

# Theory and Applications of Benes Network in Graph Theory

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**ABSTRACT:** A path partition of a graph is a collection of vertex-disjoint paths that cover all vertices of the graph. The path-partition problem is to find the path-partition number  $p(G)$  of a graph  $G$ , which is the minimum cardinality of a path partition of  $G$ . Obviously  $G$  has a Hamiltonian path if and only if  $p(G)=1$ .

**KEYWORDS:** Graph, Path, Connected Graph, complete graph, chord of a cycle.

## Preliminaries:

**Definition 1.2:** A graph is an ordered pair  $G = (V, E)$  comprising set  $V$  of vertices or nodes together with a set  $E$  of edges or lines, which are 2-element subsets of  $V$ . [9]

### Definition 1.1.21

A path is a trail in which all vertices (except possibly the first and last) are distinct.

### Definition 1.1.24

A graph is connected if there are paths containing each pair of vertices. [1]

### Definition 1.1.19

A complete graph is often called a clique. The size of the largest clique that can be made up of edges and vertices of  $G$  is called the clique number of  $G$ . [8]

### Definition 1.1.13

A chord of a cycle is an edge among its vertices that is not part of the cycle [2].

## Benes Network

The  $n$ -dimensional Benes network consists of back-to-back butterfly, denoted by  $BB(n)$  [7]. The  $BB(n)$  has  $2n + 1$  levels, each with  $2^n$  vertices. The first and last  $n + 1$  levels in the  $BB(n)$  form two  $BF(n)$ 's respectively, while the middle level in  $BB(n)$  is shared by these butterfly networks. [3] The  $n$ -dimensional Benes network  $BB(n)$  has  $(n + 1)^{2^n + 1}$  vertices and  $n2^{n+2}$  edges. It has only 2-degree vertices and 4-degree vertices, and thus, is Eulerian. In other words, the removal of the first and last levels from  $BB(n)$  results in two disjoint  $BB(n - 1)$ 's. [4]

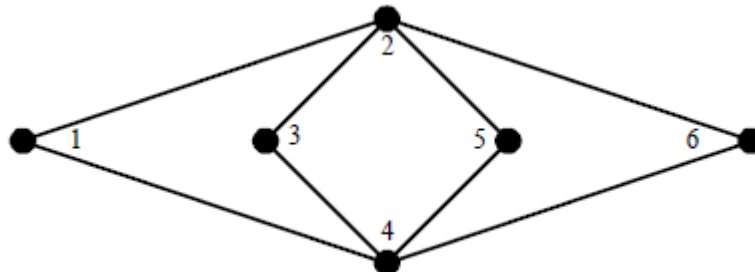


Figure 4. 5:  $BB(1)$

**Theorem 4.3.1:** For a Benes network of dimension  $n$ ,  $p(BB(n)) = 2^n$ ,  $n \geq 1$ . [5]

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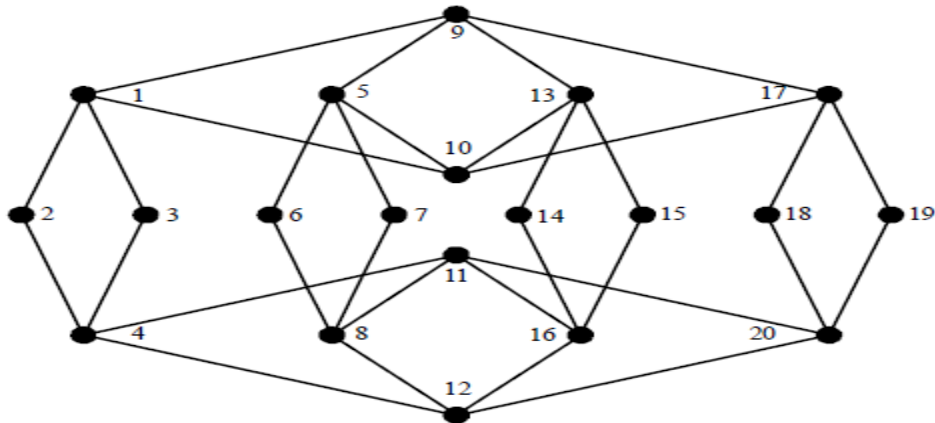


Figure 4.6:  $BB(2)$

The number of paths for  $BB(2)$  shown in Figure 4.6 is  $p(BB(2)) = 4$  and the paths are  $\{2, 1, 9, 5, 6\}$ ,  $\{3, 4, 12, 8, 7\}$ ,  $\{18, 17, 10, 13, 14\}$ ,  $\{19, 20, 11, 16, 15\}$  [6]

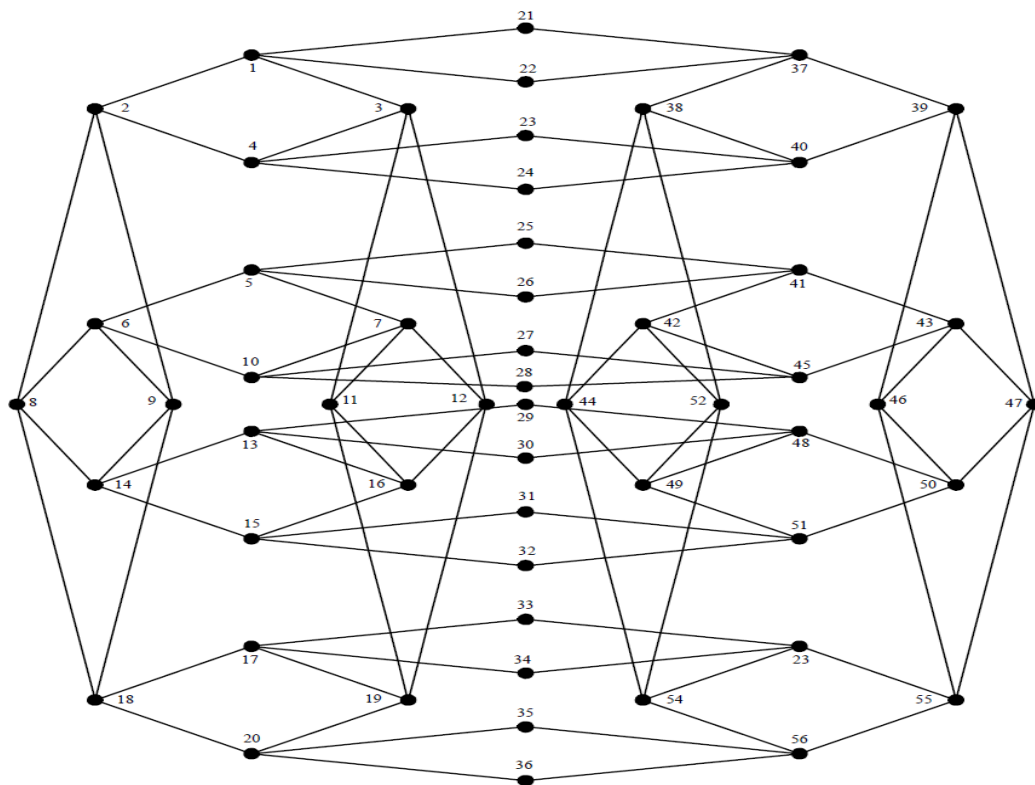


Figure 4.7:  $BB(3)$

The number of path for  $BB(3)$  shown in Figure 4.7 is  $p(BB(3)) = 8$  and paths are  $\{22, 37, 21, 1, 2, 8, 18\}$ ,  $\{24, 40, 23, 4, 3, 12, 19\}$ ,

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{35,20,36,56,54, 44, 38}, {34,17,33,53, 55, 47, 39}. {14, 9, 6, 5, 25, 41,26},  
{16,11,7,10,27,45,28}, {42,52,49,48,29,13,30}, {43, 46, 50, 51,31, 15, 32}.

## IV. CONCLUSION

There exists theoretical and practical reasons for studying special classes of Benesgraph, and it can be very interesting and worthwhile to explore a graph – algorithm problem for special types of graphs. It is very useful in Network System. In this paper we have discussed Theory and Applications of Benes Graph.

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