

A Slip-Line Field Solution for Micro-Indentation of a Rigid Conical Wedge by Numerical Technique

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ABSTRACT: Slip-line field analysis is one of the important techniques to compute force in micro-indentation. The nature of the plastic flow lines are actually analyzed with the help of the slip-line field. In the present work, the analysis of micro-indentation of a rigid conical wedge in a semi-infinite block is presented. The problem is assumed to be plane-strain in nature. Numerical technique has been adopted to find out the slip-line field. A computer program generated shows the slip-line field and also calculates the height of piled-up material for specific indentation, tangential and normal force on rigid indenter and the corresponding value of co-efficient of static friction.

KEYWORDS: micro-indentation, slip-line field, numerical technique, forces analysis.

I. INTRODUCTION

The work on slip-line field analysis was originally initiated by Hill et al. [1] in 1950's. Over past few decades, several advancements have been achieved in this area. The analytical works of Grunzweig et al. [2], Johnson et al. [3], Haddow [4] and also Hill [5][6] are significant. When the wedge is indented, there would be pile up of soft material around the wedge. All the works stated earlier consider the free surface of the flow of piled-up material to be straight one. Dodd et al. [7] analyzed slip-line field numerically by computer program. The recent work by Busquet and Torrance [8] shows numerically generated slip-line field for a sliding cylinder problem. The work considers the free surface to be a polynomial one. Works of Oxley, Challen and Torrance are pioneering in this area. Few significant works by Oxley and Challen [9][10][11] depicts how slip-lines field theory is used in wear analysis. Works of Torrance et al. [12][13] are equally important in this field.

For the present work, a computer code has been generated for finding out slip-line field based on previous works [7][8]. The specialty of this work is to give the specific value of percentage of piled-up material of original volume indented. The straight line assumption of free surface has been considered for this work as in the work of Hill et al. [1]. The program also calculates the normal and tangential force and the corresponding values of static co-efficient of friction for any specific indentation.

II. BASIC MATHEMATICAL EQUATIONS AND CONTRUCTION OF SLIP-LINE FIELD

Figure 1 shows the schematic of slip line field. α - and β -lines are shown in Figure 1. $\angle EOF$ is the semi-wedge angle. The distance EO is the indentation depth h . AB is the free surface along which material flows. OA represents the height

of the piled up material from the centre O, shown in the figure as $f * h$, where f is a factor, generally greater than 1, which represents the amount of the material piled up. The co-ordinate of A is found out as $\left[x_1 = \frac{f * h}{m_1}, y_1 = f * h \right]$, where m_1 is the slope of the wedge.

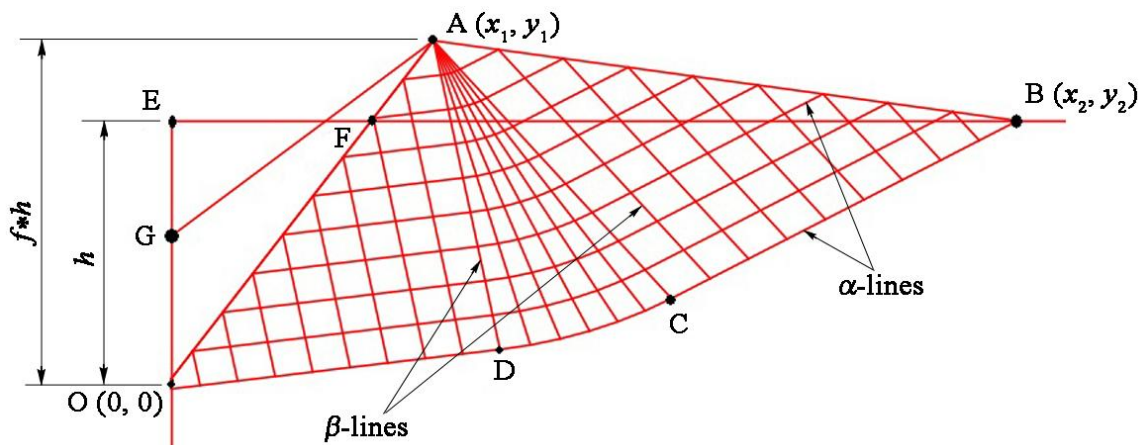


Figure 1: The description of slip-line field for conical indentation

The co-ordinate of the point B (x_2, y_2) is found by conservation of volume principle, assuming the material indented is equal to the material piled up above the plane of semi-infinite block as $\left[x_2 = \frac{h}{m_1} * \frac{f}{(f-1)}, y_2 = h \right]$. The slope of the free surface AB is, therefore, becomes $m_2 = \frac{h * (f-1)}{(x_2 - x_1)}$. The basic equations for the slip line field as given by Hencky's equations are as follows:

$$\Delta p + 2k \Delta \varphi = 0, \text{ along } \alpha\text{-lines and } \Delta p - 2k \Delta \varphi = 0, \text{ along } \beta\text{-lines} \quad \dots\dots\dots(1)$$

where, p is the hydrostatic stress acting normal to the slip lines, k is the shear flow stress of material (assumed constant), φ is the anti-clockwise angle of slip-lines measured from fixed reference axis (for the present work x -axis).

The calculation of slip-line field has been done in three stages. The field has three distinct region namely ABC, ACD, ADO as shown in Figure 1. The region ACD is called Centered Fan Region. For each type of region, the numerical solution has been discussed in the reference [14][15]. The input to the program are indenting depth (h) in μm , semi-wedge angle, the value of shear yield stress in $\text{N}/\mu\text{m}^2$. The shear yield stress value is taken to be 150MPa or $0.00015\text{N}/\mu\text{m}^2$. The values of the pressure for free surface AB is k . Now, with these inputs, the φ values are solved for all points and corresponding (x, y) co-ordinates are calculated. The program is run iteratively until the slip-line converges at O. the value of f is also found out. Once all the φ values are known, the pressures at all points are calculated using equation (1). The normal force and tangential force, F_N and F_T in ($\text{N}/\mu\text{m}$) are calculated from the pressure values on the wedge using the following relationships:

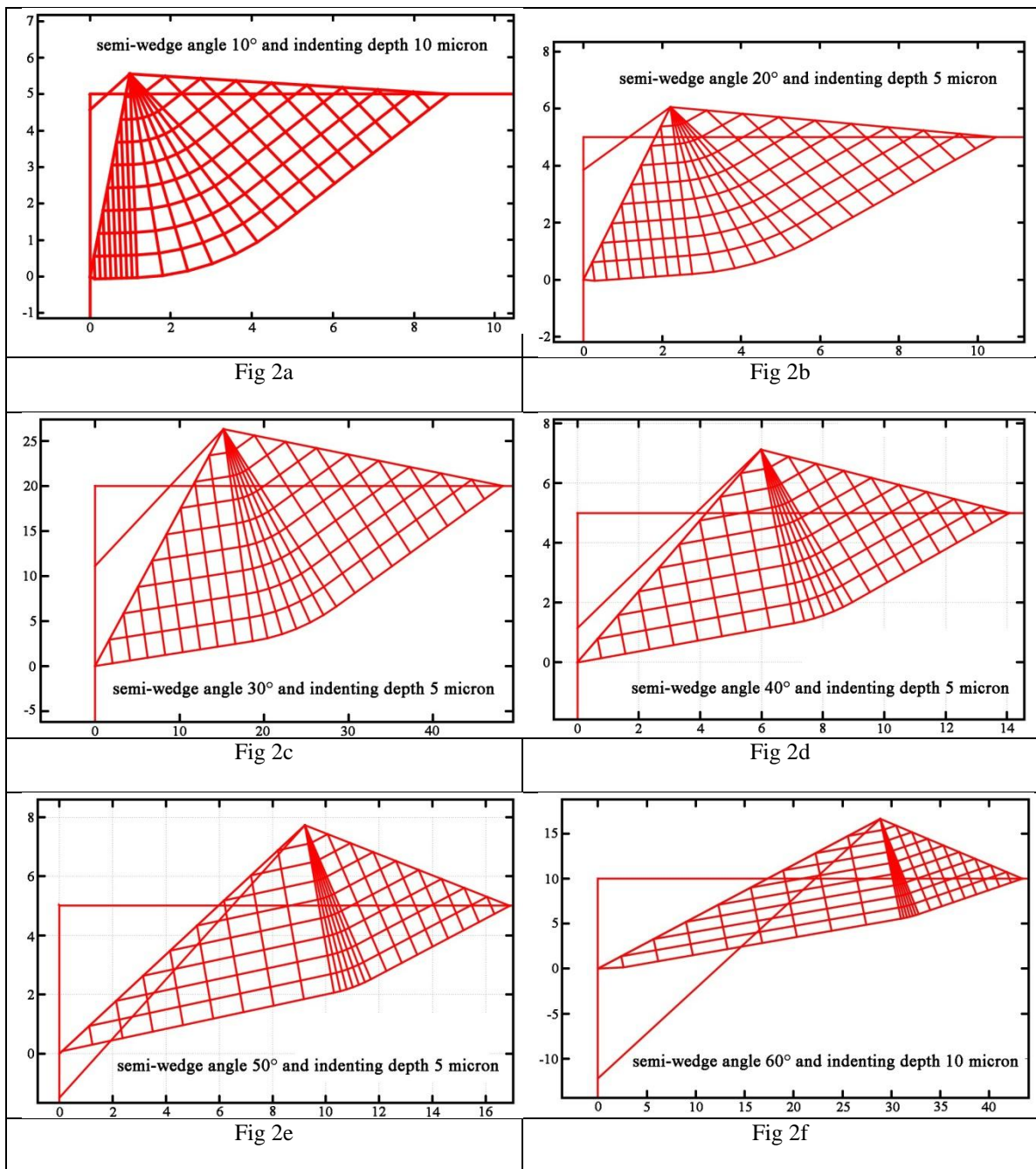
$$F_N = \sum 2.p.\Delta l.\sin \theta \quad \text{And} \quad F_T = \sum 2.p.\Delta l.\cos \theta \quad \dots\dots\dots(2)$$

Where, Δl is the length between two consecutive α -lines on the wedge and θ is the semi-wedge angle. Now, on the wedge surface value of the frictional stress is equal to the maximum yield stress value k . So, the value of the coefficient of friction $\mu = \frac{k}{p}$.

All the procedures discussed above do not hold if the semi-wedge angle is greater than 45° . In those cases, the boundary of the slip line meets below the point O not at O. The line GA makes an angle 45° with the horizontal. It is clear from the Figure 1 if the semi-wedge angle is greater than 45° point G comes below point O. In this case, slip-line meets at G. This is termed as Haddow's paradox [4]. Actually a cup of material accumulated below the wedge if the wedge angle is greater than 45° . This has been discussed in the reference [15]. For the present work semi wedge angle is gradually varied from 10° to 80° .

III. RESULTS AND DISCUSSION

Figure 2 shows different patterns of slip-line fields for different wedge angles. All the slip-line fields show that they are converged to the point O. But previous literatures [4], [7] suggest that beyond 45°, Haddow's paradox may appear and for this study it is prominent if the semi-wedge angle is greater than 68°.



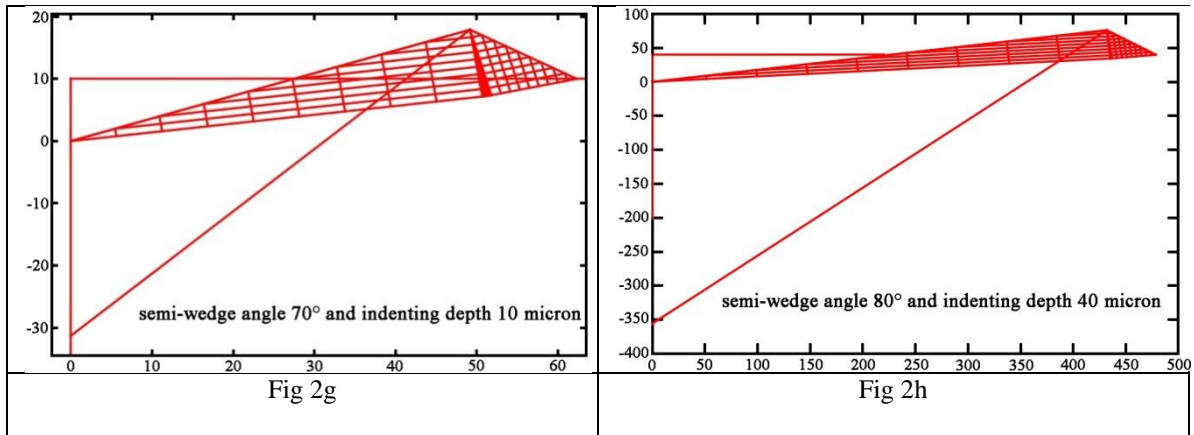


Fig 2: Various patterns of slip-line fields for different wedge angle and depth of indentation

It seems from the results that the present work contradicts previous works. If the wedge angle is small, it would be indented very easily. Zone of plastic deformation (FB in Figure 1) will be low. On the other hand, if the wedge angle is high, more material would be bulged around the wedge. Again, zone of plastic deformation is more. But, in our present work volume conservation has been assumed. Figure 3(a) shows that with the increase of semi-wedge angle, f -value increases, so more material is bulged out. As more material is bulged out, zone of plastic deformation decreases which are clear from Figure 2. There is obviously a contradiction. The value of f has been calculated by the program considering volume conservation. But previous literatures suggest that values of f have some limitation. Again, practically volume indented may not bulge out wholly [16]. Therefore, for high wedge angle, high zone of plastic deformation and again limitation on the f value leads to Haddow's paradox. That means lower boundary of slip line do not converged at O but meets at G (refer to figure 1).

TABLE 1: SHOWS THE VARIATION OF DIFFERENT PARAMETERS WITH THE CHANGE IN THE WEDGE ANGLE

| Semi-wedge angle(θ) in degree | Indentation (μm) | f , factor for piled-up depth | Average pressure on wedge ($\text{N}/\mu\text{m}^2$) | F_N ($\text{N}/\mu\text{m}$) | F_T ($\text{N}/\mu\text{m}$) | Co-efficient of friction, μ |
|--|-------------------------------|---------------------------------|--|----------------------------------|----------------------------------|---------------------------------|
| 10 | 2.5 | 1.110 | 0.001263 | 0.000247 | 0.0014 | 0.11875 |
| | 5 | 1.111 | 0.001263 | 0.000492 | 0.00279 | 0.11875 |
| 20 | 5 | 1.211 | 0.00126 | 0.00112 | 0.00307 | 0.119 |
| | 10 | 1.215 | 0.00126 | 0.00221 | 0.00607 | 0.119 |
| 30 | 5 | 1.315 | 0.001257 | 0.001932 | 0.003349 | 0.1193 |
| | 10 | 1.316 | 0.001257 | 0.003856 | 0.006683 | 0.11929 |
| 40 | 5 | 1.425 | 0.0012543 | 0.003078 | 0.00367 | 0.11959 |
| | 10 | 1.427 | 0.001254 | 0.00612 | 0.0073 | 0.1196 |
| 50 | 5 | 1.545 | 0.00125 | 0.00474 | 0.00398 | 0.1199 |
| | 10 | 1.545 | 0.00125 | 0.009486 | 0.007967 | 0.11996 |
| | 20 | 1.545 | 0.0012503 | 0.018972 | 0.0159338 | 0.11996 |
| 60 | 10 | 1.665 | 0.0012462 | 0.0152223 | 0.00879938 | 0.12307 |
| | 20 | 1.665 | 0.00124606 | 0.0302371 | 0.0174788 | 0.12308 |
| 70 | 10 | 1.789 | 0.00124091 | 0.0255571 | 0.00931995 | 0.1230879 |
| | 20 | 1.789 | 0.00124091 | 0.0511142 | 0.0186399 | 0.1230879 |
| 80 | 20 | 1.9025 | 0.00123533 | 0.167129 | 0.0295913 | 0.121425 |
| | 30 | 1.9025 | 0.00123533 | 0.222839 | 0.0394551 | 0.121425 |

Table 1 shows comparison of different parameters for different wedge angle and indentation. The figure 3(b) depicts the variation of co-efficient of friction with respect to semi wedge angle. It shows that nearly 75° , it has maximum frictional co-efficient, then decreases. With the increase of wedge angle the normal force and tangential force increases. But, the variation of normal force is very high. Again, from Table I, it is clear that normal and tangential force increase with increase of depth of indentation. But wedge surface pressure decreases with the increasing wedge angle.

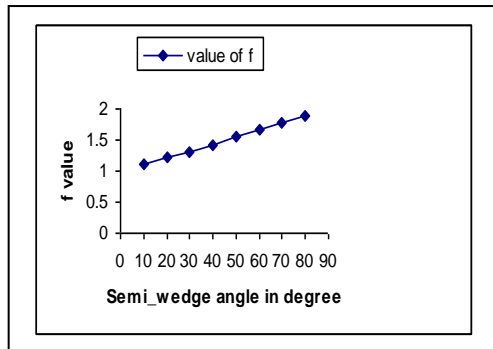


Fig 3(a)

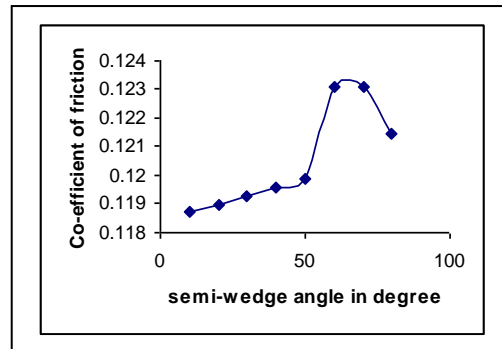


Fig 3(b)

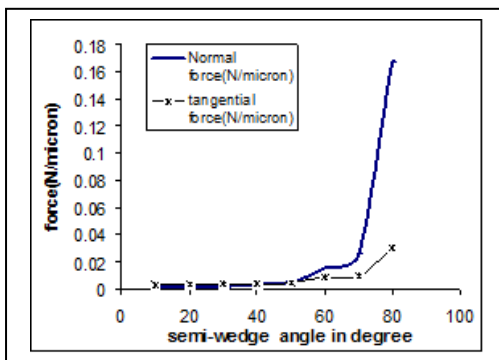


Fig 3(c)

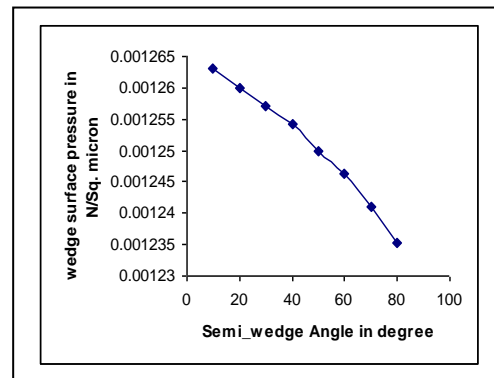


Fig 3(d)

Fig 3: Variation of different parameters with the change in wedge angle for indentation of 10 μ m

IV. CONCLUSION

The present works gives similar trends of results with the previous works in this field up to 45°. It has been mentioned earlier, why the results do not match with Haddow's paradox beyond 45°. The reasons have been detected. Further, experimentations are needed as in the work [16]. Proper experimentations would lead the nearest value of f and may be tallied with the theory of micro-plastic indentation.

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