

# New Approaches to Find the Solution for the Intuitionistic Fuzzy Transportation Problem with Ranking of Intuitionistic Fuzzy Numbers

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**ABSTRACT:** In this paper, we investigate Intuitionistic Fuzzy Transportation Problem with the Trapezoidal Intuitionistic fuzzy numbers. Methods are proposed to find the initial solution of Intuitionistic Fuzzy Transportation Problem and a method to find the optimal solution of Intuitionistic Fuzzy Transportation Problem is developed. Ranking method based on the magnitude of membership function and non-membership function of a Intuitionistic Fuzzy Number is utilized to order the Intuitionistic fuzzy numbers. Numerical example is provided to illustrate the new approaches.

**KEYWORDS:** Intuitionistic Fuzzy numbers, Trapezoidal Intuitionistic Fuzzy Numbers, Ranking of Intuitionistic fuzzy numbers, Intuitionistic fuzzy Vogel's approximation method, Intuitionistic fuzzy Hungarian approach, Intuitionistic fuzzy modified distribution method.

## I. INTRODUCTION

The Transportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh (1970) and Zadeh(1965) introduced the notion of fuzziness.

Fuzzy transportation problem is a transportation problem whose decision parameters are fuzzy numbers. Lai and Hwang (1992) used tabular method for solving Fuzzy transportation problem by means of a crisp parametric programming problem. Stephen Dinagar (2009) investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan (2010) proposed an algorithm namely fuzzy zero point method for finding an optimal solution to a fuzzy transportation problem.

The concept of Intuitionistic Fuzzy Set (IFS) (Atanassov (1986,1989)) can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. The IFSs were first introduced by Atanassov (1986) which is a generalization of the concept of fuzzy set Zadeh(1965). In this paper, Intuitionistic fuzzy Vogel's approximation method and Intuitionistic fuzzy Hungarian approach are proposed to find the initial solution of Intuitionistic Fuzzy Transportation Problem (IFTP) and a method, Intuitionistic fuzzy modified distribution is proposed to find the optimal solution of IFTP. Ranking method based on the magnitude of membership function and non-membership function of a Intuitionistic Fuzzy Number(IFN) is utilized to order the IFNs. Numerical example is provided to illustrate the new approaches.

## II. PRELIMINARIES

In this section, some basic concepts related to IFS are discussed. (Atanassov (1986, 1989) , Hassan Mishmast Nehi (2010))

Let  $X$  be a classical set of objects called the universal set, where an element of  $X$  is denoted by  $x$ .

**Definition1:** An IFS  $A$  in  $X$  is given by  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  where the functions  $\mu_A, \nu_A : X \rightarrow [0,1]$  are functions such that  $0 \leq \mu(x) + \nu(x) \leq 1 \forall x \in X$ . For each  $x$  the numbers  $\mu_A(x)$  and  $\nu_A(x)$  represent the degree of membership and degree of non-membership of the element  $x \in X$  to the set, which is a subset of  $X$ , respectively.

**Definition2:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  is called IF- normal, if there exists at least two points  $x_0, x_1 \in X$  such that  $\mu_A(x_0) = 1, \nu_A(x_1) = 1$ .

**Definition3:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  of the real line is called IF- convex if  $\forall x_1, x_2 \in R, \forall \lambda \in [0,1]$ ,  $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$  and  $\nu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \nu_A(x_1) \wedge \nu_A(x_2)$ . Thus  $A$  is IF-convex if its membership function is fuzzy convex and its non membership function is fuzzy concave.

**Definition4:** An IFS  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  of the real line is called an Intuitionistic Fuzzy Number (IFN) if (a)  $A$  is IF-normal, (b)  $A$  is IF-convex, (c)  $\mu_A$  is upper semi continuous and  $\nu_A$  is lower semi continuous, (d)  $A = \{x \in X / \nu_A(x) < 1\}$  is bounded.

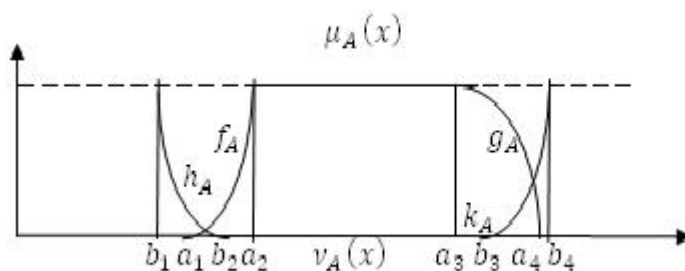


Fig 1 Intuitionistic Fuzzy Number

For any IFN  $A$  there exists eight numbers  $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$  such that  $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$  and four functions  $f_A, g_A, h_A, k_A : R \rightarrow [0,1]$ , called the sides of a fuzzy number, where  $f_A$  and  $k_A$  are nondecreasing and  $g_A$  and  $h_A$  are non increasing, such that we can describe a membership function  $\mu_A$  and non membership  $\nu_A$  in the following form

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ f_A(x) & \text{if } a_1 < x < a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ g_A(x) & \text{if } a_3 < x < a_4 \\ 0 & \text{if } a_4 \leq x \end{cases} \text{ and } \nu_A(x) = \begin{cases} 1 & \text{if } x \leq b_1 \\ h_A(x) & \text{if } b_1 < x < b_2 \\ 0 & \text{if } b_2 \leq x \leq b_3 \\ k_A(x) & \text{if } b_3 < x < b_4 \\ 1 & \text{if } b_4 \leq x \end{cases}$$

**Definition 5:**  $A$  is a Trapezoidal Intuitionistic Fuzzy Number (TIFN) with parameters  $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$  and denoted by

$$A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$$

In this case,

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 < x < a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4} & \text{if } a_3 < x < a_4 \\ 0 & \text{if } a_4 \leq x \end{cases} \text{ and } \nu_A(x) = \begin{cases} 1 & \text{if } x \leq b_1 \\ \frac{x-b_1}{b_2-b_1} & \text{if } b_1 < x < b_2 \\ 0 & \text{if } b_2 \leq x \leq b_3 \\ \frac{x-b_4}{b_3-b_4} & \text{if } b_3 < x < b_4 \\ 1 & \text{if } b_4 \leq x \end{cases}$$

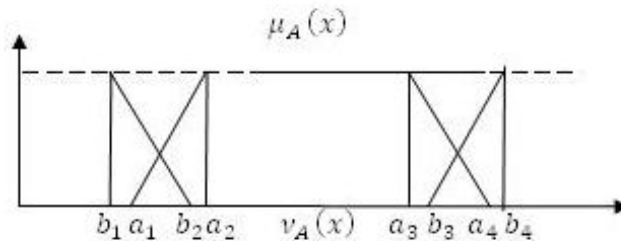


Fig. 2 Trapezoidal Intuitionistic Fuzzy Number

If in a TIFN  $A$ , we let  $b_2 = b_3$  (and hence  $a_2 = a_3$ ) then we will give a Triangular Intuitionistic Fuzzy Number (TriFN) with parameters  $b_1 \leq a_1 \leq b_2$  ( $a_2 = a_3 = b_3$ )  $\leq a_4 \leq b_4$  and denoted by  $A = (b_1, a_1, b_2, a_4, b_4)$ .

### III. ARITHMETIC OPERATIONS OF TIFNS

The Arithmetic Operations between two TIFNs defined on universal set of real numbers  $R$ , are reviewed as in Atanassov (1986, 1989), Nehi (2010), AmitKumar(2013)

(i) Let  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  and  $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$  be two TIFNs, then

$$A \oplus B = (b_1 + b'_1, a_1 + a'_1, b_2 + b'_2, a_2 + a'_2, a_3 + a'_3, b_3 + b'_3, a_4 + a'_4, b_4 + b'_4)$$

(ii) Let  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  and  $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$  be two TIFNs, then

$$A \ominus B = (b_1 - b'_4, a_1 - a'_4, b_2 - b'_3, a_2 - a'_3, a_3 - a'_2, b_3 - b'_2, a_4 - a'_1, b_4 - b'_1)$$

(iii) Let  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  be a TIFN and  $r$  be a real number then

$$rA = \begin{cases} (rb_1, ra_1, rb_2, ra_2, ra_3, rb_3, ra_4, rb_4) & \text{if } r > 0 \\ (rb_4, ra_4, rb_3, ra_3, ra_2, rb_2, ra_1, rb_1) & \text{if } r < 0 \end{cases}$$

(iv) Let  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  and  $B = (b'_1, a'_1, b'_2, a'_2, a'_3, b'_3, a'_4, b'_4)$  be two non-negative TIFNs, then

$$A \otimes B = (b_1 b'_1, a_1 a'_1, b_2 b'_2, a_2 a'_2, a_3 a'_3, b_3 b'_3, a_4 a'_4, b_4 b'_4)$$

### IV. RANKING OF INTUITIONISTIC FUZZY NUMBERS

Ranking of IFNs draws the attention of several researchers. Nehi (2010) ranked IFNs based on characteristic values of membership and non – membership functions of IFN.

In this paper, based on the magnitude of a fuzzy number introduced in Abbasbandy & Hajjari(2009), an ordering method for Intuitionistic fuzzy numbers proposed in Sagaya & Henry (2013) is used to order the Intuitionistic fuzzy numbers and it is as follows.

For an arbitrary IFN  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ , the magnitude of membership and non-membership function for IFN  $A$  denoted by  $\text{Mag}(A_\mu)$ ,  $\text{Mag}(A_\nu)$  are defined respectively, as  $\text{Mag}(A_\mu) = \frac{1}{2} \left( \int_0^1 (f_A^{-1}(r) + g_A^{-1}(r) + a_2 + a_3) f(r) dr \right)$  and  $\text{Mag}(A_\nu) = \frac{1}{2} \left( \int_0^1 (h_A^{-1}(r) + k_A^{-1}(r) + b_2 + b_3) f(r) dr \right)$  where the function  $f(r)$  is a regular reducing function on  $[0,1]$  with  $f(0)=0$ ,  $f(1)=1$  and  $\int_0^1 f(r) dr = \frac{1}{2}$ . Obviously function  $f(r)$  can be considered as a weighting function. In actual applications, function  $f(r)$  can be chosen according to the actual situation. In this chapter we use  $f(r) = r$ . In particular, let  $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$  be a TIFN.

$$\text{Then } \text{Mag}(A_\mu) = \frac{1}{12} (a_1 + 5a_2 + 5a_3 + a_4) \text{ and } \text{Mag}(A_\nu) = \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4)$$

And the algorithm of ordering is as follows

#### Algorithm

As a ranking method, two TIFNs  $A$  and  $B$  are compared using the following steps :

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**Step1** : Compute  $\text{Mag}(A_\mu)$  and  $\text{Mag}(B_\mu)$ . Then compare them. If they are equal then go to the step 2. Otherwise rank  $A$  and  $B$  according to the relative position of  $\text{Mag}(A_\mu)$  and  $\text{Mag}(B_\mu)$ .

**Step2** : Compute  $\text{Mag}(A_\gamma)$  and  $\text{Mag}(B_\gamma)$ . Then compare them. If they are equal then  $A$  and  $B$  are equal. Otherwise rank  $A$  and  $B$  according to the relative position of  $-\text{Mag}(A_\gamma)$  and  $-\text{Mag}(B_\gamma)$ .

Therefore for any two TIFNs  $A$  and  $B$ , we define the ranking of  $A$  and  $B$  by the  $\text{Mag}(\cdot)$  as follows:

- (i)  $\text{Mag}(A_\mu) > \text{Mag}(B_\mu)$  if and only if  $A > B$ ,
- (ii)  $\text{Mag}(A_\mu) < \text{Mag}(B_\mu)$  if and only if  $A < B$ ,
- (iii)  $\text{Mag}(A_\mu) = \text{Mag}(B_\mu)$  and  $\text{Mag}(A_\gamma) = \text{Mag}(B_\gamma)$  if and only if  $A \sim B$ ,
- (iv)  $\text{Mag}(A_\mu) = \text{Mag}(B_\mu)$  and  $-\text{Mag}(A_\gamma) > -\text{Mag}(B_\gamma)$  if and only if  $A > B$ .
- (v)  $\text{Mag}(A_\mu) = \text{Mag}(B_\mu)$  and  $-\text{Mag}(A_\gamma) < -\text{Mag}(B_\gamma)$  if and only if  $A < B$ .

Then the order  $\geq$  and  $\leq$  is formulated as  $A \geq B$  if and only if  $A > B$  or  $A \sim B$ ,  $A \leq B$  if and only if  $A < B$  or  $A \sim B$ .

### V. THE INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM

In this section, mathematical model for IFTP is given as follows

Consider a transportation problem with  $m$  Intuitionistic fuzzy origins (rows) and  $n$  Intuitionistic fuzzy destinations (columns).

Let  $\tilde{C}_{ij} = (c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)}, c_{ij}^{(7)}, c_{ij}^{(8)})$  be the Intuitionistic fuzzy cost of transporting one unit of the product from  $i^{\text{th}}$  Intuitionistic fuzzy origin to  $j^{\text{th}}$  Intuitionistic fuzzy destination.

$\tilde{a}_i = (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)})$  be the Intuitionistic fuzzy quantity of commodity available at Intuitionistic fuzzy origin  $i$ .

$\tilde{b}_j = (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$  be the Intuitionistic fuzzy quantity of commodity needed at Intuitionistic fuzzy destination  $j$ .

$\tilde{X}_{ij} = (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, x_{ij}^{(5)}, x_{ij}^{(6)}, x_{ij}^{(7)}, x_{ij}^{(8)})$  is the Intuitionistic fuzzy quantity transported from  $i^{\text{th}}$  Intuitionistic fuzzy origin to  $j^{\text{th}}$  Intuitionistic fuzzy destination.

The IFTP can be stated in the tabular form as

	IFD <sub>1</sub>	IFD <sub>2</sub>	...	IFD <sub>n</sub>	IFC
IFO <sub>1</sub>	$\tilde{X}_{11}$	$\tilde{X}_{12}$	...	$\tilde{X}_{1n}$	$\tilde{a}_1$
	$\tilde{C}_{11}$	$\tilde{C}_{12}$	...	$\tilde{C}_{1n}$	
IFO <sub>2</sub>	$\tilde{X}_{21}$	$\tilde{X}_{22}$	...	$\tilde{X}_{2n}$	$\tilde{a}_2$
	$\tilde{C}_{21}$	$\tilde{C}_{22}$	...	$\tilde{C}_{2n}$	
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
IFO <sub>m</sub>	$\tilde{X}_{m1}$	$\tilde{X}_{m2}$	...	$\tilde{X}_{mn}$	$\tilde{a}_m$
	$\tilde{C}_{m1}$	$\tilde{C}_{m2}$	...	$\tilde{C}_{mn}$	
IFD	$\tilde{b}_1$	$\tilde{b}_2$	...	$\tilde{b}_n$	

IFO <sub>$i$</sub>  ( $i=1,2,\dots,m$ ) – Intuitionistic Fuzzy Origin, IFD <sub>$j$</sub>  ( $j=1,2,\dots,n$ ) – Intuitionistic Fuzzy Destination, IFC – Intuitionistic Fuzzy Capacity, IFD – Intuitionistic Fuzzy Demand.

Mathematically, the problem may be stated as a linear programming problem as follows

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n (c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)}, c_{ij}^{(7)}, c_{ij}^{(8)}) (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, x_{ij}^{(5)}, x_{ij}^{(6)}, x_{ij}^{(7)}, x_{ij}^{(8)})$$

Subject to the Constraints

$$\sum_{j=1}^n (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, x_{ij}^{(5)}, x_{ij}^{(6)}, x_{ij}^{(7)}, x_{ij}^{(8)}) = (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}), i = 1, 2, \dots, m$$

$$\sum_{i=1}^m (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, x_{ij}^{(5)}, x_{ij}^{(6)}, x_{ij}^{(7)}, x_{ij}^{(8)}) = (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)}), j = 1, 2, \dots, n$$

$$(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}, x_{ij}^{(5)}, x_{ij}^{(6)}, x_{ij}^{(7)}, x_{ij}^{(8)}) \geq 0, \text{ for all } i \text{ and } j$$

The given Intuitionistic fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) = \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$$

(ie) if the total Intuitionistic fuzzy capacity is equal to the total Intuitionistic fuzzy demand.

The IFTP can be solved in two stages (i) finding an initial basic feasible solution and then (ii) attaining an optimum solution.

## VI. METHODS TO FIND THE INITIAL SOLUTION OF IFTP

There are numerous methods for finding the fuzzy initial basic feasible solution of a transportation problem. In this section, Intuitionistic Fuzzy Vogel's Approximation is developed and Intuitionistic Fuzzy Hungarian Approach is proposed to find the Initial solution of IFTP.

### (1) Intuitionistic Fuzzy Vogel's Approximation Method

To find the initial basic feasible solution of IFTP, Intuitionistic Fuzzy Vogel's Approximation Method is developed as follows

**Step1:** Set up the given IFTP in the tabular form.

**Step2:** Test whether the given IFTP is balanced or not.

If it is a balanced one (ie the sum of the destinations is equal to the sum of the origins) then go to step 3. Otherwise,

$$\text{If } \sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) > \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$$

then introduce a dummy destination in the Intuitionistic fuzzy transportation table. The cost of transporting to this destination are all set equal to Intuitionistic fuzzy zero and the requirement at this dummy destination is then assumed

to be equal to

$$\sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) - \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$$

If  $\sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) < \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$  introduce a dummy origin in the Intuitionistic fuzzy transportation table. The cost of transporting from this origin to any destination are all set equal to Intuitionistic fuzzy zero and the availability at this dummy origin is then assumed to be equal to

$$\sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)}) - \sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)})$$

**Step3 :** Find the rank of each cell  $\tilde{C}_{ij}$  by using the ranking method and determine the minimum and the next minimum Intuitionistic fuzzy costs in each row and column and then find the Intuitionistic fuzzy penalty cost, namely the Intuitionistic fuzzy difference between the minimum and next minimum Intuitionistic fuzzy costs in each row and column.

**Step4:** Choose the Intuitionistic fuzzy maximum penalty by ranking method

**Step5:** In the selected row or column as by step 4, find out the cell having the least Intuitionistic fuzzy cost. Allocate to this cell the maximum feasible amount depending on the Intuitionistic fuzzy capacity and Intuitionistic fuzzy demands. Delete the row or column which is fully exhausted.

**Step6:** Recompute the column and row Intuitionistic fuzzy penalties for the reduced IFTP and then go to step 3, repeat the procedure until all the rim requirements are satisfied

### (2) Intuitionistic Fuzzy Hungarian Approach

The steps of Intuitionistic Fuzzy Hungarian approach to find the initial solution of IFTP is as follows

**Step1:** Set up the given IFTP in the tabular form.

**Step2:** Test whether the given IFTP is balanced or not.

If it is a balanced one (ie the sum of the destinations is equal to the sum of the origins) then go to step 3. Otherwise,

$$\text{If } \sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) > \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$$

then introduce a dummy destination in the Intuitionistic fuzzy transportation table. The cost of transporting to this destination are all set equal to Intuitionistic fuzzy zero and the requirement at this dummy destination is then assumed to be equal to

$$\sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) - \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$$

If  $\sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)}) < \sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)})$  introduce a dummy origin in the Intuitionistic fuzzy transportation table. The cost of transporting from this origin to any destination are all set equal to Intuitionistic fuzzy zero and the availability at this dummy origin is then assumed to be equal to

$$\sum_{j=1}^n (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)}, b_j^{(7)}, b_j^{(8)}) - \sum_{i=1}^m (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)}, a_i^{(7)}, a_i^{(8)})$$

**Step3:** Find the row minimum by using the ranking and then subtract the row minimum from each row entry of that row.

**Step4:** Find the column minimum by using the ranking and then subtract the column minimum of the resulting IFTP from each column entry of that column. Each column and row now have at least one Intuitionistic fuzzy zero/(ranking zero).

**Step5:** Check whether each column Intuitionistic fuzzy demand is lesser than the sum of Intuitionistic fuzzy supply whose reduced Intuitionistic fuzzy cost in that column are Intuitionistic fuzzy zero. Also check whether each row Intuitionistic fuzzy supply is lesser than the sum of the column Intuitionistic fuzzy demands whose reduced costs in that row are Intuitionistic fuzzy zero. If so go to step 7 otherwise go to step 6.

**Step6:** Draw minimum number of horizontal and vertical lines to cover all Intuitionistic fuzzy zeros. Find the smallest Intuitionistic fuzzy cost not covered by any line and subtract it from all uncovered Intuitionistic fuzzy costs and add the same to all Intuitionistic fuzzy costs lying at the intersection of any two lines. Repeat this step till Intuitionistic fuzzy supply satisfies Intuitionistic fuzzy demand for all rows and columns.

**Step7:** Allocate the maximum quantity to be transported where the Intuitionistic fuzzy costs have been Intuitionistic fuzzy zero depending on the Intuitionistic fuzzy demand and Intuitionistic fuzzy supply.

**Step8:** Repeat the procedure till all Intuitionistic fuzzy supply and Intuitionistic fuzzy demand quantities are exhausted.

## VII. INTUITIONISTIC FUZZY MODIFIED DISTRIBUTION METHOD

Intuitionistic Fuzzy optimality test is applied to Intuitionistic fuzzy initial basic feasible solution having (m+n-1) occupied cells where m is the number of Intuitionistic fuzzy origins and n is the number of Intuitionistic fuzzy destinations. This proposed method is used for finding the optimal basic feasible solution in fuzzy environment and the following step by step procedure is utilized to find out the same.

**Step1:** For each occupied cell, find out a set of numbers,  $(u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, u_i^{(5)}, u_i^{(6)}, u_i^{(7)}, u_i^{(8)})$  and  $(v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, v_j^{(5)}, v_j^{(6)}, v_j^{(7)}, v_j^{(8)})$  for each row and column satisfying

$$(u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, u_i^{(5)}, u_i^{(6)}, u_i^{(7)}, u_i^{(8)}) + (v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, v_j^{(5)}, v_j^{(6)}, v_j^{(7)}, v_j^{(8)}) = (c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)}, c_{ij}^{(7)}, c_{ij}^{(8)})$$

To start with assign a Intuitionistic fuzzy zero to any row or column having maximum number of allocations. If this maximum number of allocation is more than once, choose any one arbitrary.

**Step2 :** Compute the net evaluations  $(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, z_{ij}^{(5)}, z_{ij}^{(6)}, z_{ij}^{(7)}, z_{ij}^{(8)}) =$

$$\left( (u_i^{(1)}, u_i^{(2)}, u_i^{(3)}, u_i^{(4)}, u_i^{(5)}, u_i^{(6)}, u_i^{(7)}, u_i^{(8)}) + (v_j^{(1)}, v_j^{(2)}, v_j^{(3)}, v_j^{(4)}, v_j^{(5)}, v_j^{(6)}, v_j^{(7)}, v_j^{(8)}) \right) - (c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)}, c_{ij}^{(7)}, c_{ij}^{(8)})$$

for all unoccupied basic cells. If all



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$(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, z_{ij}^{(5)}, z_{ij}^{(6)}, z_{ij}^{(7)}, z_{ij}^{(8)}) < (-1\lambda, -\frac{3}{4}\lambda, -\frac{1}{2}\lambda, -\frac{1}{4}\lambda, \frac{1}{4}\lambda, \frac{1}{2}\lambda, \frac{3}{4}\lambda, 1\lambda)$ ,  $\lambda \in [0,1]$ , then the current basic feasible solution is optimal and a unique solution exists.

If all  $(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, z_{ij}^{(5)}, z_{ij}^{(6)}, z_{ij}^{(7)}, z_{ij}^{(8)}) \leq (-1\lambda, -\frac{3}{4}\lambda, -\frac{1}{2}\lambda, -\frac{1}{4}\lambda, \frac{1}{4}\lambda, \frac{1}{2}\lambda, \frac{3}{4}\lambda, 1\lambda)$ ,  $\lambda \in [0,1]$ , then the current basic feasible solution is optimal but an alternate solution exists.

If atleast one

$(z_{ij}^{(1)}, z_{ij}^{(2)}, z_{ij}^{(3)}, z_{ij}^{(4)}, z_{ij}^{(5)}, z_{ij}^{(6)}, z_{ij}^{(7)}, z_{ij}^{(8)}) > (-1\lambda, -\frac{3}{4}\lambda, -\frac{1}{2}\lambda, -\frac{1}{4}\lambda, \frac{1}{4}\lambda, \frac{1}{2}\lambda, \frac{3}{4}\lambda, 1\lambda)$ ,  $\lambda \in [0,1]$ , select the unoccupied cell having most positive net evaluation to enter the basis and then go to next step.

**Step3** : From that unoccupied cell (having most positive net evaluation ) draw a loop starting and ending at that unoccupied cell with corner cell as occupied cell. Assign sign + and - alternatively starting from that unoccupied cell and find the Intuitionistic fuzzy minimum allocation from the cells having negative sign. This allocation should be added to the allocation of the cell with + sign and subtracted from the allocation of the cell with - sign.

**Step4** : Step3 gives a better solution by making one or more occupied cell as unoccupied and one unoccupied cell as occupied. For this new set of Intuitionistic fuzzy basic feasible allocation repeat from the step1 till a Intuitionistic fuzzy optimal feasible solution is obtained.

### VIII. NUMERICAL EXAMPLE

Consider a IFTP with rows representing three Intuitionistic fuzzy origins IFO<sub>1</sub> IFO<sub>2</sub> IFO<sub>3</sub> and columns representing four destinations IFD<sub>1</sub>, IFD<sub>2</sub>, IFD<sub>3</sub>, IFD<sub>4</sub>.

	IFD <sub>1</sub>	IFD <sub>2</sub>	IFD <sub>3</sub>	IFD <sub>4</sub>	IFC
IFO <sub>1</sub>	(1,2,4,5,7,8,10,12)	(2,3,5,6,9,10,12,14)	(1,3,5,6,9,10,11,13)	(3,4,5,6,8,9,11,12)	(2,4,5,6,10,11,14,15)
IFO <sub>2</sub>	(3,4,6,7,9,11,13,15)	(1,2,4,6,10,12,14,16)	(2,3,5,7,10,12,14,15)	(1,4,5,7,10,12,13,15)	(4,5,8,9,12,14,16,18)
IFO <sub>3</sub>	(2,4,6,8,10,11,12,14)	(2,3,4,5,8,9,10,11)	(3,4,5,7,11,12,13,14)	(1,3,4,6,9,10,11,14)	(6,8,11,15,20,23,27,33)
IFD	(4,5,7,8,11,13,15,16)	(3,4,6,8,12,13,14,18)	(2,3,5,6,9,10,14,16)	(3,5,6,8,10,12,14,16)	

Since  $\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j = (12,17,24,30,42,48,57,66)$ , the problem is balanced IFTP.

(1) Intuitionistic Fuzzy Vogel's approximation method

Using this method, the initial Intuitionistic fuzzy basic feasible solution is obtained as follows

	IFD <sub>1</sub>	IFD <sub>2</sub>	IFD <sub>3</sub>	IFD <sub>4</sub>	IFC
IFO <sub>1</sub>	(2,4,5,6,10,11,14,15)				(2,4,5,6,10,11,14,15)
IFO <sub>2</sub>	(1,2,4,5,7,8,10,12)	(2,3,5,6,9,10,12,14)	(1,3,5,6,9,10,11,13)	(3,4,5,6,8,9,11,12)	(4,5,8,9,12,14,16,18)
IFO <sub>3</sub>	(-11,-9,-4,-2,5,8,11,14)		(2,3,5,6,9,10,14,16)	(-27,-18,-11,-4,7,14,20,28)	(6,8,11,15,20,23,27,33)
IFD	(4,5,7,8,11,13,15,16)	(3,4,6,8,12,13,14,18)	(2,3,5,6,9,10,14,16)	(3,5,6,8,10,12,14,16)	

The initial Intuitionistic fuzzy transportation cost is

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$$(Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}, Z^{(5)}, Z^{(6)}, Z^{(7)}, Z^{(8)}) = (-60, -97, -18, 88, 479, 751, 1132, 1668)$$

Using Intuitionistic fuzzy modified distribution method, it is found that the above Intuitionistic fuzzy basic feasible solution is optimal.

### (2) Intuitionistic Fuzzy Hungarian Approach

Using this method, the initial Intuitionistic fuzzy basic feasible solution is obtained as follows

Using step 3 and step 4 of the proposed method the following table is obtained

$(-22, -16, -8, -4, 8, 16, 22)$	$(-25, -19, -11, -5, 8, 14, 22, 28)$	$(-25, -19, -11, -5, 7, 13, 20, 26)$	$(-23, -17, -11, -5, 6, 12, 19, 26)$
$(-24, -18, -10, -5, 5, 11, 19, 25)$	$(-30, -24, -16, -8, 8, 16, 24, 30)$	$(-28, -23, -15, -7, 7, 15, 23, 28)$	$(-29, -21, -15, -7, 7, 15, 21, 29)$
$(-20, -14, -7, -2, 7, 11, 17, 23)$	$(-23, -19, -13, -7, 7, 13, 19, 24)$	$(-22, -18, -12, -5, 9, 15, 21, 26)$	$(-24, -18, -13, -6, 7, 13, 18, 27)$

Since step 5 is not satisfied, draw minimum number of horizontal and vertical lines to cover all the Trapezoidal Intuitionistic fuzzy zeros.

<del><math>(-22, -16, -8, -4, 8, 16, 22)</math></del>	<del><math>(-25, -19, -11, -5, 8, 14, 22, 28)</math></del>	<del><math>(-25, -19, -11, -5, 7, 13, 20, 26)</math></del>	<del><math>(-23, -17, -11, -5, 6, 12, 19, 26)</math></del>
<del><math>(-24, -18, -10, -5, 5, 11, 19, 25)</math></del>	<del><math>(-30, -24, -16, -8, 8, 16, 24, 30)</math></del>	<del><math>(-28, -23, -15, -7, 7, 15, 23, 28)</math></del>	<del><math>(-29, -21, -15, -7, 7, 15, 21, 29)</math></del>
$(-20, -14, -7, -2, 7, 11, 17, 23)$	$(-23, -19, -13, -7, 7, 13, 19, 24)$	$(-22, -18, -12, -5, 9, 15, 21, 26)$	$(-24, -18, -13, -6, 7, 13, 18, 27)$

Subtracting  $(-24, -18, -13, -6, 7, 13, 18, 27)$ , the smallest trapezoidal Intuitionistic fuzzy cost not covered by any line from all uncovered trapezoidal Intuitionistic fuzzy costs and adding the same to all Intuitionistic fuzzy costs lying at the intersection of any two lines, the following table is obtained.

$(-22, -16, -8, -4, 8, 16, 22)$	$(-49, -37, -24, -11, 15, 27, 40, 55)$	$(-25, -19, -11, -5, 7, 13, 20, 26)$	$(-23, -17, -11, -5, 6, 12, 19, 26)$
$(-24, -18, -10, -5, 5, 11, 19, 25)$	$(-54, -42, -29, -14, 15, 29, 42, 57)$	$(-28, -23, -15, -7, 7, 15, 23, 28)$	$(-29, -21, -15, -7, 7, 15, 21, 29)$
$(-47, -32, -20, -9, 13, 24, 35, 47)$	$(-23, -19, -13, -7, 7, 13, 19, 24)$	$(-49, -36, -25, -12, 15, 28, 39, 50)$	$(-51, -36, -26, -13, 13, 26, 36, 51)$

Repeating the steps 5 and 6 till trapezoidal Intuitionistic fuzzy supply satisfies trapezoidal Intuitionistic fuzzy demand for all rows and columns and applying steps 7 and 8, the following initial trapezoidal Intuitionistic fuzzy transportation table is obtained.



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	IFD <sub>1</sub>	IFD <sub>2</sub>	IFD <sub>3</sub>	IFD <sub>4</sub>	IFC
IFO <sub>1</sub>	(2,45,6,10,11,14,15) (-22,-16,-8,-4,48,16,22)	(-122,-91,-57,-27,31,61,95,129)	(-98,-73,-44,-21,23,47,75,100)	(-96,-71,-44,-21,22,46,74,100)	(2,45,6,10,11,14,15)
IFO <sub>2</sub>	(-11,-9,-4,-2,5,8,11,14) (-98,-73,-44,-21,21,44,73,98)	(-54,-42,-29,-14,15,29,42,57)	(2,35,69,10,14,16) (-28,-23,-15,-7,7,15,23,28)	(-27,-18,-11,-4,7,14,20,28) (-29,-21,-15,-7,7,15,21,29)	(4,5,8,9,12,14,16,18)
IFO <sub>3</sub>	(-121,-87,-54,-25,29,57,89,120)	(3,4,6,8,12,13,14,18) (-23,-19,-13,-7,7,13,19,24)	(-49,-36,-25,-12,15,18,39,50)	(-12,-6,-2,3,12,17,23,30) (-51,-36,-26,-13,13,26,36,51)	(6,8,11,15,20,23,27,33)
IFD	(4,5,7,8,11,13,15,16)	(3,4,6,8,12,13,14,18)	(2,3,5,6,9,10,14,16)	(3,5,6,8,10,12,14,16)	

And the initial Intuitionistic trapezoidal fuzzy transportation minimum cost is  $(Z^{(1)}, Z^{(2)}, Z^{(3)}, Z^{(4)}, Z^{(5)}, Z^{(6)}, Z^{(7)}, Z^{(8)}) = (-60, -97, -18, 88, 479, 751, 1132, 1668)$

Since the number of occupied cells is 6, there exists a non degenerate trapezoidal Intuitionistic fuzzy basic feasible solution.

Using Intuitionistic fuzzy modified distribution method, it is found that the above Intuitionistic fuzzy basic feasible solution is optimum.

## IX. CONCLUSION

Here, IFTP with cost, demand and supply as TIFN is considered. Ranking method based on magnitude of membership function and non-membership function of IFN is used to order the TIFNs. Methods, Intuitionistic Fuzzy Vogel's Approximation and Intuitionistic Fuzzy Hungarian are proposed to find the initial solution of IFTP and Intuitionistic Fuzzy Modified Distribution Method is proposed to find the optimal solution. Finally a numerical example is provided to discuss the methods and one can apply these approaches to solve any IFTP

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