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On arresting the complex growth rates in ferromagnetic convection in a rotating porous medium

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Abstract: It is proved analytically that the complex growth rate $\omega = \omega_r + i\omega_i$ (ω_r and ω_i are respectively the real and imaginary parts of ω) of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection in a rotating porous medium for the case of free boundaries, must lie inside a semicircle in the right half of the $\omega_r\omega_i$ -plane whose centre is origin and $(radius)^2$ = greater of $\left\{\frac{RM_1}{P_r},\ T_a\right\}$, where R is the Rayleigh number, M_1 is the magnetic number, P_r is the Prandtl number and T_a is the Taylor number. Further, bounds for the case of rigid boundaries are also derived separately.

Keywords: Ferrofluid, Rotation, Porous medium, Oscillatory motion, Darcy Model, Ferroconvection.

I. INTRODUCTION

A ferrofluid is a colloidal suspension of a surfacted single domain of magnetic particles, with typical dimensions of 10nm dispersed in a non conducting liquid (Rosenweig [1]). In the last Millennium, the investigation on ferromagnetic fluids attracted researchers because of the increase of applications in area such as instrumentation, lubrication, vacuum technology, vibration damping, metals recovery, acoustics etc. Some of the applications of ferrofluids are magnetic drug targeting hyperthermia, contrast enhancement of magnetic Resonance Imaging (MRI), novel zero-leakage rotary shaft seals used in computer disk drives, pressure seals for compressors and blowers etc. (Rosenweig [1], Vaidyanathan *et al* [2] and Odenbach [3]).

Experimental and theoretical physicists and engineers gave significant contributions to Ferrohydrodynamics (Odenbach [3]). An authoritative introduction to the research on ferrofluids has been discussed in detail by Rosenweig [1]. This book reviews several applications of heat transfer through ferrofluids. One such phenomenon is enhanced convective cooling having a temperature dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of magnetic field, temperature and density of the fluid. Any variation of these quantities can induce a change of body force distribution in the fluid. This causes convection in ferrofluids in the presence of magnetic field gradient. This mechanism is known as ferroconvection, which is similar to Benard convection (Chandrasekhar [4]). Finlayson [5] investigated convective instability of a ferrofluid layer heated from below in the presence of a uniform vertical magnetic field by using linear stability theory and predicted the critical temperature gradient for the onset of convection when both buoyancy and magnetic forces are considered . Thermoconvective stability of ferrofluids without considering buoyancy effects has been studied by Lalas and Carmi

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[6], whereas Shliomis [7] studied the linear relation for magnetized perturbed quantities at the limit of instability. Schwab *et al* [8] experimentally investigated the problem of Finlayson in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number verses the Rayleigh number. Later this problem was extended by Stiles and Kagan [9] to allow for the dependence of effective shear viscosity on temperature and colloid concentration. The Benard convection in ferromagnetic fluids has been considered by many researchers (Gupta and Gupta [10], Auernhammer and Brand [11], Aniss *et al* [12] and Sunil and Mahajan [13]). In recent years many researchers (Vaidyanathan *et al* [14], Borglin *et al* [15], Nanjundappa *et al* [16] and Sekar *et al* [17]) have shown their keen interest in analyzing the onset of ferroconvection in a fluid layer subjected to a vertical temperature gradient in a porous medium owing to its importance in controlled emplacement of liquids or treatment of chemicals and emplacement of geophysically imageable liquids in to particular zones for subsequent imaging etc.

The problem of obtaining bounds for the complex growth rate of an arbitrary oscillatory motion of growing amplitude is important especially when both the boundaries are not dynamically free so that exact solutions in the closed form are not obtainable. Recently Jyoti Prakash [18] has shown that the 'principle of the exchange of stabilities' is not, in general valid in ferromagnetic convection in a ferrofluid layer heated from below and derived the upper bounds for the complex growth rates . As a further step in the present analysis upper bounds for the complex growth rate of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection, in a rotating porous medium, for the cases of free and rigid boundaries, are obtained.

II. MATHEMATICAL FORMULATION

Consider a ferromagnetic Boussinesq fluid layer of infinite horizontal extension and finite vertical thickness 'd' saturating a rotating (with uniform angular velocity $\vec{\Omega}$ about the vertical) porous medium heated from below which is kept under the action of a uniform vertical magnetic field $\vec{H}$. A Cartesian coordinate system (x, y, z) is used with the origin at the middle of the layer and z-axis is directed vertically upward as shown in Fig. 1.

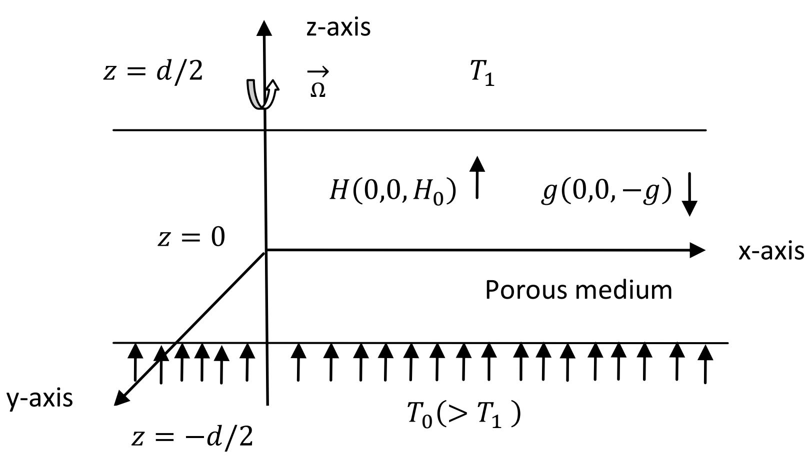

Fig. 1 Geometrical configuration

The temperatures at the bottom and top surfaces $z = \mp\frac{1}{2}d$ are T_0 and T_1 and a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of

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porosity $\in$ and medium permeability k_0. The governing non dimensional equations for the above model (Darcy model) are given by (Vaidyanathan *et al.* [14], Sunil *et al.* [19]).

$$\left(\frac{1}{k_o}+\omega\right)(D^2-a^2)w=-aR^{\frac{1}{2}}\left[(1+\ M_1)\theta - M_1D\phi\ \right]-\ T_a^{\frac{1}{2}}\,D\zeta, \tag{1}$$

$$(D^2-a^2-P_r\omega)\theta-\ P_r^{'}M_2\omega D\phi\ =\ -\,(1-M_2)aR^{1/2}w, \tag{2}$$

$$\left(\frac{1}{k_o}+\omega\right)\zeta\quad=\quad T_a^{\frac{1}{2}}Dw, \tag{3}$$

$$(D^2-a^2M_3)\phi\ =\ D\theta, \tag{4}$$

where z is the real independent variable such that $-1/2\le z\le 1/2$, D is differentiation with respect to z, a^2 is square of the wave number, $P_r > 0$ is the Prandtl number, ω is the complex growth rate, $R>0$ is the Rayleigh number, $T_a>0$ is the Taylor number, $M_1>0$ is the magnetic number, $M_3>0$ is the measure of the nonlinearity of magnetization, $M_2>0$ is a nondimensional parameter; $\omega=\omega_r+i\omega_i$ is a complex constant in general such that ω_r and ω_i are real constants and as a consequence the dependent variables $w(z)=w_r(z)+i\,w_i(z)$, $\theta(z)=\theta_r(z)+i\,\theta_i(z)$, $\phi(z)=\phi_r(z)+i\,\phi_i(z)$ and $\zeta(z)=\zeta_r(z)+i\zeta_i(z)$ are complex valued functions of the real variable z such that $w_r(z)$, $w_i(z)$, $\theta_r(z)$, $\theta_i(z)$, $\phi_r(z)$, $\phi_i(z)$, $\zeta_r(z)$ and $\zeta_i(z)$ are real valued functions of the real variable z.

since, M_2 is of very small order Finlayson [5], it is neglected in the subsequent analysis and thus Eq. (2) takes the form

$$(D^2-a^2-P_r\omega)\theta=-aR^{1/2}w\,. \tag{5}$$

The constant temperature ferromagnetic boundaries are considered to be either free or rigid. Hence the boundary conditions are:

$$w=0=\theta=D^2w=D\zeta=D\phi \quad \text{at} \quad z=-\tfrac{1}{2} \text{ and } z=\tfrac{1}{2} \quad \text{(Both the boundaries are free)} \tag{6}$$

$$\text{or } w=0=\theta=Dw=\zeta=\phi \quad \text{at } z=-\tfrac{1}{2} \text{ and } z=\tfrac{1}{2}. \quad \text{(Both the boundaries are rigid)} \tag{7}$$

It may further be noted that Eqs. (1) and (3) - (7) describe an eigenvalue problem for ω and govern ferromagnetic convection, in a rotating porous medium heated from below.

III. MATHEMATICAL ANALYSIS

we now derive upper bounds for the complex growth rates for ferromagnetic convection in a rotating porous medium of the arbitrary oscillatory motions of neutral or growing amplitude for the cases of free and rigid boundaries separately.

Theorem1: If $R>0$, $M_1>0$, $T_a>0$, $\omega_r\ \ge\ 0$, $\omega_i\ \ne\ 0$, then a necessary condition for the existence of a nontrivial solution $(w\,,\theta\,,\phi\,,\zeta\,,\omega)$ of the equations (1) and (3) $-$ (5) together with the boundary conditions (6) is that $|\omega|^2 <\ greater\ of\ \left\{\frac{RM_1}{P_r}\,,\,T_a\right\}$.

Proof: Multiplying equation (1) by w^*(the superscript $*$ henceforth denotes the complex conjugation throughout and integrating the resulting equation over the vertical range of z, we get

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$$\int_{-1/2}^{1/2} w^*\left(\frac{1}{k_o}+\omega\right)(D^2-a^2)w\,dz = -aR^{1/2}(1+M_1)\int_{-1/2}^{1/2} w^*\,\theta dz + aR^{1/2}M_1\int_{-1/2}^{1/2} w^*\,D\phi\,dz - T_a^{1/2}\int_{-1/2}^{1/2} w^*\,D\zeta\,dz \tag{8}$$

using Eqs. (5), (3) and (4) and the boundary conditions(6), we can write

$$-aR^{1/2}(1+M_1)\int_{-1/2}^{1/2} w^*\,\theta dz = (1+M_1)\int_{-1/2}^{1/2}\theta\,(D^2-a^2-P_r\omega^*)\theta^* dz\,, \tag{9}$$

$$aR^{1/2}M_1\int_{-1/2}^{1/2} w^*\,(D\phi)dz = -M_1\int_{-1/2}^{1/2} D\phi\,(D^2-a^2-P_r\omega^*)\theta^*\mathrm{dz}$$

$$= M_1\int_{-1/2}^{1/2} D^2\phi(D^2-a^2M_3)\,\phi^* dz - M_1(a^2+P_r\omega^*)\int_{-1/2}^{1/2}\phi(D^2-a^2M_3)\phi^* dz, \tag{10}$$

and $-T_a^{\frac{1}{2}}\int_{-1/2}^{1/2} w^*\,(D\zeta)dz = T_a^{\frac{1}{2}}\int_{-1/2}^{1/2}\zeta(Dw^*)dz = \int_{-1/2}^{1/2}\zeta\left(\frac{1}{k_o}+\omega^*\right)\zeta^* dz\,.$ (11)

Combining equations (8) - (11), we get

$$\left(\frac{1}{k_o}+\omega\right)\int_{-1/2}^{1/2}(|Dw|^2+a^2|w|^2)dz = (1+M_1)\int_{-1/2}^{1/2}(|D\theta|^2+a^2|\theta|^2+P_r\omega^*|\theta|^2)dz - M_1\int_{-1/2}^{1/2}(|D^2\phi|^2+ a^2M_3|D\phi|^2)dz - M_1(a^2+P_r\omega^*)\int_{-1/2}^{1/2}(|D\phi|^2+a^2M_3|\phi|^2)dz - \left(\frac{1}{k_o}+\omega^*\right)\int_{-1/2}^{1/2}|\zeta|^2 dz\,. \tag{12}$$

Integrating the various terms of Eq.(12) by parts for an appropriate number of times and making use of the boundary conditions (6) and the equality

$$\int_{-1/2}^{1/2}\psi^*\,D^{2n}\psi dz = (-1)^n\int_{-1/2}^{1/2}|D^n\psi|^2 dz\,,\quad \text{where } \psi = w,\theta,\phi\ \ (n=1), \tag{13}$$

we may write equation (12) in the form

$$\left(\frac{1}{k_o}+\omega\right)\int_{-1/2}^{1/2}(|Dw|^2+a^2|w|^2)dz = (1+M_1)\int_{-1/2}^{1/2}(|D\theta|^2+a^2|\theta|^2+P_r\omega^*|\theta|^2)dz - M_1\int_{-1/2}^{1/2}(|D^2\phi|^2+ a^2M_3|D\phi|^2)dz - M_1(a^2+P_r\omega^*)\int_{-1/2}^{1/2}(|D\phi|^2+a^2M_3|\phi|^2)dz - \left(\frac{1}{k_o}+\omega^*\right)\int_{-1/2}^{1/2}|\zeta|^2 dz \tag{14}$$

Equating imaginary parts of both sides of (14) and cancelling $\omega_i\ (\neq 0)$ throughout from the resulting equation, we get

$$\int_{-1/2}^{1/2}(|Dw|^2+a^2|w|^2)dz = -(1+M_1)P_r\int_{-1/2}^{1/2}|\theta|^2 dz + M_1\,P_r\int_{-1/2}^{1/2}(|D\phi|^2+a^2M_3|\phi|^2)dz + \int_{-1/2}^{1/2}|\zeta|^2 dz\,. \tag{15}$$

Multiplying equations (5) and (3) by their respective complex conjugates and integrating over the vertical range of z for an appropriate number of times and using the boundary conditions(6), we obtain

$$\int_{-1/2}^{1/2}(|D^2\theta|^2+2a^2|D\theta|^2+a^4|\theta|^2)dz + 2P_r\omega_r\int_{-1/2}^{1/2}(|D\theta|^2+a^2|\theta|^2)dz + P_r^2|\omega|^2\int_{-1/2}^{1/2}|\theta|^2 dz = R\,a^2\int_{-1/2}^{1/2}|w|^2 dz\,, \tag{16}$$

and $\left(\frac{1}{k_o^2}+2\,\frac{\omega_r}{k_o}+|\omega|^2\right)\int_{-1/2}^{1/2}|\zeta|^2 dz = T_a\int_{-1/2}^{1/2}|Dw|^2 dz.$ (17)

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since $\omega_r \geq 0$, we obtain from Eq. (16) that

$$\int_{-1/2}^{1/2} |\theta|^2 dz \ \leq \ \frac{R\,a^2}{P_r^2|\omega|^2} \int_{-1/2}^{1/2} |w|^2 dz \,, \tag{18}$$

and from Eq. (17), we have

$$\int_{-1/2}^{1/2} |\zeta|^2 dz \ \leq \ \frac{T_a}{|\omega|^2} \int_{-1/2}^{1/2} |Dw|^2 dz. \tag{19}$$

Now multiplying Eq. (4) by ϕ^* and integrating over the vertical range of z, we get

$$\int_{-1/2}^{1/2} \phi^*(D^2 - a^2 M_3)\phi dz \ = \ \int_{-1/2}^{1/2} \phi^*(D\theta) dz \ ,$$

which gives $\int_{-1/2}^{1/2} (|D\phi|^2 + a^2 M_3 |\phi|^2)\, dz \ = - \int_{-1/2}^{1/2} \phi^*(D\theta) dz \ = \ \int_{-1/2}^{1/2} \theta\,(D\phi^*) dz \ \leq \left| \int_{-1/2}^{1/2} \theta\,(D\phi^*) dz \right|$

$\leq \int_{-1/2}^{1/2} |\theta||D\phi^*| dz \ \leq \int_{-1/2}^{1/2} |\theta||D\phi| dz \leq \left(\int_{-1/2}^{1/2} |\theta|^2 dz\right)^{1/2} \left(\int_{-1/2}^{1/2} |D\phi|^2 dz\right)^{1/2}$, using Schwartz inequality (20)

which implies that $\int_{-1/2}^{1/2} |D\phi|^2\, dz \leq \left(\int_{-1/2}^{1/2} |\theta|^2 dz\right)^{1/2} \left(\int_{-1/2}^{1/2} |D\phi|^2 dz\right)^{1/2}$,

and thus $\left(\int_{-1/2}^{1/2} |D\phi|^2 dz\right)^{1/2} \leq \left(\int_{-1/2}^{1/2} |\theta|^2 dz\right)^{1/2}$, (21)

Combining inequalities (20) and (21), we get

$$\int_{-1/2}^{1/2} (|D\phi|^2 + a^2 M_3 |\phi|^2)\, dz \ \leq \int_{-1/2}^{1/2} |\theta|^2 \ dz \,. \tag{22}$$

Now using inequalities (18), (19) and (22), in equation (15), we obtain

$$\int_{-1/2}^{1/2} (|Dw|^2 + a^2 |w|^2) \mathrm{dz} \ + (1 + \ M_1) P_r \int_{-1/2}^{1/2} |\theta|^2 dz \leq \frac{RM_1 a^2}{P_r|\omega|^2} \int_{-1/2}^{1/2} |w|^2 dz \ + \ \frac{T_a}{|\omega|^2} \int_{-1/2}^{1/2} |Dw|^2 dz \,,$$

which can be rearranged as

$$\left(1 - \frac{T_a}{|\omega|^2}\right) \int_{-1/2}^{1/2} |Dw|^2 dz + \left(1 - \frac{RM_1}{P_r|\omega|^2}\right) a^2 \int_{-1/2}^{1/2} |w|^2 dz + (1 + \ M_1) P_r \int_{-1/2}^{1/2} |\theta|^2 dz \leq \ 0. \tag{23}$$

It follows from inequality (23) that $|\omega|^2 \ \leq \ \ greater\ of \left\{\frac{RM_1}{P_r}, T_a\right\}$. (24)

This completes the proof of the theorem.

Above theorem may be stated in an equivalent form as: the complex growth rate of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection in a rotating porous medium, for the case of free boundaries, lies inside a semicircle in the right half of the $\omega_r\omega_i$ - plane whose centre is at the origin and $(radius)^2 =$ greater of $\left\{\frac{RM_1}{P_r}, T_a\right\}$.

we now obtain the critical value of Rayleigh number numerically which may be used in inequality (24) to plot the bounds for the growth rate.

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Following the analysis of Finlayson [5], the exact solution satisfying the boundary condition (6) is then

$$w = A\cos\pi z, \quad \theta = B\cos\pi z, \quad \phi = \frac{C}{\pi}\sin\pi z, \; \zeta = \frac{D}{\pi}\sin\pi z, \quad D\phi = C\cos\pi z, \; D\zeta = D\cos\pi z,$$

where A, B, C and D are constants. Substitution of above solutions in Eqs. (1) and (3)-(5) yield a system of four linear homogeneous algebraic equations in the unknowns A, B, C and D. For the existence of non-trivial solutions of this system, the determinant of the coefficients of A, B, C and D must vanish. This determinant on simplification yields

$$\mathrm{U}\omega^3+\mathrm{V}\omega^2+\mathrm{W}\omega+\mathrm{X}=0 \tag{25}$$

where $U = (\pi^2 + a^2M_3)P_r k^2$, (26)

$$V = (\pi^2 + a^2M_3)\left(\frac{2k^2P_r}{\mathrm{k_0}} + k^4\right), \tag{27}$$

$$W = \left((\pi^2 + a^2M_3)\left(\frac{2k^4}{\mathrm{k_0}} + \frac{k^2P_r}{\mathrm{k_0}^2} + \pi^2T_aP_r\right)\right) - Ra^2\left(\pi^2 + a^2M_3(1+M_1)\right), \tag{28}$$

$$X = \left((\pi^2 + a^2M_3)\left(\frac{k^4}{\mathrm{k_0}^2} + \pi^2T_ak^2\right)\right) - \frac{Ra^2}{\mathrm{k_0}}\left(\pi^2 + a^2M_3(1+M_1)\right), \tag{29}$$

and $k^2 = (\pi^2 + a^2)$.

Substituting $\omega = i\omega_i$ in Eq. (25). Real and non-zero ω_i gives the condition of overstability.

From Eq. (25), the Rayleigh number for oscillatory instability can be easily written as

$$R_1 = (1 + x\mathrm{M_3})\frac{\left\{\frac{2}{\mathrm{k_0}}(1+x)^4 + \frac{4\mathrm{P_r}}{\pi^2\mathrm{k_0}^2}(1+x)^3 + \frac{2\mathrm{P_r}^2}{\pi^4\mathrm{k_0}^3}(1+x)^2 + \frac{2\mathrm{P_r}^2\mathrm{T_a}}{\pi^4\mathrm{k_0}}(1+x)\right\}}{\left[x\left(1+x\mathrm{M_3}(1+\mathrm{M_1})\right)\right]\left[(1+x)\frac{\mathrm{P_r}}{\mathrm{k_0}^2} + \pi^2(1+x)^2\right]} \tag{30}$$

where $R_1 = \frac{R}{\pi^4}$, $x = \frac{a^2}{\pi^2}$. To find the minimum value of $R(= R_c$, the critical Rayleigh number) with respect to wave number, equation (30) is differentiated with respect to x and equated to zero and the following polynomial in x is obtained.

$$\left\{\left(x + x^2M_3(1+M_1)\right)\left[(1+x)\frac{\mathrm{P_r}}{\mathrm{k}^2} + \pi^2(1+x)^2\right]\right\}\left\{\frac{2}{\mathrm{k_0}}\left[4(1+x\mathrm{M_3})(1+x)^3 + (1+x)^4M_3\right] + 4\frac{\mathrm{P_r}}{\pi^2\mathrm{k_0}^2}\left[3(1+xM_3)(1+x)^2 + (1+x)^3\mathrm{M_3}\right] + \frac{2\mathrm{P_r}^2}{\pi^4\mathrm{k_0}^3}\left[2(1+xM_3)(1+x) + (1+x)^2\mathrm{M_3}\right] + \frac{2\mathrm{T_a}\mathrm{P_r}^2}{\pi^4\mathrm{k_0}}\left[(1+x\mathrm{M_3}) + (1+x)\mathrm{M_3}\right]\right\} - \left\{(1+x\mathrm{M_3})\left[\frac{2}{\mathrm{k_0}}(1+x)^4 + 4\frac{\mathrm{P_r}}{\pi^2\mathrm{k_0}^2}(1+x)^3 + \frac{2\mathrm{P_r}^2}{\pi^4\mathrm{k_0}^3}(1+x)^2 + \frac{2\mathrm{T_a}\mathrm{P_r}^2}{\pi^4\mathrm{k_0}}(1+x)\right]\right\}\left\{\left(x + x^2M_3(1+M_1)\right)\left[\frac{\mathrm{P_r}}{\mathrm{k_0}^2} + 2\pi^2(1+x)\right] + \left[(1+x)\frac{\mathrm{P_r}}{\mathrm{k_0}^2} + \pi^2(1+x)^2\right]\left(1 + 2xM_3(1+M_1)\right)\right\} = 0$$

The above equation is solved numerically for various values of M_1, M_3 and T_a, and the minimum value of x is obtained each time, hence the critical wave number is obtained. Using this in equation (30), we obtain the critical Rayleigh number, above which the instability sets in as oscillatory motion.

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Table 1: Marginal stability of ferromagnetic convection in a rotating porous medium heated from below for oscillatory mode having $M_1 = 1000, M_2 = 0, P_r = 0.9,\ k_0 = 0.001$.

Taylor no. T_a	Magnetization M_3	Critical wave no. x_c	Critical Rayleigh no. R_c	$\frac{R_c M_1}{P_r}$
10^5	1	9.8308	2.7514	3057.1
	3	8.0281	2.5668	2852.0
	5	7.5895	2.5248	2805.3
	7	7.3894	2.506	2784.4
10^7	1	21.758	3.935	4372.2
	3	20.758	3.8173	4241.4
	5	20.504	3.7929	4214.3
	7	20.407	3.7825	4202.8

Now using the above table for the different values of T_a and $\frac{R_c M_1}{P_r}$ in the inequality (24) we obtain the following graphs which give upper bounds for the complex growth rate ω.

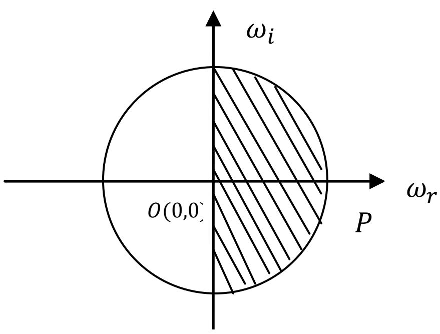

Fig 2a

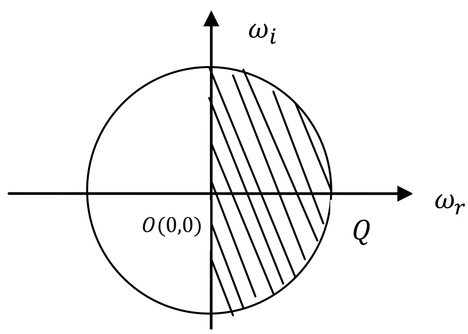

Fig 2b

Shaded region shows the region of the complex growth rate. $OP = T_a = 10^5$ (Fig 2a) and $OQ = T_a = 10^7$(Fig 2b)

we now derive an upper bound to the complex growth rate for the case of rigid boundaries in the form of following theorem.

Theorem 2: If $R > 0, M_1 > 0, T_a > 0, \omega_r \geq 0$, then a necessary condition for the existence of a nontrivial solution $(w, \theta, \phi, \zeta, \omega)$ of the equations (1) and (3) − (5) together with the boundary conditions (7) is that

$$|\omega|^2 \omega_i^2 < \text{ greater of } \left\{ \left(\frac{RM_1}{P_r}\right)^2, {T_a}^2 \right\}$$

Proof: Multiplying equation (1) by w^* throughout and integrating the resulting equation over the vertical range of z, we get

$$\int_{-1/2}^{1/2} w^* \left(\frac{1}{k_o} + \omega\right)(D^2 - a^2) w dz =$$
$$-aR^{1/2}(1 + M_1) \int_{-1/2}^{1/2} w^* \theta \, dz + aR^{1/2} M_1 \int_{-1/2}^{1/2} w^* \, d\phi \, dz - T_a^{1/2} \int_{-1/2}^{1/2} w^* D\zeta \, dz . \quad (31)$$

using equations (5) and (3) and the boundary conditions (7) we can write

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$$-aR^{1/2}(1+\ M_1)\int_{-1/2}^{1/2} w^*\theta\, dz \ = \ (1+\ M_1)\int_{-1/2}^{1/2} \theta\,(D^2-a^2-P_r\omega^*)\theta^* dz \tag{32}$$

and $-T_a^{\frac{1}{2}}\int_{-1/2}^{1/2} w^* D\zeta\, dz \ = \ T_a^{\frac{1}{2}}\int_{-1/2}^{1/2} \zeta\, Dw^* dz = \int_{-1/2}^{1/2} \zeta\left(\frac{1}{k_o}+\omega^*\right)\zeta^* dz$ (33)

combining equations (31) − (33), we obtain

$$\int_{-1/2}^{1/2} w^*\left(\frac{1}{k_o}+\omega\right)(D^2-a^2)w dz = (1+\ M_1)\int_{-1/2}^{1/2} \theta\,(D^2-a^2-P_r\omega^*)\theta^* dz + \ a\,R^{1/2}\,M_1\int_{-1/2}^{1/2} w^* D\phi\, dz - \int_{-1/2}^{1/2} \zeta\left(\frac{1}{k_o}+\omega^*\right)\zeta^* dz. \tag{34}$$

Integrating the various terms of equation(34) by parts for an appropriate number of times and making the use of the boundary conditions (7) and the equality (13) , we obtain

$$\left(\frac{1}{k_o}+\omega\right)\int_{-1/2}^{1/2}(|Dw|^2+a^2|w|^2)dz = (1+\ M_1)\int_{-1/2}^{1/2}(|D\theta|^2+a^2|\theta|^2\ +P_r\omega^*|\theta|^2)dz - aR^{1/2}\,M_1\ \int_{-1/2}^{1/2} w^*(D\phi)dz - \left(\frac{1}{k_o}+\omega^*\right)\int_{-1/2}^{1/2}|\zeta|^2 dz \quad . \tag{35}$$

Equating imaginary parts of both sides of (35) and dividing the resulting equation by ω_i ($\neq 0$), we get

$$\int_{-1/2}^{1/2}(|Dw|^2+a^2|w|^2)dz \ = \ -\ (1+\ M_1)P_r\int_{-1/2}^{1/2}|\theta|^2\, dz \ - \ \frac{aR^{1/2}\,M_1}{\omega_i}\text{imaginary part of}\int_{-1/2}^{1/2} w^*(D\phi)dz + \int_{-1/2}^{1/2}|\zeta|^2 dz\ . \tag{36}$$

Now $-\ \frac{aR^{\frac{1}{2}}\,M_1}{\omega_i}$imaginary part of $\int_{-\frac{1}{2}}^{\frac{1}{2}} w^*(D\phi)dz \quad \leq \left|-\frac{aR^{1/2}\,M_1}{\omega_i}\text{imaginary part of}\int_{-1/2}^{1/2} w^*(D\phi)\, dz\right|$

$$\leq aR^{1/2}\,M_1\left|\frac{1}{\omega_i}\int_{-1/2}^{1/2} w^*(D\phi)dz\ \right| \quad \leq \frac{aR^{1/2}\,M_1}{|\omega_i|}\ \left|\int_{-1/2}^{1/2} w^*(D\phi)dz\right| \ \leq \frac{aR^{1/2}\,M_1}{|\omega_i|}\ \int_{-1/2}^{1/2}|w^*D\phi|\, dz$$

$$\leq \frac{aR^{1/2}\,M_1}{|\omega_i|}\ \int_{-1/2}^{1/2}|w||D\phi|\, dz \ \leq \frac{aR^{1/2}\,M_1}{|\omega_i|}\left(\int_{-1/2}^{1/2}|w|^2dz\right)^{1/2}\left(\int_{-1/2}^{1/2}|D\phi|^2dz\right)^{1/2} \text{(using Schwartz inequality)} \tag{37}$$

From inequalities (18) and (21), we have

$$\left(\int_{-1/2}^{1/2}|D\phi|^2dz\right)^{1/2} \leq \frac{aR^{1/2}}{P_r|\omega|}\ \left(\int_{-1/2}^{1/2}|w|^2dz\right)^{1/2}\ . \tag{38}$$

Combining inequalities (37) and (38), we obtain

$$-\ \frac{aR^{1/2}\,M_1}{\omega_i}\text{imaginary part of}\int_{-1/2}^{1/2} w^* D\phi\, dz \leq \frac{a^2RM_1}{P_r\ |\omega||\omega_i|}\ \int_{-1/2}^{1/2}|w|^2dz \tag{39}$$

Multiplying equation (3) by ζ^* and integrating the resulting equation by parts for an appropriate number of times over the vertical range of z, we obtain from imaginary part of the final equation

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$$\int_{-1/2}^{1/2}|\zeta|^2 dz = \frac{1}{\omega_i}\text{imaginary part of } T_a^{1/2} \int_{-1/2}^{1/2}\zeta^*(Dw)dz$$
$$\leq T_a^{1/2}\left|\frac{1}{\omega_i}\text{ imaginary part of}\int_{-1/2}^{1/2}\zeta^*(Dw)dz\right|$$

$$\leq \frac{T_a^{1/2}}{|\omega_i|}\left|\int_{-1/2}^{1/2}\zeta^*(Dw)dz\right| \leq \frac{T_a^{1/2}}{|\omega_i|}\int_{-1/2}^{1/2}|\zeta||Dw|\, dz$$

$$\leq \frac{T_a^{1/2}}{|\omega_i|}\left(\int_{-1/2}^{1/2}|\zeta|^2 dz\right)^{1/2}\left(\int_{-1/2}^{1/2}|Dw|^2 dz\right)^{1/2} \text{(using Schwartz inequality)}$$
$$\leq \frac{T_a}{|\omega||\omega_i|}\int_{-1/2}^{1/2}|Dw|^2 dz\,.\text{(using inequality (19))} \tag{40}$$
utilizing inequalities (39) and (40) in equation (36) , we finally have

$$\left(1-\frac{T_a}{|\omega||\omega_i|}\right)\int_{-1/2}^{1/2}|Dw|^2 dz + a^2\left(1-\frac{RM_1}{P_r|\omega||\omega_i|}\right)\int_{-1/2}^{1/2}|w|^2 dz + (1+M_1)P_r\int_{-1/2}^{1/2}|\theta|^2 dz \leq 0\,,$$

which clearly implies that

$$|\omega|^2\omega_i^2 < greater\ of\ \left\{\left(\frac{RM_1}{P_r}\right)^2, T_a^2\right\} \tag{41}$$

Above theorem may be stated in an equivalent form as: the complex growth rate of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection in a rotating porous medium, for the case of rigid boundaries, must lies inside the region represented by inequality (41).

IV. CONCLUSIONS

The linear stability theory is used to derive the bounds for the complex growth rates in ferromagnetic convection in a rotating porous medium heated from below in the presence of a uniform vertical magnetic field. These bounds are important especially when both the boundaries are not dynamically free so that exact solutions in the closed form are not obtainable. Further, since the results derived herein involve only the non – dimensional quantities and are not wave number dependent are, thus, of uniform applicability.

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