.276, 0.284, 0.294, 0.307) | (0.219, 0.228, 0.246, 0.262) | (0.282, 0.294, 0.294, 0.316) | (0.269, 0.276, 0.292, 0.312) |

Table-5 Decision Matrix  $R^{(2)}$

|                | G <sub>1</sub>               | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.198, 0.221, 0.237, 0.266) | (0.194, 0.205, 0.218, 0.239) | (0.235, 0.270, 0.298, 0.324) | (0.177, 0.190, 0.204, 0.217) |
| X <sub>2</sub> | (0.220, 0.247, 0.261, 0.291) | (0.204, 0.216, 0.228, 0.239) | (0.173, 0.186, 0.196, 0.210) | (0.184, 0.197, 0.219, 0.224) |
| X <sub>3</sub> | (0.259, 0.274, 0.285, 0.302) | (0.254, 0.271, 0.284, 0.297) | (0.207, 0.218, 0.233, 0.260) | (0.267, 0.283, 0.300, 0.316) |
| X <sub>4</sub> | (0.211, 0.231, 0.244, 0.266) | (0.271, 0.284, 0.295, 0.308) | (0.276, 0.291, 0.309, 0.328) | (0.285, 0.297, 0.311, 0.327) |

Table-6 Decision Matrix  $R^{(3)}$

|                | G <sub>1</sub>                | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|-------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.213, 0.227, 0.246, 0.2680) | (0.183, 0.201, 0.230, 0.256) | (0.250, 0.275, 0.289, 0.338) | (0.165, 0.178, 0.195, 0.217) |
| X <sub>2</sub> | (0.225, 0.243, 0.263, 0.282)  | (0.167, 0.207, 0.241, 0.267) | (0.149, 0.179, 0.190, 0.204) | (0.182, 0.200, 0.214, 0.236) |
| X <sub>3</sub> | (0.257, 0.270, 0.287, 0.299)  | (0.243, 0.259, 0.284, 0.310) | (0.203, 0.221, 0.234, 0.262) | (0.263, 0.287, 0.305, 0.327) |
| X <sub>4</sub> | (0.206, 0.224, 0.242, 0.261)  | (0.267, 0.279, 0.302, 0.329) | (0.277, 0.300, 0.311, 0.335) | (0.281, 0.302, 0.320, 0.343) |

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Step 3 Calculate the values of  $T_{ij}^{(1)}, T_{ij}^{(2)}, T_{ij}^{(3)}$  based on equations (6) and (7)

$$T_{ij}^{(1)} = \begin{bmatrix} 1.000 & 1.000 & 1.000 & 1.000 \\ 1.000 & 1.000 & 1.000 & 1.000 \\ 1.000 & 1.000 & 1.000 & 1.000 \\ 1.000 & 1.000 & 1.000 & 1.000 \end{bmatrix} \quad T_{ij}^{(2)} = \begin{bmatrix} 0.930 & 0.850 & 0.645 & 0.745 \\ 0.665 & 0.890 & 0.715 & 0.763 \\ 0.790 & 0.965 & 0.948 & 0.933 \\ 0.975 & 0.845 & 0.983 & 0.980 \end{bmatrix}$$

$$T_{ij}^{(3)} = \begin{bmatrix} 0.630 & 0.523 & 0.505 & 0.402 \\ 0.499 & 0.567 & 0.381 & 0.437 \\ 0.652 & 0.767 & 0.604 & 0.746 \\ 0.683 & 0.703 & 0.823 & 0.821 \end{bmatrix}$$

Step 4 Utilize the TFPWM operator and the decision information matrices  $\bar{R}_k$

to get the overall aggregated decision matrix  $\bar{R} = (\bar{r}_{ij})_{4 \times 4} = [r_{ij}^{(\alpha)}, r_{ij}^{(\beta)}, r_{ij}^{(\gamma)}, r_{ij}^{(\eta)}]$  as shown in Table 7

**Table 7 Decision Matrix  $\bar{R}$**

|                | G <sub>1</sub>               | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.224, 0.241, 0.257, 0.280) | (0.200, 0.216, 0.233, 0.254) | (0.212, 0.234, 0.250, 0.278) | (0.184, 0.196, 0.213, 0.229) |
| X <sub>2</sub> | (0.201, 0.222, 0.237, 0.255) | 0.207, 0.226, 0.243, 0.258)  | (0.180, 0.198, 0.242, 0.222) | (0.184, 0.207, 0.226, 0.242) |
| X <sub>3</sub> | (0.242, 0.254, 0.267, 0.282) | 0.251, 0.266, 0.282, 0.299)  | (0.231, 0.244, 0.257, 0.279) | (0.259, 0.278, 0.294, 0.313) |
| X <sub>4</sub> | (0.234, 0.249, 0.262, 0.280) | 0.249, 0.261, 0.278, 0.296)  | (0.278, 0.295, 0.305, 0.326) | (0.278, 0.291, 0.307, 0.326) |

Step 5 Using eqn.(8) calculate the values of  $T_{ij}(i=1,2,\dots,m, j=2,3,\dots,n)$

$$T_{ij} = \begin{bmatrix} 1.000 & 0.251 & 0.226 & 0.244 \\ 1.000 & 0.228 & 0.233 & 0.210 \\ 1.000 & 0.261 & 0.274 & 0.253 \\ 1.000 & 0.257 & 0.271 & 0.301 \end{bmatrix}$$

Step 6 Calculate the overall trapezoidal fuzzy preference values  $\tilde{r}_i(i=1,2,\dots,m)$  of the candidates  $X_i$  by aggregating

all trapezoidal fuzzy preference values  $\tilde{r}_{ij}(j=1,2,\dots,n)$  calculated using TFPWM operator.

$$\tilde{r}_1 = (0.213, 0.233, 0.242, 0.269) \quad \tilde{r}_2 = (0.197, 0.214, 0.228, 0.249)$$

$$\tilde{r}_3 = (0.244, 0.255, 0.267, 0.288) \quad \tilde{r}_4 = (0.250, 0.269, 0.274, 0.297)$$

Step 7 Calculate the expected values  $E(\tilde{r}_i)$  of the candidate  $X_i$

$$E(\tilde{r}_1) = 0.239 \quad E(\tilde{r}_2) = 0.222 \quad E(\tilde{r}_3) = 0.263 \quad E(\tilde{r}_4) = 0.272$$

Step 8 Rank all the candidates  $X_i$  ( $i=1, 2, 3, 4$ ) with respect to the expected values  $E(\tilde{r}_i)$  of the overall fuzzy preference values  $\tilde{r}_i$ . Hence  $X_3 > X_4 > X_2 > X_1$ . Therefore, the best candidate is  $X_3$

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Second method based on the TFPWG operator is described as follows:

Step1-3 Refer Step 1-3 of first method

Step 4 Utilize the TFPWG operator and the decision information matrices  $\bar{R}_k$

to get the overall aggregated decision matrix  $\bar{R} = (\bar{r}_{ij})_{4 \times 4} = \left[ r_{ij}^{(\alpha)}, r_{ij}^{(\beta)}, r_{ij}^{(\gamma)}, r_{ij}^{(\eta)} \right]$  as shown in Table 8

**Table 8 Decision Matrix  $\bar{R}$**

|                | G <sub>1</sub>               | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.227, 0.240, 0.256, 0.280) | (0.199, 0.216, 0.233, 0.253) | (0.210, 0.230, 0.246, 0.273) | (0.183, 0.196, 0.213, 0.229) |
| X <sub>2</sub> | (0.199, 0.220, 0.235, 0.253) | (0.205, 0.225, 0.243, 0.258) | (0.179, 0.197, 0.237, 0.221) | (0.184, 0.206, 0.225, 0.242) |
| X <sub>3</sub> | (0.241, 0.253, 0.266, 0.281) | (0.251, 0.266, 0.282, 0.298) | (0.229, 0.242, 0.256, 0.278) | (0.259, 0.277, 0.294, 0.312) |
| X <sub>4</sub> | (0.232, 0.248, 0.261, 0.279) | (0.248, 0.259, 0.276, 0.294) | (0.278, 0.295, 0.304, 0.326) | (0.278, 0.291, 0.307, 0.326) |

Step 5 Using eqn. (8) calculate the values of  $T_{ij}(i=1,2,\dots,m, j=2,3,\dots,n)$

$$T_{ij} = \begin{bmatrix} 1.000 & 0.225 & 0.239 & 0.205 \\ 1.000 & 0.233 & 0.209 & 0.214 \\ 1.000 & 0.274 & 0.251 & 0.286 \\ 1.000 & 0.270 & 0.301 & 0.300 \end{bmatrix}$$

Step 6 Calculate the overall trapezoidal fuzzy preference values  $\tilde{r}_i(i=1,2,\dots,m)$  of the candidates  $X_i$  by aggregating

all trapezoidal fuzzy preference values  $\tilde{r}_{ij}(j=1,2,\dots,n)$  calculated using TFPWG operator.

$$\tilde{r}_1 = (0.206, 0.221, 0.237, 0.259) \quad \tilde{r}_2 = (0.192, 0.212, 0.235, 0.244)$$

$$\tilde{r}_3 = (0.245, 0.260, 0.275, 0.293) \quad \tilde{r}_4 = (0.260, 0.274, 0.288, 0.307)$$

Step 7 Calculate the expected values  $E(\tilde{r}_i)$  of the candidate  $X_i$

$$E(\tilde{r}_1) = 0.231 \quad E(\tilde{r}_2) = 0.221 \quad E(\tilde{r}_3) = 0.268 \quad E(\tilde{r}_4) = 0.282$$

Step 8 Rank all the candidates  $X_i$  ( $i=1, 2,3,4$ ) with respect to the expected values  $E(\tilde{r}_i)$  of the overall fuzzy

preference values  $\tilde{r}_i$ . Here  $X_3 > X_4 > X_2 > X_1$ . Therefore, the best candidate is  $X_3$

Third method based on the TFPWH operator is described as follows:

Step1-3Refer Step 1-3 of first method

Step 4 Utilize the TFPWH operator and the decision information matrices  $\bar{R}_k$

to get the overall aggregated decision matrix  $\bar{R} = (\bar{r}_{ij})_{4 \times 4} = \left[ r_{ij}^{(\alpha)}, r_{ij}^{(\beta)}, r_{ij}^{(\gamma)}, r_{ij}^{(\eta)} \right]$  as shown in Table 9

**Table 9 Decision Matrix  $\bar{R}$**

|                | G <sub>1</sub>               | G <sub>2</sub>               | G <sub>3</sub>               | G <sub>4</sub>               |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| X <sub>1</sub> | (0.221, 0.239, 0.255, 0.282) | (0.199, 0.202, 0.216, 0.253) | (0.208, 0.226, 0.241, 0.267) | (0.183, 0.195, 0.212, 0.228) |
| X <sub>2</sub> | (0.198, 0.219, 0.233, 0.251) | (0.199, 0.202, 0.216, 0.253) | (0.178, 0.196, 0.232, 0.221) | (0.184, 0.206, 0.225, 0.241) |

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|       |                              |                              |                              |
|-------|------------------------------|------------------------------|------------------------------|
| $X_3$ | (0.240, 0.253, 0.265, 0.282) | (0.199, 0.202, 0.216, 0.253) | (0.227, 0.240, 0.254, 0.277) |
| $X_4$ | (0.230, 0.246, 0.260, 0.278) | (0.247, 0.248, 0.259, 0.293) | (0.278, 0.295, 0.304, 0.326) |

Step 5 Using eqn. (8) calculate the values of  $T_{ij}(i=1,2,\dots,m, j=2,3,\dots,n)$

$$T_{ij} = \begin{bmatrix} 1.000 & 0.249 & 0.218 & 0.235 \\ 1.000 & 0.225 & 0.226 & 0.207 \\ 1.000 & 0.260 & 0.268 & 0.249 \\ 1.000 & 0.254 & 0.262 & 0.301 \end{bmatrix}$$

Step 6 Calculate the overall trapezoidal fuzzy preference values  $\tilde{r}_i(i=1,2,\dots,m)$  of the candidates  $X_i$  by aggregating all trapezoidal fuzzy preference values  $\tilde{r}_{ij}(j=1,2,\dots,n)$  calculated using TFPWH operator.

$$\tilde{r}_1 = (0.210, 0.225, 0.240, 0.267) \quad \tilde{r}_2 = (0.194, 0.213, 0.231, 0.246)$$

$$\tilde{r}_3 = (0.242, 0.254, 0.268, 0.287) \quad \tilde{r}_4 = (0.245, 0.259, 0.273, 0.293)$$

Step 7 Calculate the expected values  $E(\tilde{r}_i)$  of the candidate  $X_i$

$$E(\tilde{r}_1) = 0.235, \quad E(\tilde{r}_2) = 0.221, \quad E(\tilde{r}_3) = 0.263, \quad E(\tilde{r}_4) = 0.268$$

Step 8 Rank all the candidates  $X_i$  ( $i = 1, 2, 3, 4$ ) with respect to the expected values  $E(\tilde{r}_i)$  of the overall fuzzy preference values  $\tilde{r}_i$ . Here  $X_3 > X_4 > X_2 > X_1$ . Therefore, the best candidate is  $X_3$

## VIII.CONCLUSION

We have investigated the multi-attribute decision making problems under trapezoid fuzzy environment and developed an approach to handle situations where the attribute and decision makers are at different priority level. We have introduced the Trapezoid Fuzzy Prioritized Weighted Mean (TFPWM) operator, Trapezoid Prioritized Geometric Average (TFPWG) operator and Trapezoid Fuzzy Prioritized Harmonic Average (TFPWH) operator that can aggregate the individual trapezoid fuzzy variables. We have also studied some properties of these operators and illustrated their operational procedure in detail. The utility of these operators can be extended to other domains of data mining, cluster analysis, etc.

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