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Elements of Point Set Theory

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ABSTRACT: In this paper, we study some important application of mathematical analysis using open intervals and closed intervals. We define some definitions on interior point, open interval.

KEYWORDS: Open interval, interior point, open set, closed set.

1. INTRODUCTION

This concepts related in some way to real numbers, so we begin our study of analysis with a discussion of the real-number system. Several methods[4] are used to introduce real numbers. One method starts with positive integers 1,2,3,..... as undefined concepts and uses them to build a largest system, the prime, Mersenne prime, Fermat prime & Fibonacci numbers. The set of all points between a and b is called an *interval*. Sometimes it is important to distinguish between intervals which include their endpoints and intervals which do not. The notation $\{x: x \text{ satisfies } P\}$ will be used to designate the set of all real numbers x satisfy property p. Assume $a < b$. the *open interval* (a,b) is defined to be the set $(a,b) = \{x: a < x < b\}$. The *closed interval* [a,b] is the set $\{x: a \leq x \leq b\}$. This section describes that every open interval in the set of all real numbers (E_1) is an open set and that every closed interval is a closed set.

Definition 1.1. Let E_1 denote the set of all real numbers (the real line). Let E_2 denote the set of all complex numbers (the complex plane). An open interval (a, b) is defined as[5] $(a, b) = \{x \mid a < x < b\}$

Definition 1.2. Let S be a set in E_1 and assume that $x \in S$. Then x is called an interior point of S if there is some neighbourhood $N(x)$ all of whose points belong to S.

Definition 1.3. If S is a set in E_1 , S is an open set if every point of S is an interior point of S.

II. MAIN RESULTS.

Theorem. Every open interval in the set of all real numbers E_1 is an **open** set and that a closed interval is a closed set[3].

Proof.

To prove that an open interval in E_1 is an open set, we need to prove that every point in an open interval is an interior point. To do this, we must prove that every point in an open interval has an associated neighbourhood, all of whose points belong to the open interval. Consider an arbitrary open interval (a, b). Let x be any point in this interval, $x \in (a, b)$. A neighbourhood of x is given by the interval $(x-h, x+h)$ [6], where $h > 0$. What we have to show is that one of these neighbourhoods is entirely contained within the interval (a, b)[2], i.e. we have to show that $(x-h, x+h) \subset (a, b)$ for every x and for some $h > 0$. To do this, we have to show that for every[7]

$$y \in (x-h, x+h), \text{ we also have } y \in (a, b).$$

Let us choose h to be given by half the value of the minimum distance from x to each of the two endpoints, so that

$$h = \frac{1}{2} \min(|x-a|, |x-b|).$$

Because we know that $x > a$ and that $x < b$, we can rewrite h as

$$h = \frac{1}{2} \min(x-a, b-x).$$

From this definition, we see that

(i) $h \leq \frac{1}{2}(x-a)$, and that

(ii) $h \leq \frac{1}{2}(b-x)$.

Let us now analyse what we know about the point y. First of all, we know that y sits in the interval $(x-h, x+h)$, [8] so that $x-h < y < x+h$. In order to show that $a < y < b$, which is what we want to prove, all we have to show is that

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- (1) $a < x - h$, and that
(2) $x + h < b$.

From (i) we can do the abovefollowing manipulation:

$$\begin{aligned} h &\leq \frac{1}{2}(x - a) \\ \Rightarrow x - h &\geq x - \frac{1}{2}(x - a) \\ \Rightarrow x - h &\geq \frac{1}{2}(x + a) \\ \Rightarrow x - h &> \frac{1}{2}(a + a) \quad (\text{because } a < x) \\ &\Rightarrow x - h > a \end{aligned}$$

From (ii) we can do the above following manipulation:

$$\begin{aligned} h &\leq \frac{1}{2}(x - b) \\ \Rightarrow x + h &\leq x + \frac{1}{2}(x - b) \\ \Rightarrow x + h &\leq \frac{1}{2}(3x - b) \\ \Rightarrow x + h &\leq \frac{1}{2}(3b - b) \\ \Rightarrow x + h &> b \end{aligned}$$

Because we have shown that $y \in (x - h, x + h) \Rightarrow y \in (a, b)$, then we have proved that an open interval is an open set. To show that a closed interval is a closed set w.k.t the complement of an open set is a closed set[1].

Let us consider an arbitrary closed interval $[a, b]$. The complement of this interval is given by

$E_1 - [a, b] = (-\infty, a) \cup (b, \infty)$, the union of two open intervals, which we know are open sets by the above. Further, the union of two open sets is also an open set, so that $E_1 - [a, b]$ is an open set. So, as the complement of a closed interval is an open set, then a closed interval must therefore be a closed set[9].

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