

# Two-Dimensional Deformation of a Uniform Half-Space by A Buried Tensile Fault at an Arbitrary Location

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**ABSTRACT:** The studies related to the deformation of a uniform half-space caused by various faults placed at origin and at depth  $d$  has been done earlier. The purpose of the present paper is to study such deformation due to a buried tensile fault when the upper edge of the fault is placed at depths  $(d_1, d_2)$  in horizontal and vertical directions respectively. Closed-form analytical expressions for the stresses and displacements are obtained using the Airy stress function approach. Then the variation of surface displacement fields i.e. horizontal displacement and uplift with the horizontal distance from the fault is studied numerically for the different values of dip angle  $\delta$ , and depths  $d_1$  and  $d_2$ . The parameters  $d_1$  and  $d_2$  occur explicitly in the final expressions of displacements.

**KEYWORDS:** Buried tensile fault, Half-space, surface displacements, dip angle.

## I. INTRODUCTION

Continuum mechanics, in which matter is approximated as a continuously distributed medium, provides the general conceptual framework for analyzing elastic deformations in a solid. The elastic dislocation theory allows to determine deformation of an elastic medium due to slip localized on a fault. The theory is widely used to model geodetic strain due to coseismic or interseismic deformation. Deformations of the medium can be described by a three-dimensional, time-dependent displacement field  $u(x, t)$  and the associated infinitesimal symmetric strain tensor  $e_{ij} = (u_{i,j} + u_{j,i})/2$ . But for the mathematical description of the static deformation of the earth's crust, two-dimensional dislocation models have been developed by several investigators. Dislocation theories have been applied as the main tool in the mathematical and physical description of mechanics of earthquakes. Steketee (1958a, b) introduced the elasticity theory of dislocations in the field of seismology. Since then numerous studies related to the deformation of an isotropic homogeneous elastic medium have been done with generality of source type and geometry. However, the studies related to tensile dislocations are scarce. The problem of deformation of a half-space caused by a tensile fault has been studied by Maruyama (1964), Davis (1983), Yang and Davis (1986), Okada (1992), Bonafede and Rivalta (1999), Singh and Singh (2000), Singh et al. (2002), Kumar et al. (2005), Malik et al. (2012), Singh et al. (2013) etc.. In the present study, the problem of deformation of an homogeneous elastic uniform half-space due to a long inclined tensile fault when the upper edge of the fault is placed at  $(d_1, d_2)$ . The Airy stress function approach is used to derive closed-form analytical expressions for stresses and displacements at an arbitrary point of the half-space caused by a long inclined tensile fault of arbitrary dip  $\delta$  and finite width  $L$ . Then the variation of displacement fields with distance from the fault is studied analytically for different values of dip angle  $\delta$  and depths  $d_1$  and  $d_2$ . It is noted that the results obtained by Singh and Singh (2000) and Singh et al. (2002) are particular cases of the results obtained in this paper.

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## II.THEORY

Consider a two-dimensional plane strain problem in which the displacement components  $u_i$  ( $i = 1,2,3$ ) are independent of  $x_1$  such that  $\partial/\partial x_1 \equiv 0$ . The stresses and displacements in terms of the Airy Stress function  $U$  are given by

$$\tau_{22} = \frac{\partial^2 U}{\partial x_3^2}, \tau_{23} = -\frac{\partial^2 U}{\partial x_2 \partial x_3}, \tau_{33} = \frac{\partial^2 U}{\partial x_2^2}, \quad (1)$$

$$\nabla^2 \nabla^2 U = 0, \quad (2)$$

where  $\tau_{ij}$  ( $i, j = 2,3$ ) are the components of stress and  $\nabla^2 \equiv \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$ ,

and

$$\begin{aligned} 2\mu u_2 &= -\frac{\partial U}{\partial x_2} + \frac{1}{2\alpha} \int \nabla^2 U dx_2, \\ 2\mu u_3 &= -\frac{\partial U}{\partial x_3} + \frac{1}{2\alpha} \int \nabla^2 U dx_3, \end{aligned} \quad (3)$$

where  $u_i$  are displacement components and  $\alpha = \frac{\lambda+\mu}{\lambda+2\mu} = \frac{1}{2(1-\sigma)}$ ,  $\sigma$  = Poisson's Ratio,  $\mu$  = Rigidity.

The traction-free boundary conditions are

$$\tau_{23} = \tau_{33} = 0 \text{ at } x_3 = 0.$$

Following Rani et al. (1991) and Singh & Singh (2000), we obtain the Airy stress function for an inclined tensile fault of finite width  $L$  and infinite length placed at an arbitrary position ( $d_1, d_2$ ) (where  $d_1$  is the distance from  $x_3$ -axis and  $d_2$  is the distance from  $x_2$ -axis) in a uniform half-space  $x_3 \geq 0$ ;

$$U = \frac{\mu b}{2\pi(1-\sigma)} \left[ \{s - X_2 \cos\delta - X_3 \sin\delta\} \log\left(\frac{S}{R}\right) + \frac{2X_3 \{X_2(X_2 \sin\delta + d_2 \cos\delta) + x_3 X'_3 \sin\delta\}}{S^2} + \frac{2sX_3 \{(X'_3 \sin\delta - X_2 \cos\delta) \sin\delta - d_2\}}{S^2} \right] \Bigg|_{s_1}^{s_2}, \quad (4)$$

where

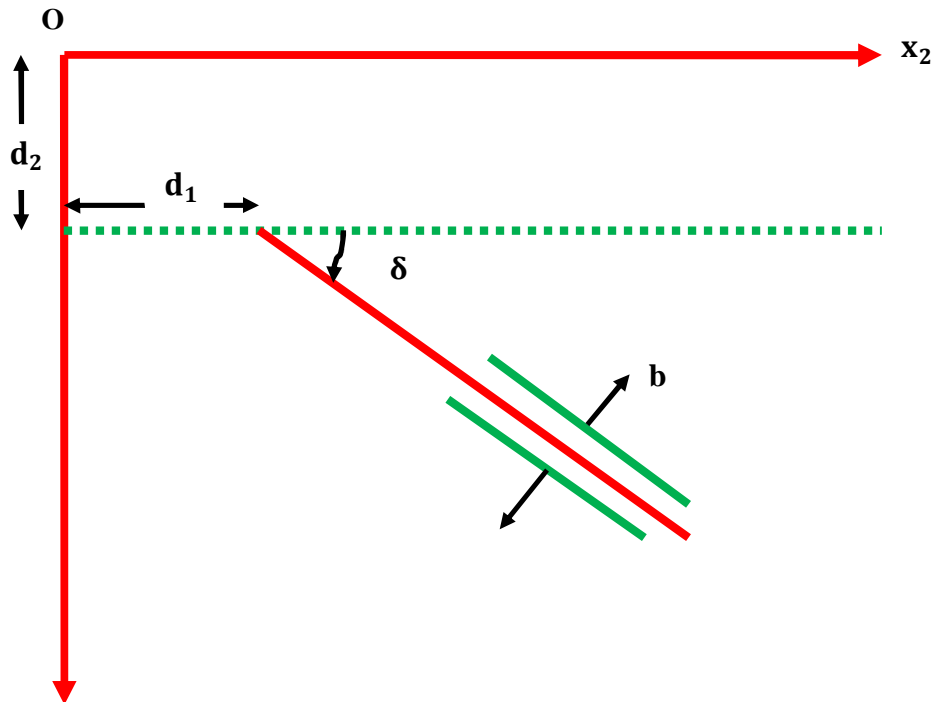
$$\begin{aligned} X_2 &= x_2 - d_1, \quad X_3 = x_3 - d_2, \quad X'_3 = x_3 + d_2, \\ R^2 &= (X_2 - s \cos\delta)^2 + (X_3 - s \sin\delta)^2, \\ S^2 &= (X_2 - s \cos\delta)^2 + (X'_3 + s \sin\delta)^2. \end{aligned} \quad (5)$$

Using equations (1) and (4), the expressions for the stresses are obtained as:

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**Figure 1. Geometry of a blind tensile fault of infinite length and finite depth at an arbitrary location ( $d_1, d_2$ ) below the earth surface.**

$$\begin{aligned} \tau_{22} = \frac{\mu b}{2\pi(1-\sigma)} & \left[ \{3X_3 \sin\delta + X_2 \cos\delta - s(1 + 2\sin^2\delta)\} \left( \frac{1}{R^2} - \frac{1}{S^2} \right) - \frac{8\sin\delta(d_2 + s\sin\delta)}{S^2} \right. \\ & + 2(s - X_2 \cos\delta - X_3 \sin\delta) \left\{ \frac{(X_3 - s\sin\delta)^2}{R^4} - \frac{(X'_3 + s\sin\delta)^2}{S^4} \right\} \\ & + 4(d_2 + s\sin\delta) \left\{ X_3 (3X'_3 \sin\delta - X_2 \cos\delta + s(1 + 2\sin^2\delta)) \right. \\ & + 2(X'_3 + s\sin\delta)(X'_3 \sin\delta - X_2 \cos\delta + s) \left. \right\} \frac{1}{S^4} \\ & \left. - \frac{16X_3(d_2 + s\sin\delta)(X'_3 \sin\delta - X_2 \cos\delta + s)(X'_3 + s\sin\delta)^2}{S^6} \right] \Bigg|_{s_1}^{s_2}, \end{aligned} \quad (6)$$

$$\begin{aligned} \tau_{23} = \frac{\mu b}{2\pi(1-\sigma)} & \left[ (X_3 \cos\delta - X_2 \sin\delta) \left( \frac{1}{R^2} - \frac{1}{S^2} \right) + 2(X_2 \sin\delta - X_3 \cos\delta) \left\{ \frac{(X_3 - s\sin\delta)^2}{R^4} - \frac{(X'_3 + s\sin\delta)^2}{S^4} \right\} \right. \\ & - \frac{4X_3(d_2 + s\sin\delta)(X_2 \sin\delta + 3X'_3 \cos\delta + s\sin 2\delta)}{S^4} \\ & \left. + \frac{16X_3(d_2 + s\sin\delta)(X_2 \sin\delta + X'_3 \cos\delta)(X'_3 + s\sin\delta)^2}{S^6} \right] \Bigg|_{s_1}^{s_2}, \end{aligned} \quad (7)$$

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$$\tau_{33} = \frac{\mu b}{2\pi(1-\sigma)} \left[ (X_2 \cos \delta - X_3 \sin \delta - s \cos 2\delta) \left( \frac{1}{R^2} - \frac{1}{S^2} \right) + 2(X_2 \cos \delta + X_3 \sin \delta - s) \left\{ \frac{(X_3 - s \sin \delta)^2}{R^4} - \frac{(X_3 + s \sin \delta)^2}{S^4} \right\} + \frac{4x_3(d_2 + s \sin \delta)\{X_2 \cos \delta - 3X_3' \sin \delta - s(1 + 2\sin^2 \delta)\}}{S^4} + \frac{16x_3(d_2 + s \sin \delta)(X_3' \sin \delta - X_2 \cos \delta + s)(X_3' + s \sin \delta)^2}{S^6} \right] \Bigg|_{s_1}^{s_2} \quad (8)$$

$$\tau_{11} = \sigma(\tau_{22} + \tau_{33}) = \frac{\mu b \sigma}{\pi(1-\sigma)} \left[ (X_2 \cos \delta + X_3 \sin \delta - s) \left( \frac{1}{R^2} - \frac{1}{S^2} \right) - \frac{4 \sin \delta (d_2 + s \sin \delta)}{S^2} + \frac{4(d_2 + s \sin \delta)(X_3' + s \sin \delta)(X_3' \sin \delta - X_2 \cos \delta + s)}{S^4} \right] \Bigg|_{s_1}^{s_2} \quad (9)$$

Corresponding to the stresses given by equations (6)-(9) and using equation (3), the expressions for the displacement components are obtained as;

$$u_2 = \frac{b}{4\pi(1-\sigma)} \left[ (1-2\sigma) \cos \delta \log \left( \frac{R}{S} \right) + 2(1-\sigma) \sin \delta \left\{ \tan^{-1} \left( \frac{X_2 - s \cos \delta}{X_3 - s \sin \delta} \right) - \tan^{-1} \left( \frac{X_2 - s \cos \delta}{X_3' + s \sin \delta} \right) \right\} + (X_3 \cos \delta - X_2 \sin \delta) \left\{ \frac{X_3 - s \sin \delta}{R^2} - \frac{X_3' + s \sin \delta}{S^2} \right\} + \frac{2(d_2 + s \sin \delta)\{(1-2\sigma)(X_2 \sin \delta + d_2 \cos \delta) + 2(1-\sigma)x_3 \cos \delta\}}{S^2} - \frac{4x_3(d_2 + s \sin \delta)(X_3' + s \sin \delta)(X_2 \sin \delta + X_3' \cos \delta)}{S^4} \right] \Bigg|_{s_1}^{s_2} \quad (10)$$

$$u_3 = \frac{b}{4\pi(1-\sigma)} \left[ (1-2\sigma) \sin \delta \log \left( \frac{R}{S} \right) - 2 \sin \delta + 2(1-\sigma) \cos \delta \left\{ \tan^{-1} \left( \frac{X_3 - s \sin \delta}{X_2 - s \cos \delta} \right) - \tan^{-1} \left( \frac{X_3' + s \sin \delta}{X_2 - s \cos \delta} \right) \right\} + (X_3 - s \sin \delta)(s - X_2 \cos \delta - X_3 \sin \delta) \left( \frac{1}{R^2} - \frac{1}{S^2} \right) + \frac{2(d_2 + s \sin \delta)\{2(1-\sigma)(X_2 \cos \delta - X_3' \sin \delta - s) + 3x_3 \sin \delta\}}{S^2} - \frac{4x_3(d_2 + s \sin \delta)(X_3' + s \sin \delta)(X_3' \sin \delta - X_2 \cos \delta + s)}{S^4} \right] \Bigg|_{s_1}^{s_2} \quad (11)$$

### IV. NUMERICAL RESULTS AND DISCUSSION

For numerical calculations, we take  $\sigma = \frac{1}{4}$  and define the following dimensionless quantities:

$$Y = \frac{x_2}{L}, Z = \frac{x_3}{L}, D_i = \frac{d_i}{L}, U_j = \frac{\pi u_j}{b}, (i = 1, 2; j = 2, 3) \quad (12)$$

where b is the displacement discontinuity normal to the fault.

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Thus,  $Y$  is the dimensionless horizontal distance from the upper edge of the fault,  $Z$  the dimensionless depth,  $U_2$  the dimensionless horizontal displacement and  $U_3$  the dimensionless vertical displacement. We will study the variation of surface displacements with respect to distance from the upper edge of the fault.

On putting  $x_3 = 0$  in equations (10)-(11) and using equation (12), the expressions for dimensionless horizontal and vertical surface displacements are obtained as:

$$U_2 = \sin\delta \left\{ \tan^{-1} \left( \frac{Y - D_1}{D_2} \right) - \tan^{-1} \left( \frac{Y - D_1 - \cos\delta}{D_2 + \sin\delta} \right) \right\} + \{(Y - D_1)\sin\delta + D_2\cos\delta\} \left( \frac{D_2 + \sin\delta}{A_0^2} - \frac{D_2}{B_0^2} \right), \quad (13)$$

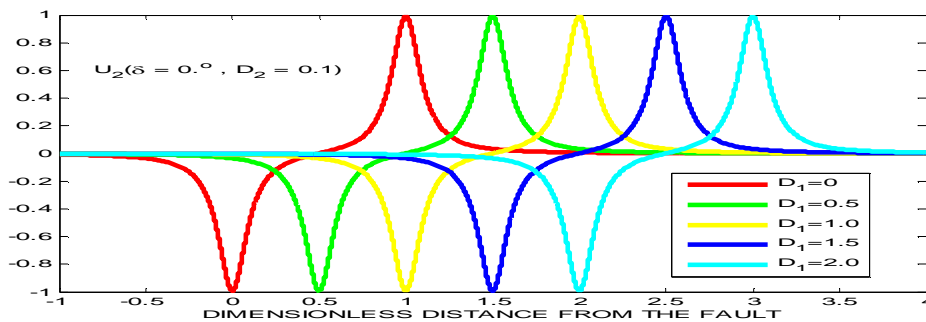
$$U_3 = \cos\delta \left\{ \tan^{-1} \left( \frac{D_2}{Y - D_1} \right) - \tan^{-1} \left( \frac{D_2 + \sin\delta}{Y - D_1 - \cos\delta} \right) \right\} + \frac{(D_2 + \sin\delta)\{(Y - D_1)\cos\delta - D_2\sin\delta - 1\}}{A_0^2} + \frac{D_2\{(Y - D_1)\cos\delta - D_2\sin\delta}}{B_0^2}, \quad (14)$$

where

$$\begin{aligned} A_0^2 &= (Y - D_1 - \cos\delta)^2 + (D_2 + \sin\delta)^2, \\ B_0^2 &= (Y - D_1)^2 + D_2^2. \end{aligned} \quad (15)$$

The variation of the horizontal surface displacement  $U_2$  with the horizontal distance from the upper edge of the fault is shown in figures 2(a)-2(d), 3(a)-3(d) and 4(a)-4(d) for different dip angles  $\delta = 0^\circ, 15^\circ, 60^\circ, 90^\circ$  respectively. In each case,  $D_1$  takes values 0, 0.5, 1.0, 1.5 and 2.0 while  $D_2$  takes value 0.1 in figures 2(a)-2(d), 0.5 in figures 3(a)-3(d) and 1.0 in figures 4(a)-4(d). In figures 2(a)-2(d), it is noticed that for each value of  $D_1$ , the pattern of variation of  $U_2$  is same for every dip angle. From figures 3(a)-3(d), it is concluded that there is significant change near the fault but as we away from the fault, the change is negligible. It is noticed that for all the values of  $D_2$ , as  $\delta$  increases from  $0^\circ$  to  $60^\circ$ , the magnitude of maximum displacement increases and after that it starts decreasing. It is also observed that the pattern of variation of  $U_2$  remains same for all the values of  $D_1$  for a particular value of  $D_2$ .

Figures 5(a)-5(d), 6(a)-6(d) and 7(a)-7(d) show the variation of the surface uplift  $-U_3$  with the horizontal distance from the upper edge of the fault for different dip angles  $\delta = 0^\circ, 15^\circ, 60^\circ, 90^\circ$  respectively by taking  $D_1 = 0, 0.5, 1.0, 1.5, 2.0$  for all the cases and  $D_2$  varies over 0.1, 0.5 and 1.0 respectively. It is noticed that the surface uplift is discontinuous at the surface points for dip angles  $\delta = 0^\circ, 15^\circ, 60^\circ$  for all the values of  $D_1$  and  $D_2$  but for a vertical tensile fault i.e.  $\delta = 90^\circ$ , the variation of  $-U_3$  is smooth for all the cases.

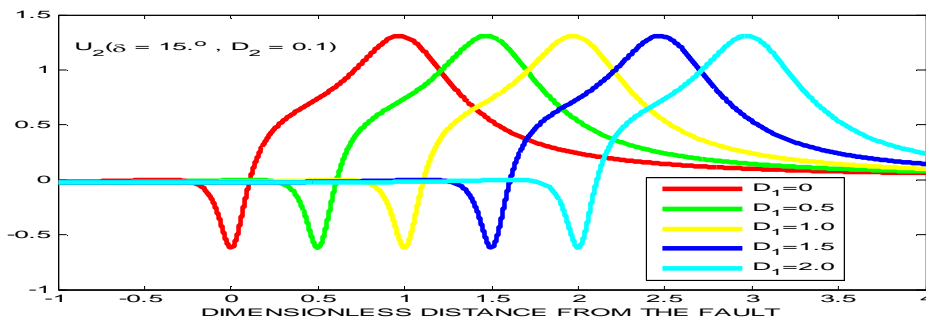


2(a)

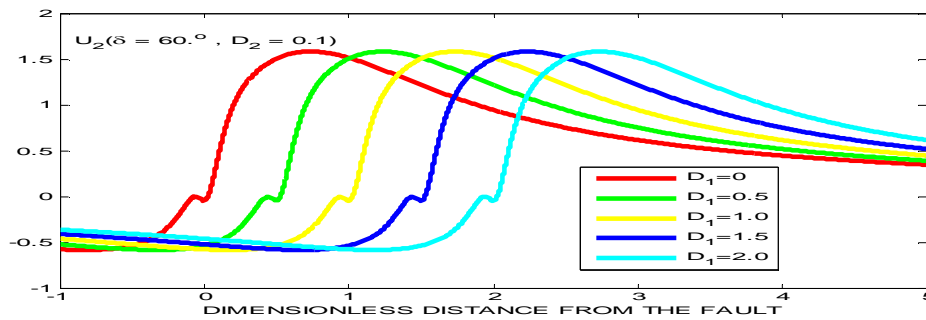
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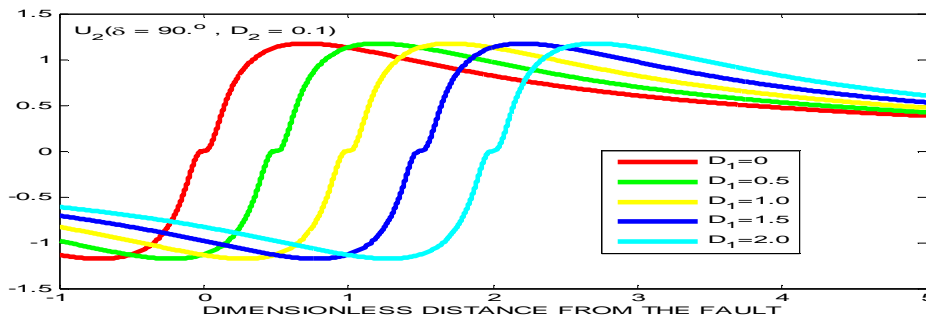
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2(b)

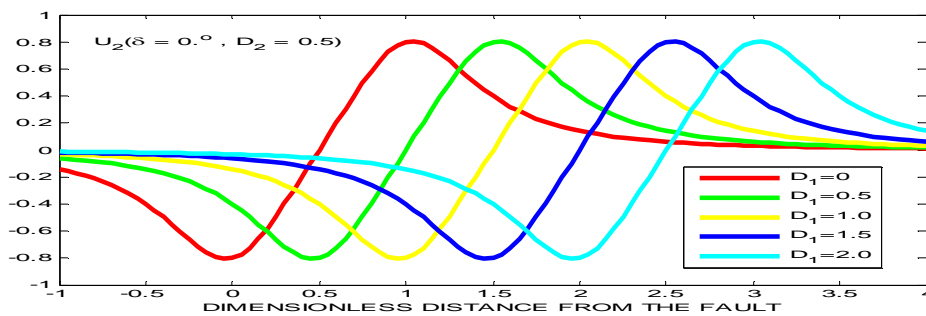


2(c)



2(d)

Figure 2: Variation of the dimensionless horizontal surface displacement  $U_2$  with distance from the fault for  $D_2 = 0.1$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .

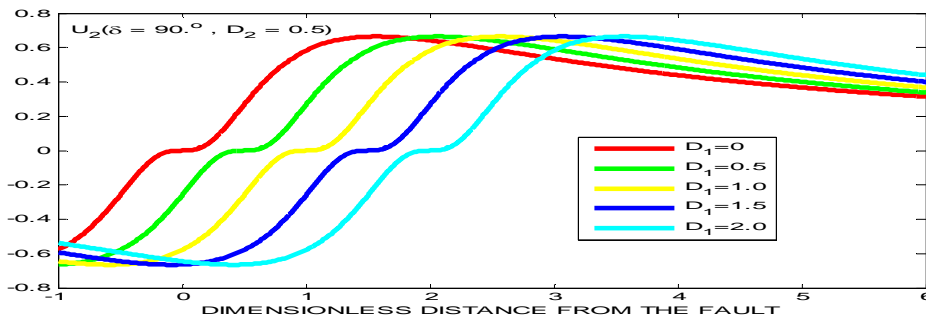
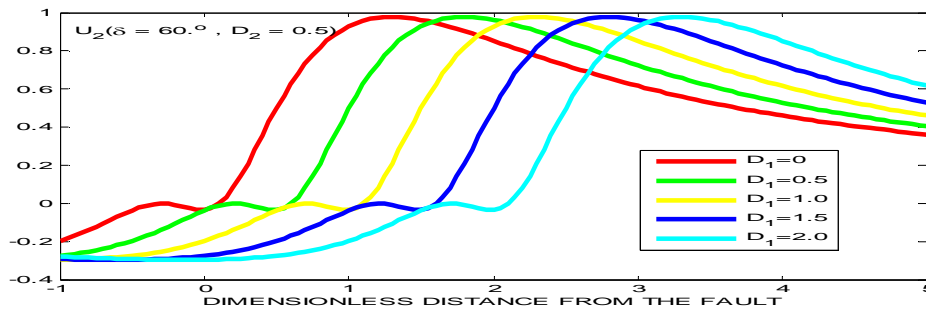
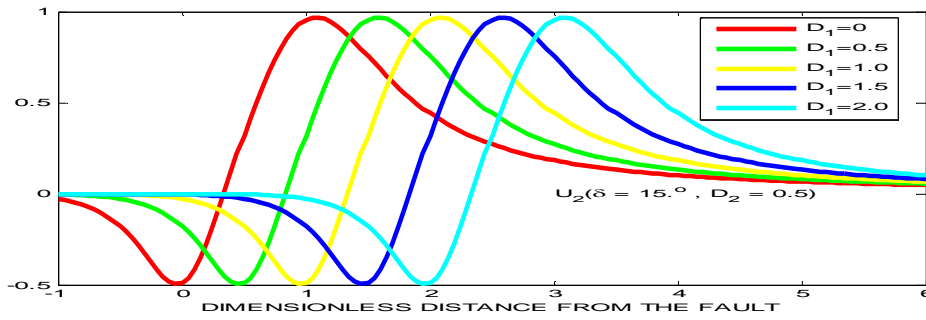


3(a)

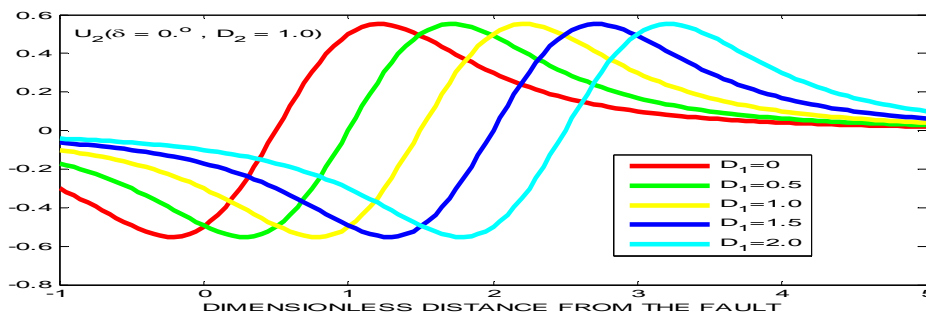
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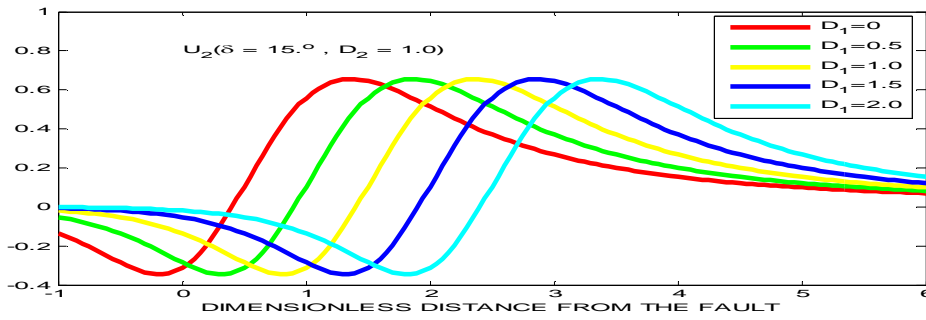
**Figure 3: Variation of the dimensionless horizontal surface displacement  $U_2$  with distance from the fault for  $D_2 = 0.5$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .**



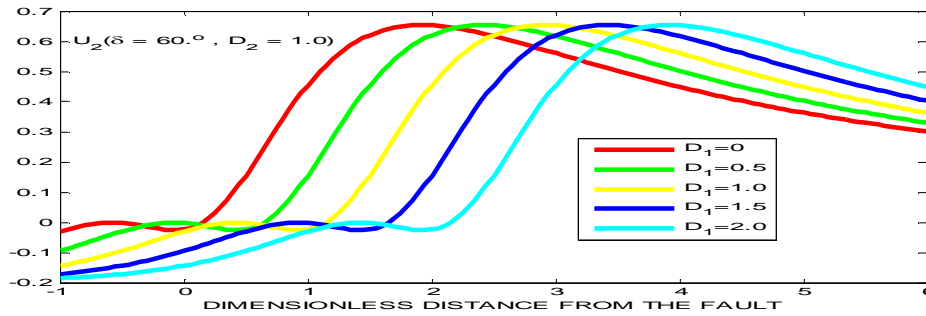
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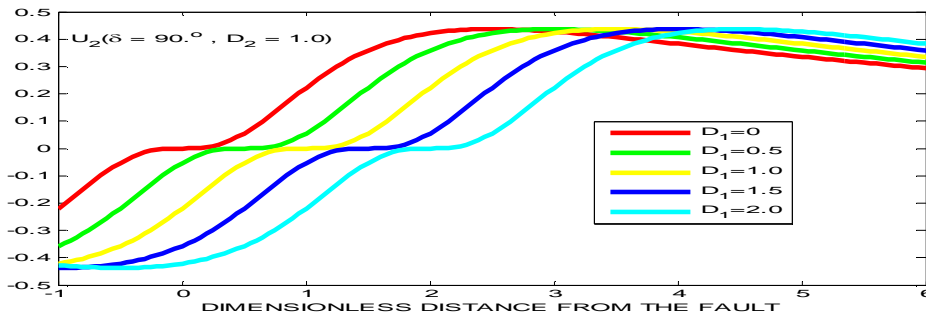
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4(b)

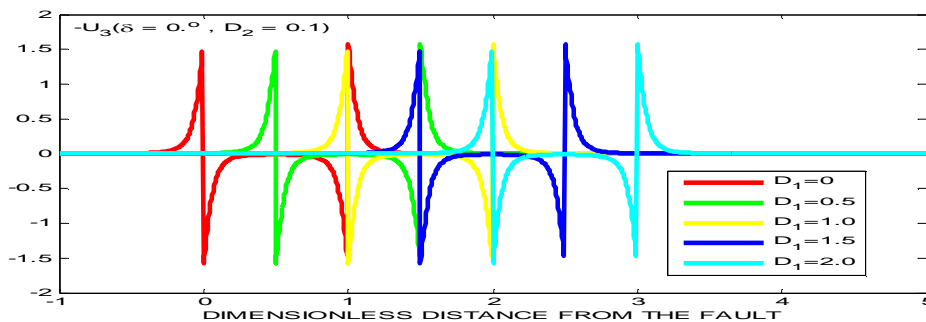


4(c)



4(d)

**Figure 4: Variation of the dimensionless horizontal surface displacement  $U_2$  with distance from the fault for  $D_2 = 1.0$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .**



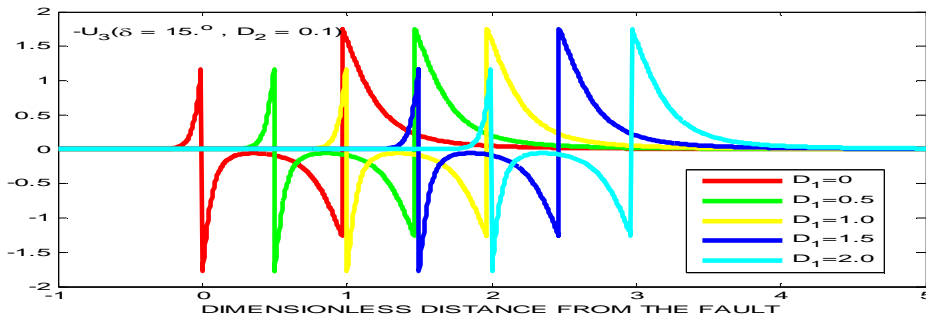
5(a)



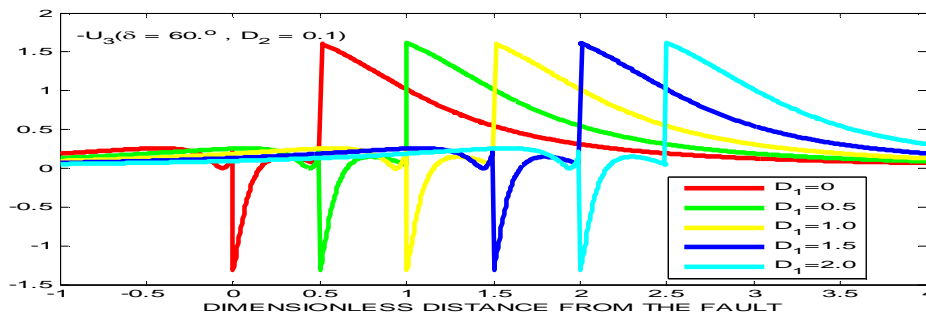
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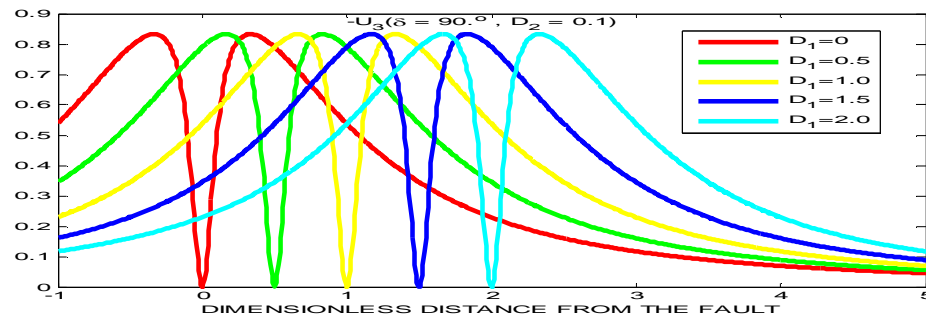
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5(b)

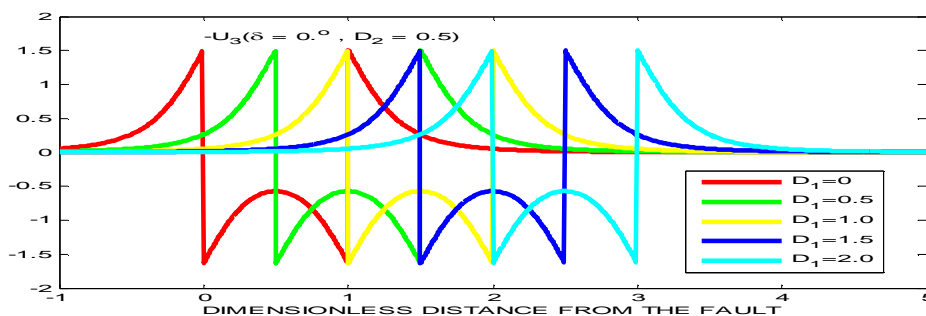


5(c)



5(d)

**Figure 5: Variation of the dimensionless surface uplift  $-U_3$  with distance from the fault for  $D_2 = 0.1$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .**

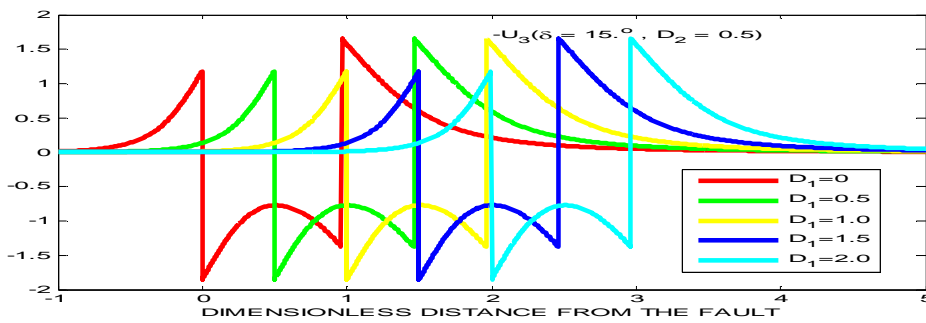


6(a)

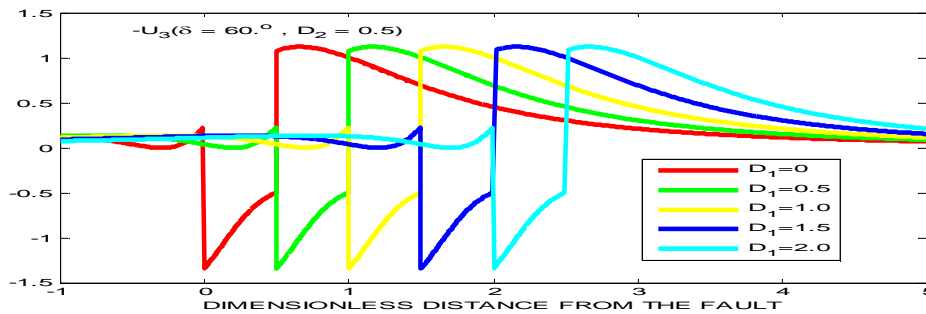
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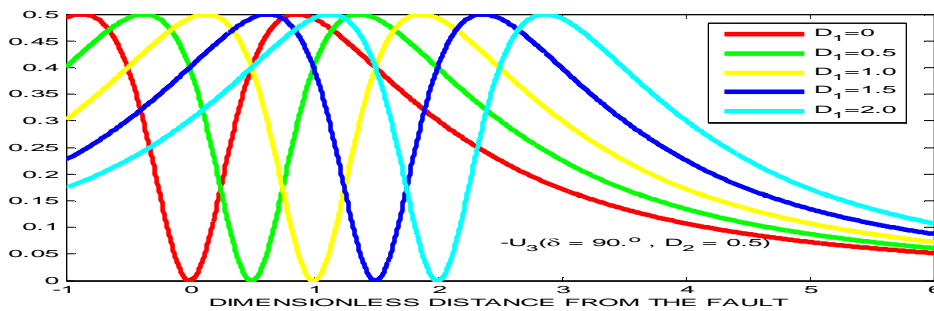
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6(b)

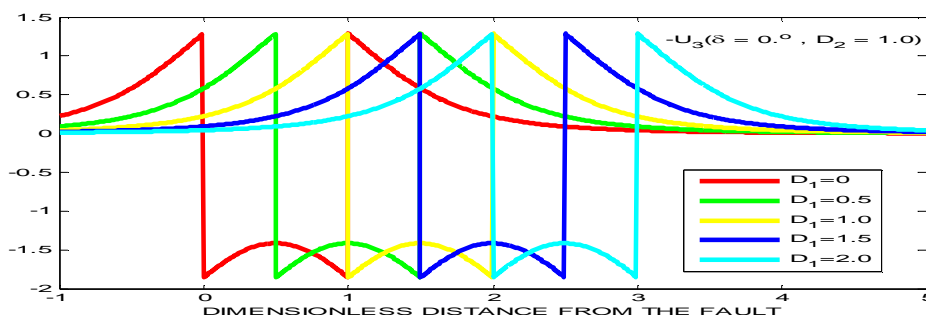


6(c)

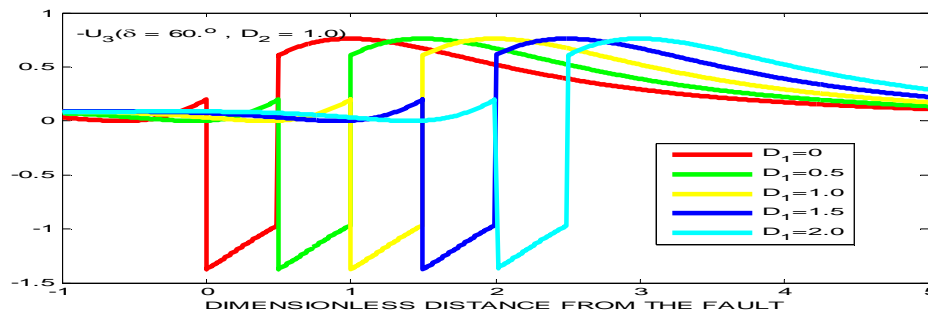
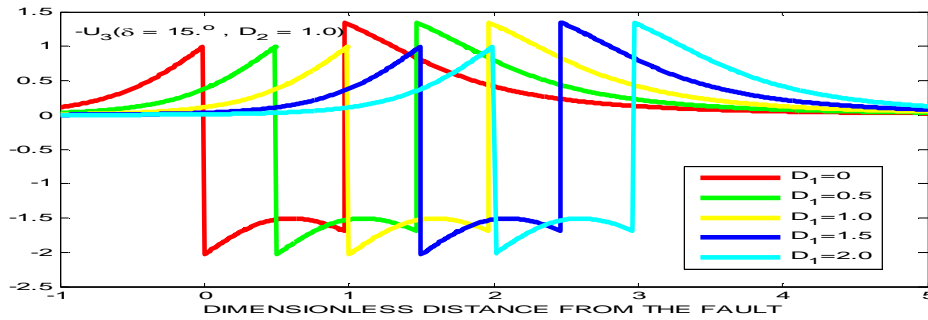


6(d)

**Figure 6: Variation of the dimensionless surface uplift  $-U_3$  with distance from the fault for  $D_2 = 0.5$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .**



7(a)



**Figure 7: Variation of the dimensionless surface uplift  $-U_3$  with distance from the fault for  $D_2 = 1.0$  for (a)  $\delta = 0^\circ$  (b)  $\delta = 15^\circ$  (c)  $\delta = 60^\circ$  (d)  $\delta = 90^\circ$ .**

### V. CONCLUSION

After analyzing the figures, it is concluded that both the horizontal surface displacement  $U_2$  and the surface uplift  $-U_3$  tend to zero as  $x_2$  tends to infinity. The variation of  $U_2$  is smooth for all the cases but the variation of  $-U_3$  is abrupt for the dip angles  $0^\circ$ ,  $15^\circ$  and  $60^\circ$ . Further, it has been verified that the study done by Singh et al. (2002) becomes a particular case of the results obtained in the present study by substituting  $d_1 = 0$  in equations (4)-(11). Also, by putting  $d_1 = d_2 = 0$  and  $\delta = 90^\circ$  in equations (4)-(11), the results match with the results derived by Singh and Singh (2000).

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