

Scalar Field in Stationary Rotating String World Models

Dr. Lakshmi S. Desai

Department of Mathematics, Faculty of Technology, Dharmsinh Desai University, Nadiad, Gujarat, India

ABSTRACT: In this paper, rotating string world models with a space-dependent scalar field are studied considering a Gödel type line element. A number of exact solutions of Einstein field equations corresponding to energy momentum tensor are obtained. The physical plausibility of the solutions is also discussed. In absence of scalar field, the solution reduces to those discussed by Patel and Dadhich [1] in connection with stationary rotating string world models. Gödel [2] Universe is derived as a particular case.

KEYWORDS: Cosmology, Gödel Universe, Strings, Scalar field

I. INTRODUCTION

Many researchers have taken keen interest in the study of cosmic strings. The present day observations of the Universe do not rule out the possibility of the existence of large scale networks of strings in the early Universe Kibble [3]. Ogawa et. al [4] obtained all solutions describing stationary rotating strings in flat spacetime. It is believed that cosmic strings give rise to density perturbations which lead to galaxy formations Zeldovich [5]. Strings possess stress-energy and hence couple to gravitational field. Detailed accounts of various features of cosmic strings are given by Vilenkin [6], Gott [7], and Garfinkle [8].

Cosmic strings as source of gravitational field in general relativity was discussed by Letelier [9] and Stachel [10]. Letelier [11] has further constructed string cosmological models for Bianchi I and Kantowski-Sachs space-times by introducing the energy momentum tensor as the source term in Einstein's equations.

$$M_{ik} = \rho v_i v_k - \lambda \omega_i \omega_k, v_i v^k = 1 = -\omega^i \omega_i, v^i \omega_i = 0. \quad 1.1$$

Here ρ denotes the density of a cloud of strings with particles attached to them and λ is the string tension density. The unit time-like vector v^i is the flow vector and the unit space-like vector ω^i specifies the direction of the strings. The particle density associated with the configuration is given by,

$$\rho_p = \rho - \lambda \quad 1.2$$

The energy conditions imply $\rho \geq 0, \rho_p \geq 0$ leaving the sign of the string density λ unrestricted. The energy momentum tensor for a scalar field ϕ is given by,

$$N_{ik} = \phi_i \phi_k - \left(\frac{1}{2}\right) g_{ik} \phi_a \phi^a \quad 1.3$$

where $\phi_i = \partial\phi/\partial x^i$ and ϕ satisfies the field equation

$$\phi^i_{;i} = 0 \quad 1.4$$

Here, semicolon indicates covariant differentiation. In the present paper Einstein field equations are used in the form,

$$R_{ik} = \left(\frac{1}{2}\right) R g_{ik} = -8\pi [M_{ik} + N_{ik}] - \Lambda g_{ik} \quad 1.5$$

where Λ is the cosmological constant.

Note that the rotation plays an important role in the structure and equilibrium configurations of elementary particles, and also in astrophysical bodies. For example the nature of pulsars, which are widely accepted as rotating neutron stars, can only be properly described by including the effects of rotation. During the last three decades many researchers have

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

introduced rotation in the general theory of relativity in the discussions of rotating cosmological models. Rotation is likely to significantly affect the behavior of the gravitational field in the early Universe. As an attempt to model the early Universe in terms of network of strings, rotation should be included in the analysis to determine its effect on the field.

Patel and Dadhich [1] have obtained one parameter class of stationary rotating string cosmological models of which the well known Gödel Universe Gödel [2] is a particular case. In their models, the rotation hinges on the cosmological constant turns out to be negative. Patel and Maharaj [12] have presented some new physically relevant stationary rotating solutions of string cosmology with an axial magnetic field. The purpose of the present investigation is to introduce stationary scalar field in the string models discussed by Patel and Dadhich [1].

II. THE METRIC AND THE FIELD EQUATIONS

Consider a Gödel type space-time described by the metric,

$$ds^2 = (dt + H dy)^2 - \alpha^2 dy^2 - dx^2 - dz^2 \quad 2.1$$

where H and α are functions of x only.

Here, the orthonormal tetrad is introduced as,

$$\theta^1 = dx, \theta^2 = \alpha dy, \theta^3 = dz, \theta^4 = dt + Hdy \quad 2.2$$

to write the metric in the form,

$$ds^2 = (\theta^4)^2 - (\theta^1)^2 - (\theta^2)^2 - (\theta^3)^2 = g_{(ab)} \theta^a \theta^b \quad 2.3$$

Here and in what follows the bracketed indices indicate tetrad components. Thus $g_{(ab)} = \text{diag}\{-1, -1, -1, 1\}$ are tetrad components of the metric tensor g_{ik} . Using Cartan's equations of structure one can easily obtain tetrad component for the metric $R_{(ab)}$ of Ricci tensor for the metric 2.1. The non-vanishing $R_{(ab)}$ are listed below for ready reference.

$$R_{(11)} = R_{(22)} = \frac{\alpha''}{\alpha} - \frac{H'^2}{2\alpha^2}, \quad R_{(44)} = \frac{-H'^2}{2\alpha^2}, \quad R_{(24)} = \frac{-1}{2\alpha} \left\{ H'' - \frac{H' \alpha'}{\alpha} \right\} \quad 2.4$$

Here and in what follows a prime indicates differentiation with respect to x.

It is easy to see that the field equations 1.5 can be expressed in the tetrad basis as,

$$R_{(ab)} = -8\pi [\rho v_{(a)} v_{(b)} - \lambda \omega_{(a)} \omega_{(b)}] + 4\pi (\rho + \lambda) g_{(ab)} - 8\pi \phi_{(a)} \phi_{(b)} + \Lambda g_{(ab)} \quad 2.5$$

Comoving coordinates are used and the string direction is taken as z –direction. Therefore,

$$v_{(a)} = (0, 0, 0, 1), \quad \omega_{(a)} = (0, 0, 1, 0) \quad 2.6$$

It is easy to check that v_i and ω_i given by 2.6 satisfy the conditions given in 1.1.

It is assumed that the zero mass scalar field ϕ is a function of z only. The equation 1.4 can be easily integrated to get,

$$\phi = a z \quad 2.7$$

where a is a constant of integration. One can easily obtain the tetrad components $\phi_{(a)}$ of ϕ_i .

They are given by,

$$\phi_{(a)} = (0, 0, a, 0) \quad 2.8$$

In view of 2.6 and 2.8, the field equations 2.5 give the following non-trivial relations,

$$R_{(24)} = 0 \quad 2.9$$

$$R_{(11)} = R_{(22)} = -4\pi (\rho + \lambda) - \Lambda \quad 2.10$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

$$\Lambda = 4 \pi (\lambda - \rho) - 8 \pi a^2 \quad 2.11$$

$$R_{(44)} = 4 \pi (\lambda - \rho) + \Lambda \quad 2.12$$

where $R_{(ab)}$ are given by 2.4.

The equations 2.9 is easily integrable. The solution of this equation is,

$$\frac{H'}{\alpha} = \text{constant} = m, \quad 2.13$$

From the equations 2.11, 2.12 and 2.13; Λ is obtained as,

$$\Lambda = \left(\frac{-1}{4}\right) m^2 - 4 \pi a^2 \quad 2.14$$

It is interesting to note that the cosmological constant Λ turns out to be negative.

From the equations 2.10 and 2.11,

$$8 \pi \lambda = \frac{m^2}{2} + 8 \pi a^2 - \frac{\alpha''}{\alpha} \quad 2.15$$

$$8 \pi \rho = \frac{-\alpha''}{\alpha} + m^2 \quad 2.16$$

From equations 2.15 and 2.16 one can easily see that the strings satisfy the equation of state,

$$\rho = \lambda + \frac{m^2}{16 \pi} - a^2 \quad 2.17$$

The particle density ρ_p is obtained as,

$$8 \pi \rho_p = \frac{m^2}{2} - 8 \pi a^2 \quad 2.18$$

$$\rho_p \geq 0, \text{ requires } m^2 \geq 16 \pi a^2 .$$

It is verified that the velocity field v_i given by 2.6 is expansion-free, shear-free and acceleration-free. But the radiation Ω of the velocity field v_i is non-zero and it is found to have the expression,

$$\Omega = \sqrt{2} m \quad 2.19$$

As the metric potential α remains undetermined, many rotating string cosmological models with a scalar field can be obtained. In the next session, some physically relevant explicit solutions of the equations 2.13 – 2.16 are given.

From 2.18 it is clear that when $m^2 = 16 \pi a^2$, ρ_p vanishes and consequently string-dust cosmological models are regained.

Here, it should be noted that when the scalar field is switched off, the above equations reduce to those discussed by Patel and Dadhich [1] in connection with the stationary string cosmological models.

III. EXPLICIT SOLUTIONS

SOLUTION 1 :

The simplest case will obviously be $\alpha = x$. i.e. $\alpha'' = 0$.

In this case,

$$8 \pi \rho = m^2, \quad 8 \pi \lambda = \left(\frac{m^2}{2}\right) + 8 \pi a^2, \quad 8 \pi \rho_p = \left(\frac{m^2}{2}\right) - 8 \pi a^2, \quad H = \frac{m x^2}{2} \quad 3.1$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

Then the above metric reads as,

$$ds^2 = -dx^2 - x^2 dy^2 - dz^2 + \left(dt + \frac{m x^2}{2} dy\right)^2 \quad 3.2$$

SOLUTION 2 :

Take $\lambda = 0$. Therefore,

$$\frac{\alpha''}{\alpha} = \frac{m^2}{2} + 8\pi a^2 = b^2, \quad 3.3$$

The solution of the equation 3.3 is,

$$\alpha = e^{bx} \quad 3.4$$

In this case,

$$8\pi\rho = 8\pi\rho_p = \left(\frac{m^2}{2}\right) - 8\pi a^2, \quad H = \frac{m}{b} e^{bx} \quad 3.5$$

The metric of above solution is,

$$ds^2 = \left(dt + \frac{m}{b} e^{bx} dy\right)^2 - e^{2bx} dy^2 - dx^2 - dz^2 \quad 3.6$$

The metric 3.6 represents the Gödel Universe with a scalar field discussed by Chakraborty and Bandyopadhyay [13]. Further if $m^2 = 16\pi a^2$ then $\rho = \rho_p = 0$ and consequently a rotating Universe which is filled with purely a scalar field is obtained. The metric for this particular case is,

$$ds^2 = \left(dt + e^{4ax\sqrt{\pi}} dy\right)^2 - e^{8ax\sqrt{\pi}} dy^2 - dx^2 - dz^2 \quad 3.7$$

SOLUTION 3:

$$\frac{\alpha''}{\alpha} = -n^2 = \text{constant}$$

In this case,

$$8\pi\rho = m^2 + n^2, \quad 8\pi\lambda = \frac{m^2}{2} + 8\pi a^2 + n^2, \quad \alpha = \cos nx, \quad H = \frac{m}{n} \sin nx \quad 3.8$$

The geometry of this solution is described by the line element,

$$ds^2 = \left(dt + \frac{m}{n} \sin nx dy\right)^2 - \cos^2 nx dy^2 - dx^2 - dz^2 \quad 3.9$$

When $m = a = 0$, $8\pi\rho = 8\pi\lambda = n^2$ and the metric takes the simple form,

$$ds^2 = dt^2 - dx^2 - dz^2 - \cos^2 nx dy^2 \quad 3.10$$

Thus the metric 3.10 describes a geometric string ($\rho = \lambda$). Patel and Vaidya [14] have shown that the metric 3.10 describes a simple magnetic Universe with a non-zero cosmological constant. That is the space-time has two different physical visualizations.

SOLUTION 4:

It is assumed that the string distribution has the equation of state,

$$\rho = (1 + k) \lambda \quad 3.11$$

where k is a positive constant. Such strings are known as Takabayashi strings.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

In this case,

$$8\pi\rho = \frac{(k+1)(m^2 - 16\pi a^2)}{2k} \quad 3.12$$

$$8\pi\lambda = \frac{(m^2 - 16\pi a^2)}{2k} \quad 3.13$$

As $k \geq 0$ and $m^2 - 16\pi a^2 \geq 0$, the energy conditions are satisfied.

By substituting,

$$m^2 (k - 1) + (1 + k)16\pi a^2 = 2k \varepsilon b^2 \quad 3.14$$

where ε is +1, -1 or 0.

The equation 3.11 leads to,

$$\frac{\alpha''}{\alpha} = \varepsilon b^2 \quad 3.15$$

Therefore,

$$\alpha = \begin{cases} x \\ e^{bx} \\ \cos bx \end{cases} \quad \text{when } \varepsilon = \begin{cases} 0 \\ 1 \\ -1 \end{cases} \quad 3.16$$

In each case, the explicit forms of the line elements can be written. For the sake of brevity, these details are not given here. When the scalar field is switched off (i.e. $a = 0$), the solution for Takabayashi strings discussed by Patel and Dadhich [1] is obtained.

SOLUTION 5:

Note that the solutions presented above are all homogeneous. Raychaudhuri and Thakurta [15] have investigated the conditions for the metric (2.1) to be homogeneous from Killing equations without reference to the field equations.

These conditions are,

$$\alpha'' \alpha - \alpha'^2 = \text{constant}, \quad \frac{H'}{\alpha} = \text{constant} \quad 3.17$$

All the four solutions cited above satisfy the conditions 3.17. But the metric function α is arbitrary and therefore it is not necessary to satisfy the first condition in 3.17. For example, take α in the form,

$$\alpha = \cosh^2\left(\frac{mx}{2}\right) \quad 3.18$$

In this case,

$$8\pi\rho = \frac{m^2}{2} \operatorname{sech}^2\frac{mx}{2}, \quad 8\pi\lambda = 8\pi a^2 - \frac{m^2}{2} + \frac{m^2}{2} \operatorname{sech}^2\left(\frac{mx}{2}\right), \quad H = \frac{m}{2} \left[x + \frac{1}{m} \sinh mx \right] \quad 3.19$$

The geometry of this inhomogeneous rotating string model with a scalar field is described by the line element,

$$ds^2 = \left[dt^2 + \frac{m}{2} \left(x + \frac{1}{m} \sinh mx \right) dy \right]^2 - dx^2 - dz^2 - \cosh^4 \frac{mx}{2} dy^2 \quad 3.20$$

This is just an illustration of inhomogeneous string models. There are few exact solutions to Einstein equations for cosmic strings available in the literature. An attempt is made to obtain more exact solutions so that the understanding of cosmic strings is improved.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 10, October 2015

IV. CONCLUSION

In this paper, one parameter class of stationary rotating string cosmological models with a scalar field, of which the scalar field generalizations of Gödel Universe is a particular case; is obtained. By suitably choosing the free-parameter function, it is always possible to satisfy the energy conditions. By various choices of the free function some homogeneous as well as inhomogeneous rotating string world models with scalar field are obtained.

REFERENCES

- [1] Patel, L. K. and Dadhich, N., "A class of stationary rotating string cosmological models", *Pramana – J. Phys.* Vol. 47, no. 5, pp. 387-392, 1996.
- [2] Gödel, K., "An example of a new type of cosmological solutions of Einstein's field equations of gravitation", *Rev. Mod. Phys.*, Vol. 21, pp. 447-450, 1949.
- [3] Kibble, T. W. S., "Topology of cosmic domains and strings", *J. Phys.* Vol. A9, pp. 1387 – 1398, 1976.
- [4] Ogawa, K., Ishihara, H., Kozaki, H., Nakano, H., and Saito, S., "Stationary rotating strings as relativistic particle mechanics", *Phys. Rev.* Vol. D 78, no. 2, pp. 023525, 2008.
- [5] Zeldovich, Y. B., "Cosmological fluctuations produced near singularity", *Mon. Not. R. Astron. Soc.*, Vol. 192, pp. 663 – 667, 1980.
- [6] Vilenkin, A., "Cosmological density fluctuations produced by vacuum strings", *Phys. Rev. Letter*, Vol. 46, no. 17, pp. 1169 – 1172, 1981.
- [7] Gott, J. R., "Gravitational lensing effects of vacuum strings: exact solutions", *Astrophys. J.*, Vol. 46, pp. 1169 – 1172, 1981.
- [8] Garfinkle, D., "General relativistic strings", *Phys. Rev.*, Vol. D32, pp. 1323 – 1329, 1985.
- [9] Letelier, P. S., "Clouds of strings in general relativity", *Phys. Rev.*, Vol. D20, pp. 1294 – 1302, 1979.
- [10] Stachel, J., "Thickening the string. I. The string perfect dust", *Phys. Rev.*, Vol. D21, pp. 2171-2181, 1980.
- [11] Letelier, P.S., "String cosmologies", *Phys. Rev.*, Vol. D28, pp. 2414 – 2419, 1983.
- [12] Patel, L. K. and Maharaj, S. D., "Stationary rotating string world models with a magnetic field", *Pramana – J. Phys.*, Vol. 47, no. 1, pp. 33-39, 1996.
- [13] Chakraborty, S. K. and Bandyopadhyay, N., "Gödel-type universe with a perfect fluid and a scalar field", *J. Math. Phys.*, Vol. 24, pp.129 – 132, 1983.
- [14] Patel, L. K. and Vaidya, P. C., "A simple magnetic universe", *Letters to the Editor, Current Science*, Vol. 40, pp. 288, 1971.
- [15] Raychaudhuri, A. K. and Thakurta, G. S., "Homogeneous space-times of the Gödel type", *Phys. Rev.*, Vol. D22, pp. 802 – 806, 1980.

BIOGRAPHY



Dr. Lakshmi S. Desai obtained her doctoral degree in the field of Applied Mathematics from Gujarat University, Ahmedabad, India. She has over 17 years of extensive experience at undergraduate and postgraduate levels at Faculty of Technology, Dharmsinh Desai University, Nadiad (formerly known as Dharmsinh Desai Institute of Technology - DDIT). Her area of research is Applied Mathematics, General Relativity, Cosmology, and Visual Cryptography.