

# Study of Imbibition Phenomenon through Homogeneous Porous Media Using Finite Difference Method

Priti V.Tandel

Department of Mathematics, Veer Narmad South Gujarat University, Surat, Gujarat, India

**ABSTRACT:**The present paper contains the solution of phenomenon of imbibition using finite difference method. Such a phenomenon arise due to the different wetting abilities of the fluid in two immiscible phase flow through homogeneous porous media. A porous medium, filled with some fluid which preferentially wets the medium and the another fluid is injected into the medium, there is a natural flow of the resident fluid from the media, which is called counter-current Imbibition.

**KEYWORDS:** Homogeneous Porous media, Imbibition,,Immiscible, S.O.R.

## I. INTRODUCTION

The imbibition phenomenon takes place in immiscible double phase flow through homogeneous porous media [1]. Water and oil are considered in the occurrence of immiscible flow. When two fluids are together in contact with a rock surface, the angle, measured through water is called the contact angle. The porous medium is described as being water wet, if contact angle is acute, whereas if contact angle is obtuse, it is oil wet. The wet ability is defined by the angle. This angle is measured of which fluid preferentially adhere to the rock.

Many Authors, have discussed in different ways. Verma [2,3] has discussed for a cracked porous medium. He [4] also obtained Perturbation solution of imbibition phenomena. Verma and Rammohan [5] has discussed the existence and uniqueness of similarity of imbibition equation. Mehta and Verma [6] obtained Singular perturbation solution of double phase flow due to differential wettability. Mishra, Verma & Shah [7] have discussed imbibition for double phase flow through porous medium with capillary pressure. Handy [8] discussed determination of effective capillary pressures for porous media from imbibition and Li K.W.[9] have discussed Scaling of spontaneous imbibition data with wettability included.

## II. STATEMENT OF THE PROBLEM

A finite cylindrical piece of homogenous porous matrix of length L is assume to be fully saturated with an oil. Its boundaries are completely bordered by an impervious surface except for one end which is uncovered to an adjacent formation of injected water. It is assumed that injected water is favourably more wetting than that of native liquid (oil). Due to this arrangement there is a unstructured linear flow of water into the medium. Also there is a counter flow of the resident fluid (oil) from the medium, which is known as the phenomenon of linear counter-current Imbibition.

The governing equation to this phenomenon is a partial differential equation. The solution is obtained by Successive over Relaxation Method.

## III. MATHEMATICAL FORMULATION

Using Darcy's law for the the equations of seepage velocity of flowing fluids are written as:

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \quad (1)$$

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$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \quad (2)$$

Where  $V_w$  and  $V_o$  are seepage velocity of water and oil respectively,  $k$  is the permeability of the homogeneous medium,  $k_w$  and  $k_o$  are relative permabilities of water and oil respectively,  $P_w$  and  $P_o$  are the pressures and  $\mu_o$  and  $\mu_w$  are viscosities of water and oil respectively.

The equations of continuity for the flowing phase are:

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$\varphi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

Where  $\varphi$  is the porosity of the medium and  $S_w$  and  $S_o$  are water and oil saturation respectively.

For imbibition phenomenon an analytic condition is given by

$$V_w = -V_o \quad (5)$$

$$P_c = P_o - P_w \quad (6)$$

From equations (1),(2) and (5), we have

$$\frac{k_o}{\mu_o} \frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w} \frac{\partial P_w}{\partial x} = 0 \quad (7)$$

From (6) and (7), we have

$$\frac{k_o}{\mu_o} \left\{ \frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x} \right\} + \frac{k_w}{\mu_w} \frac{\partial P_w}{\partial x} = 0$$

$$\therefore \frac{\partial P_w}{\partial x} = \frac{-k_o/\mu_o}{\left\{ \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} \right\}} \frac{\partial P_c}{\partial x} \quad (8)$$

From (1) and (3), we get

$$\varphi \frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x} \left\{ \frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \right\} = 0 \quad (9)$$

From equations (8) and (9), we get

$$\varphi \frac{\partial s_w}{\partial t} + \frac{\partial}{\partial x} \left\{ k \frac{k_o k_w}{k_o \mu_w + k_w \mu_o} \frac{dP_c}{dS_w} \frac{\partial S_w}{\partial x} \right\} = 0 \quad (10)$$

$$\text{Setting } D(S_w) = \frac{k_o k_w}{k_o \mu_w + k_w \mu_o} = D$$

$D$  is called co-efficient of saturation which is assumed to be a constant.

Using the relation between capillary pressure and phase saturation as

$$P_c = -\beta S_w \quad (11)$$

Equation (10) can be written as

$$\varphi \frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x} \left[ k D \beta \frac{\partial s_w}{\partial x} \right] = 0 \quad (12)$$

With  $s_w(0, t) = s_0$ ,  $s_w(L, t) = s_1$ ,  $s_w(L, t) = 0$ ,  $0 \leq x \leq L$

$$\frac{\partial s_w}{\partial t} - k D \beta \frac{\partial^2 s_w}{\partial x^2} = 0 \quad (13)$$

$$\text{Let } \xi = \frac{x}{L}, \quad T = \frac{k D \beta}{L^2} t$$

$$\frac{\partial s_w}{\partial T} - \frac{\partial^2 s_w}{\partial \xi^2} = 0 \quad (14)$$

With  $s_w(\xi, 0) = s_0$ ,  $0 \leq \xi \leq 1$ ,  $s_w(0, T) = s_1$ ,  $s_w(1, T) = 0$

## IV. MATHEMATICAL SOLUTION

Using S.O.R. method [10], we have

$$S_{w_{i,j+1}} = S_{w_{i,j}} + \frac{k}{2h^2} \left( S_{w_{i+1,j}} - 2S_{w_{i,j}} + S_{w_{i-1,j}} + S_{w_{i+1,j+1}} - 2S_{w_{i,j+1}} + S_{w_{i-1,j+1}} \right)$$

$$\text{Let } r = \frac{k}{h^2}$$

$$(1+r)S_{w_{i,j+1}} = S_{w_{i,j}} + \frac{r}{2} \left( S_{w_{i+1,j}} - 2S_{w_{i,j}} + S_{w_{i-1,j}} + S_{w_{i+1,j+1}} + S_{w_{i-1,j+1}} \right)$$

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$$\lambda_i = s_{w_{i,j}} + \frac{r}{2} (s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}})$$

$$s_{w_{i,j+1}} = (1 - \omega)s_{w_{i,j}} + \omega \left[ \frac{r}{2(1+r)} (s_{w_{i+1,i}} + s_{w_{i-1,j+1}}) + \frac{\lambda_i}{(1+r)} \right]$$

Choose  $k = 0.01, h = 0.05, \omega = 1.58, s_0 = 0, s_1 = 1$

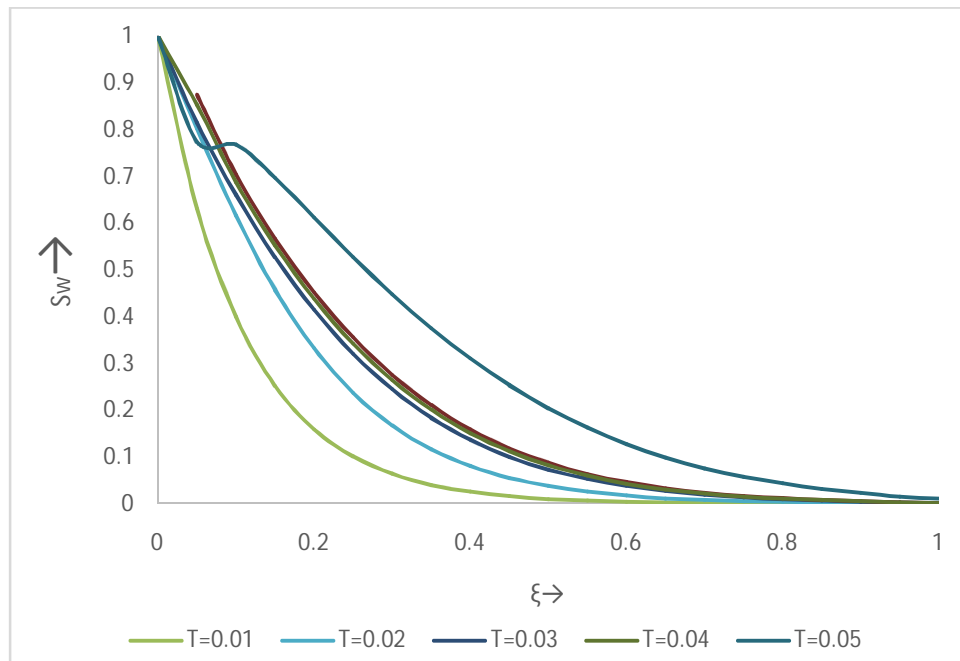
$$s_{w_{i,j+1}} = -0.58s_{w_{i,j}} + 1.58 \left[ \frac{2}{5} (s_{w_{i+1,j}} + s_{w_{i-1,j+1}}) + \frac{\lambda_i}{5} \right]$$

Where  $\lambda_i = s_{w_{i,j}} + 2 (s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}}) = -3s_{w_{i,j}} + 2 (s_{w_{i+1,j}} + s_{w_{i-1,j}})$

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

T→	T=0.01	T=0.02	T=0.003	T=0.04	T=0.05
$\xi \downarrow$	Sw				
0	1	1	1	1	1
0.05	0.632	0.803175936	0.815106214	0.852974425	0.774402717
0.1	0.399424	0.615790383	0.660171456	0.688188518	0.768168726
0.15	0.252435968	0.457551299	0.527402095	0.552705849	0.697889348
0.2	0.159539532	0.332383384	0.414651527	0.438614684	0.61230983
0.25	0.100828984	0.237375627	0.320859153	0.343045031	0.527502537
0.3	0.063723918	0.167280892	0.244618898	0.264356521	0.448235179
0.35	0.040273516	0.116629525	0.183993851	0.20088083	0.37589065
0.4	0.025452862	0.080603717	0.136728756	0.150695942	0.310991066
0.45	0.016086209	0.055298467	0.10051365	0.111742726	0.25379215
0.5	0.010166484	0.037702203	0.073182307	0.081999027	0.204324832
0.55	0.006425218	0.02556805	0.052826217	0.059613052	0.162361025
0.6	0.004060738	0.01725885	0.037839458	0.042977091	0.127424921
0.65	0.002566386	0.011602694	0.026917283	0.030751418	0.098850726
0.7	0.001621956	0.007772206	0.019028443	0.021854922	0.075860368
0.75	0.001025076	0.005189674	0.013375741	0.015437332	0.057638426
0.8	0.000647848	0.003455342	0.009354045	0.010843805	0.043391835
0.85	0.00040944	0.002294672	0.006510957	0.0075787	0.031806877
0.9	0.000258766	0.001520319	0.004512602	0.005140408	0.02261078
0.95	0.00016354	0.001005136	0.003010929	0.003189947	0.014424892
1	0.000103357	0.000580673	0.001650884	0.001283263	0.011141761

### V. GRAPHICAL REPRESENTATION



### VI. CONCLUDING REMARKS

From the table we can say that Keeping time constant, as length increases the saturation decreases. Also keeping length constant, it is clear that as time increases saturation increases. For different values of time the saturation increases as time increases. The solution obtained assuming co-efficient of saturation to be constant.

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