

Some Euler Sequence Spaces in Non-Archimedean Fields

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ABSTRACT: The aim of this paper is to introduce some Euler sequence spaces in a Complete, Non-trivially valued, Non-Archimedean fields and prove a few inclusion theorems on it.

KEYWORDS: Non-Archimedean fields, Euler sequence space, sequence space.

I. INTRODUCTION

Throughout this paper, K denotes complete, non-trivially valued, non-archimedean fields. Infinite matrices and sequences considered in the sequel have entries in K . A general reference for the study of summability methods in classical one is [9]. For analysis, in non-archimedean fields, see [3].

If $A = (a_{n,k})$, $a_{n,k} \in K$, $n, k = 0, 1, 2, \dots$ is an infinite matrix, then the A -transform of a sequence $x = \{x_k\}$, $x_k \in K$, $k = 0, 1, 2, \dots$, is the sequence, where [16]

$$(Ax)_n = \sum_{k=0}^n a_{nk} x_k, \quad n = 0, 1, 2, \dots, \text{ it being assumed that the series on the right converge.}$$

In 1969, Maddox [4] studied Continuous and Kothe-Toeplitz duals of certain sequence spaces and later he worked with Lascarides [5] to determine the matrices of the classes $(\ell(p), \ell_\infty)$, $(\ell(p), c)$ and $(\ell_\infty, \ell_\infty)$. In classical analysis, Nanda [8] studied some sequence spaces over valued fields [10].

Wang [11] studied sequence spaces for Nörlund (1978), In 1981, Ahmad and Saraswat [10] obtained the necessary and sufficient conditions to characterize the matrices of the classes C_0 , c , ℓ_∞ and ℓ_p . Altay et.al. [1] also introduced an Euler sequence spaces e'_0 and e'_c of nonabsolute type that are BK -spaces and constructed the bases for the duals of the spaces e'_0 and e'_c [15]. Finally, they established the necessary and sufficient conditions of an infinite matrix belonging to the classes (e'_0, ℓ_p) and (e'_c, c) , where $1 \leq p \leq \infty$.

We introduce some Euler sequence spaces and prove a few inclusion theorems over Non-Archimedean Fields [13].

If $x = \{x_k\}$ is a sequence defined over K , then the sequence spaces are:

$$c = \left\{ x = \{x_k\} \in K : \lim_{k \rightarrow \infty} x_k \text{ exists} \right\}, \quad \ell_\infty = \left\{ x = \{x_k\} \in K : \sup_k |x_k| < \infty \right\} \text{ and}$$

$$\ell_p = \left\{ x = \{x_k\} \in K : \sum_{k=0}^n |x_k|^p < \infty, p \geq 1 \right\}$$

We introduce some Euler sequence spaces over K as [14]

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$$e_c^r = \left\{ x = \{x_k\} \in K : \lim_{n \rightarrow \infty} \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k \text{ exists} \right\}$$

$$e_p^r = \left\{ x = \{x_k\} \in K : \sum_{k=0}^n \left| nC_k r^k (1-r)^{n-k} x_k \right|^p < \infty \right\}$$

$$e_\infty^r = \left\{ x = \{x_k\} \in K : \sup_{n,k} \left| nC_k r^k (1-r)^{n-k} x_k \right| < \infty \right\}$$

The Euler matrix $(e_{n,k}^r)$ is defined by

$$e_{n,k}^r = \begin{cases} nC_k r^k (1-r)^{n-k}, & k \leq n \\ 0, & k > n \end{cases}$$

for all $k, n = 0, 1, 2, \dots$, and the method E^r is regular for $|1-r| < 1$. Atlay et al [] redefined the spaces e_p^r , e_c^r and e_∞^r as [11]

$$e_p^r = (\ell_p)_{E^r} e_c^r = (c)_{E^r} \text{ and } e_\infty^r = (\ell_\infty)_{E^r}$$

Theorem 2. The sets e_p^r , e_c^r and e_∞^r are linear spaces with co-ordinatewise addition and scalar multiplication that are BK- spaces with norm $\|x\|_{e_c^r} = \|x\|_{e_p^r} = \|E^r x\|_{\ell_\infty}$,

where $1 \leq p \leq \infty$ [12].

Proof Since (1) holds and c, ℓ_∞, ℓ_p are the BK-spaces with respect to their norms in \mathbf{K} and $e_{n,n}^r \neq 0, e_{n,k}^r = 0$ ($k > n$), for $k, n = 0, 1, 2, \dots$, Wilansky [10] gives the fact that the spaces e_p^r, e_c^r and e_∞^r are the BK- spaces, $1 \leq p \leq \infty$.

Definition 1. Let $y_n = \sum_{k=0}^n e_{n,k}^r x_k$ be Euler transform $[E, r]$ of sequence $\{x_k\}$, where $[E, r] = (e_{n,k}^r)$ which we define by [09]

$$y_n = [E, r] x_k = \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k$$

II. SOME INCLUSION THEOREMS ON EULER SEQUENCE SPACES

Theorem 3. The inclusion $\ell_p \subset e_p^r$ strictly holds for $1 \leq p \leq \infty$

Proof. Let us take any $x \in \ell_p$. Now

$$y_k = \sum_{j=0}^k kC_j r^j (1-r)^{k-j} x_j$$

$$|y_k| = \left| \sum_{j=0}^k kC_j r^j (1-r)^{k-j} x_j \right|$$

$$\leq \max \left\{ |kC_0| |r|^0 |1-r|^k |x_0|, |kC_1| |r| |1-r|^{k-1} |x_1|, \dots, |kC_k| |r|^k |1-r|^0 |x_k| \right\}$$

$$= |r|^k |x_k| \quad \text{since, } |r| = 1$$

$$< |x_k|, \quad \text{since } |r| < 1$$

$$|y_k|^p < |x_k|^p$$

$$\sum_{k=0}^n |y_k|^p < \sum_{k=0}^n |x_k|^p$$

This shows that $\ell_p \subset e_p^r, 1 \leq p \leq \infty$

Theorem 4. The inclusion $c \subset e_c^r$ strictly holds.

Proof. Let us take $x \in c$. Now

$$y_n = \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k$$

$$\lim_{n \rightarrow \infty} y_n = \lim_{n \rightarrow \infty} \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k$$

$$\lim_{n \rightarrow \infty} |y_n| = \lim_{n \rightarrow \infty} \left| \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k \right|$$

$$\leq \lim_{n \rightarrow \infty} \left(|r^0 (1-r)^n x_0|, |r^1 (1-r)^{n-1} x_1|, \dots, |r^n (1-r)^0 x_n| \right)$$

$$= \lim_{n \rightarrow \infty} |r|^n |x_n|$$

$$= \lim_{n \rightarrow \infty} |x_n|, (|r| = 1)$$

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$$\lim_{n \rightarrow \infty} y_n = y \text{ (say)}$$

$$\Rightarrow x \in e_c^r$$

(i.e.). $C \subset e_c^r$ Strictly holds.

Theorem 5. The inclusion $\ell_\infty \subset e_\infty^r$ hold strictly.

Proof. Let us take $x \in \ell_\infty$. Now,

$$y_n = \sum_{k=0}^n nC_k r^k (1-r)^{n-k} x_k$$

$$|y_n| \leq \max \left\{ \left| nC_0 r^0 (1-r)^n x_0 \right|, \left| nC_1 r^1 (1-r)^{n-1} x_1 \right|, \dots, \left| nC_n r^n (1-r)^0 x_n \right| \right\}$$

$$= |r|^n |x_n| < |x_n|, (|r| = 1)$$

$$\sup_n |y_n| < \sup_n |x_n|$$

$$\Rightarrow x \in e_\infty^r, \text{ i.e., } \ell_\infty \subset e_\infty^r.$$

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