

Recent Spectral Decomposition Techniques and Its Applications in Analysis of Seismological Data: A Review

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ABSTRACT: The Spectral decomposition of seismic data was incepted earlier in the second half of the 19th century, however limited research work has been done in this field because of lack of availability of advanced tools/ techniques for analysis of transient signal like earthquake time history records. Although, there are several conventional techniques like Fourier Transform (FT), Fourier Series (FS) that are available and used for analysis and processing in seismic data. FT decomposes the signal into its constituent frequency components. But its simple interpretation of pure frequencies, it is not always the suitable tool to analyse the non-stationary signals. FT can identify the frequencies that are present in signal but does not reveal where changes in the frequency contents occur i.e., no temporal information of frequencies. Consequently, for the proper interpretation of seismic data in terms of varying frequency content, we need time-frequency representation techniques jointly. This paper reviews the linear and quadratic time-frequency representations techniques and their applications in analysis and processing of seismic data.

KEYWORDS: Time-frequency analysis, spectral decomposition, STFT, GT and Gabor expansion.

I. INTRODUCTION

Spectral decomposition of seismic data is a mathematical tool of transforming seismic data from time domain to time vs frequency domain. It converts one dimensional seismic trace in time domain to two dimensional time vs frequency domain representation called as Time-Frequency Representation (TFR). Consequently, two dimensional time-frequency representation provides information of evolution of frequency contents over time in seismic signals. The frequency domain representation of seismic data illustrates many features that are not apparent in time domain representation and hence time-frequency distribution serves as useful tool for interpretation of seismic data. Time-Frequency representation technology mainly consists of two categories viz., linear time-frequency transforms and quadratic time-frequency transforms. The selection of these transform technologies depends upon the area applications. The difference between is that the linear transform can be inverted to reconstruct the signal, thus it is suitable for signal processing such as noise suppression and time varying filtering. Whereas, quadratic transform describe the energy distribution of the signal in the time frequency domain, which is useful to analyse, classify and detect features in the signals. In view of above this paper reviews the recent time-frequency representation technologies and their applications. The first section introduces basic concept of the time domain and frequency domain as defined by the Fourier Transform. The second section describe the linear transform methodology of the TFR viz., the Short time Fourier Transform (STFT), Gabor transform (GT) & Gabor expansion, and time frequency version of the Wavelet transform (WT) and their applications. The third section review the quadratic (bilinear) transform category of the TFA based on energy densities and correlation functions of the Wigner-Ville Distribution (WVD), Spectrogram (SP) and Pseudo-Wigner-Ville (PWV) and its application. Finally, the aforementioned categories of transform is compared and summarised by the example and conclusion. The paper is summarised by comparing the transform categories and detail discussion on STFT, GT and Gabor expansion.

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II. TIME DOMAIN AND FREQUENCY DOMAIN

Fourier analysis has been used for decades for representing the plane of frequency in seismic signals. There are two alternative ways to observe a signal viz., time domain and frequency domain. The time domain function indicates how amplitude of signal changes over time whereas, frequency domain function shows the pattern of such changes. Thus, Fourier Transform and its inverse establish a bridge between the time domain and the frequency domain. The fundamental idea behind the Fourier Transform was to decompose a signal as the sum of the weighted sinusoidal function. Despite their simple interpretation of pure frequencies, it is not always the suitable tool to analysis the non-stationary seismic signal in which change in frequency content with time is the main the feature. For analysis of stationary signals (Signals whose frequency content does not change in time), it is not necessary to know change in spectral content a function of time, since all frequency components present at all times. FT can identify the frequencies constitute in a signal but does not reveal temporal information of these frequencies. Therefore, unsuitability to analyse non stationary signal is one of the primary limitation of the FT in signal processing and synthesis.

$$\text{Time domain (signal):} \quad x(t) = \int_f X(f) e^{j2\pi ft} df$$

$$\text{Frequency domain (spectrum):} \quad X(t) = \int_t x(t) e^{-j2\pi tf} dt$$

III. LINEAR TIME-FREQUENCY TRANSFORM AND ITS APPLICATION

All linear TFRs satisfy the principle of superposition which states that if $x(t)$ is a linear combination of some signal components, the TFRs of $x(t)$ is the same linear combination of the TFRs of each of the signal components. Linearity is a desirable property in any application involving multi component signals like biomedical signal, seismic records and speech signals etc. Linear time-frequency representation consist Short Time Fourier Transform (STFT), Gabor Transform (GT) & Gabor Expansion and wavelet Transform (WT).

The Short Time Fourier Transform (STFT): Although, the Fourier Transform does not explicitly show the time localization of frequency components. Such time localization can be obtained by suitably pre-windowing the signal $x(t)$ as shown in fig.1. The width of selected window must be equal to the segment of the signal where analyse signal is being assumed stationary. The STFT of the signal is obtained based on the FFT algorithms. Then the amplitude distribution is described by the short-time Fourier transform through the linear integral as

$$\text{STFT}(\tau, f) = \int_{-\infty}^{+\infty} s(t) \gamma(\tau - t) e^{-j2\pi ft} dt$$

Where, $s(t)$ is the time domain seismogram, $\gamma(\tau - t)$ windowing function and $e^{-j2\pi ft}$ is Fourier kernel.

STFT is the local spectrum of signal $x(t)$ around time t , selected by localization window $h(t)$. For discrete signals, with bounded duration and frequency band, considering that $t_n = n \cdot \Delta t$, $f_k = k \cdot \Delta f$, $\tau_m = m \cdot \Delta t$, where Δt and Δf are sampling time and frequency intervals.

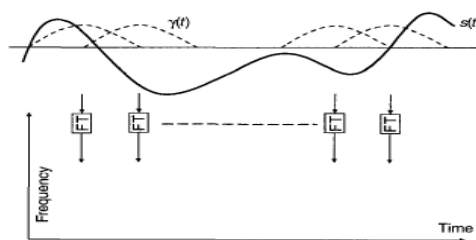


Fig 1 The STFT analysis of a signal

Gabor Transform (GT) and Expansion: In 1946, Dennis Gabor suggested representation a signal into a set of function that are concentrated in both time and frequency domain. The Gabor transform is a special case of short time Fourier transforms where window function is chosen as Gaussian function. Since the Gaussian function signals is more

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concentrated than the rectangular function in frequency domain, the frequency resolution of the Gabor transform is much better than short time Fourier transforms.

The Gabor expansion is a very useful tool in the signal processing. Gabor transform maps the time domain into time-frequency domain whereas Gabor expansion reconstructs the time domain signal after some modification being made in time-frequency domain Fig.2. For signal $x(t)$ Gabor expansion is define as

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_{m,n} h_{m,n}(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_{m,n} h(t - mT) e^{jn\Omega t}$$

Where, T and Ω denote the time and frequency sampling intervals, and the sampling cell $T\Omega$ must be small enough to satisfy $T\Omega \leq 2\pi$. The function $h(t)$ is called synthesis window. The coefficients $c_{m,n}$ are the so-called Gabor coefficients, which are computed by the Gabor transform or the sampled STFT.

$$c_{m,n} = \int_{-\infty}^{+\infty} x(t) \gamma_{m,n}^* (t - mT) e^{-j2\Omega t} dt$$

Where, the function $\gamma^*(t)$ is called analysis window, and is a dual function of the synthesis window $h(t)$.

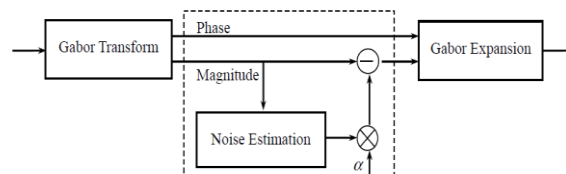


Fig.2 The diagram show noise suppresses and reconstruction of signal using the linear Gabor transform based spectral subtraction.

Wavelet Transform (WT): Wavelet analysis is a recent technique in signal analysis and processing. It overcomes the shortcomings of the other transform methods and in the recent years enormous interest in application of engineering has been observed. STFT gives a fixed window resolution at all times, whereas WT gives a variable width window resolution i.e. higher frequencies are better resolved in time, and lower frequencies are better resolved in frequency.

A Wavelet is a wave form of effectively limited duration which has one special property that is an average value of zero (equation 1). They have their energy concentrated in time and are suited to analysis of transient signals.

$$\int_{-\infty}^{+\infty} \psi(x) dx = 0 \quad \dots 1$$

The wavelets are generated from a single basic wavelet $\psi(x)$, called the mother wavelet which is given by Equation 2 by means of scaling and shifting.

$$\psi_{(s,\tau)}(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x-\tau}{s}\right) \quad \dots 2$$

The variables s and τ in Equation 2 corresponds to the scaling and shifting parameters respectively. The term $\frac{1}{\sqrt{s}}$ is the normalizing term and is used to maintain the same energy before and after the transformation process. Signal analysis and processing using wavelet transform technique obtained by multiplying the signal with a wavelet function and then its transform is computed for each wavelet function generated segments. Wavelet transform (WT) is define

$$X_{WT}(s, \tau) = \frac{1}{\sqrt{s}} \int x(t) \psi\left(\frac{x-\tau}{s}\right) dt$$

Where, $x(t)$ is the signal to be analyzed and $\psi(t)$ is the mother wavelet or the basis function.

Wavelet transform comprises as continuous wavelet transform (CWT) and discrete wavelet transform (DWT). The continuous wavelet transform was computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times. Whereas, in the discrete wavelet transform (DWT) case filters of different cut-off-frequencies are used to analyze the signal at different scales. The signal is passed through a series of

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high-pass filters to analyze the highfrequencies, and it is passed through a series of low-pass filters to analyze the low frequencies.

Application of the Linear TFRs:The STFT, GT and WT have been applied to signal processing problems in many different areas. Major applications includes time varying signal analysis, system identification, spectral estimation, signal detection, speaker identification& speech coding, estimation of group delay or the instantaneous frequency of the signal. Another important application of linear time-frequency analysis is to reduce noise in noise-corrupted signal in time frequency domain and then reconstruct or synthesise time domain signal which has a higher signal to noise ratio. Chakraborty and Okaya [1995] have performed time-frequency analysis of seismic traces using STFT,continuous wavelet Transform(CWT) and matching pursuit decomposition.Kareem and Gurley [1999] presented example applications of wavelet transforms in ocean, windand earthquake engineering. Continuous and discrete wavelet transform has been utilized to analyse, identify, characterization and simulation of random signal processes.Zhou and Adeli [2003] decomposed earthquake records using waveletstransform (Mexican Hat typewavelets). Sinha et al. [2003] have developed a novel method of calculating time-frequency spectrum using CWT which reported as TFCWT. Spectral decompositions with TFCWT are used to detect low frequency shadows caused by hydrocarbonsand to identify subtle stratigraphic features for reservoir characterization.Qiang and Wen [2010] have used deconvolutive short time Fourier Transform for spectral decomposition of seismic signals.Puryear et al. [2012] developed an algorithm called "constrained least-squares spectral analysis"for time-frequency analysis of seismic trace.

IV. QUADRATIC (BILINEAR) TIME –FREQUENCY TRANSFORM AND APPLICATIONS

Besides the linear TFR, another important method for joint time frequency analysis is quadratic TFR that interpret a TFR as a time- frequency energy distribution ‘ $T_x(t, f)$ ’ or instantaneous power spectrum of a signal ‘ $x(t)$ ’. Since energy is a quadratic representation, energetic TFRs look for combine the concept of the instantaneous power ‘ $x(t)$ ’ and the spectral energy density ‘ $X(f)$ ’.

$$\text{Energy spectral density: } |X(f)|^2 = \int_{-\infty}^{+\infty} T_x(t, f) dt$$

$$\text{Instantaneous power: } |x(t)|^2 = \int_{-\infty}^{+\infty} T_x(t, f) df$$

Analogous to the conventional power spectrum, a straightforward approach of computing the time dependent spectrum is to take the square of the magnitude of linear TFRs and designated as spectrogram ‘ SP ’ and scalogram ‘ $SCAL$ ’.

$$SP_x^h(t, f) = |STFT(t, f)|^2$$

$$SCAL_x^h(t, f) = |Wt(t, f)|^2$$

Apart from the energetic interpretation of quadratic TFRs, there exists another interpretation in terms of correlation function. A correlative TFR look for combine the temporal correlation and the spectral correlation.

The recent developed quadratic time-frequency distribution (also known as Cohen,s class Transforms) are; Wigner-Ville (WV), Spectrogram (SP), Pseudo-Wigner-Ville (PWV), Smoothed Pseudo Wigner-Ville (SPWV), Butterworth (BU), Born-Jordan (BJ), Morgenau-Hill (MH), PseudoMorgenau-Hill (PMH), Reduced interference (RI) with different kernels (Bessel, Hanning), Choi-Williams (CW), Rihaczec, Page (PAG), Zhao-Atlass-Mark (ZAM). Each of them has its characteristics merits and demerits. Different techniques are required in different applications. Among all Wigner-Ville (WV) distribution satisfies an exceptionally large number of desirable properties and summarized here.

Wigner-Ville (WV) distribution:Wigner-Ville Distribution (WVD) was developed in the area of quantum mechanics by P. Wigner in 1932 and later it was introduced in area of signal processing by J. Ville.

Wigner-Ville Distribution (WVD) is define as

$$W_s(\tau, f) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} x(t + \tau/2)x^*(t - \tau/2)e^{-j2\pi f\tau} dt$$

Where; $x(t + \tau/2)x^*(t - \tau/2)$ is average instantaneous correlations function, ‘ τ ’ is time lag and ‘*’ represent the complex conjugate of signal ‘ $x(t)$ ’. Due to averaging the instantaneous correlations or bilinearity of the quadratic transformations, WVD suffered from cross-terms interference effect.

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Spectrogram (SP): Spectrogram (SP) is the short-time quadratic integral measure of the energy distribution and defined as

$$SP_x^h(t, f) = |STFT(t, f)|^2 = \left| \int_{-\infty}^{+\infty} x(\tau) \cdot h(\tau - t) \cdot e^{-j2\pi f\tau} d\tau \right|^2$$

Application of the Quadratic (Bilinear) TFRs: The WVD has served as a useful tool in the area of quantum mechanics, optics, acoustics, bio-engineering, image processing and oceanography. Moreover, many researchers have used this technology to analyse time varying systems and highly non-stationary signals.

Lopez et al. [2000] has been studied the Mexico City Valley seismic records ranging from strong to moderate with spectrogram (SP), Wigner Ville (WVD), The Choi-Williams (CWD) and the reduce interference (RID) distributions. Li and Zheng [2008] have used smoothed Wigner-Ville distribution (SWVD) based spectral decomposition on seismic data from the Central Tarim Basin. Zhang [2008] introduced chirplet transform and combined it with WVD to do spectral decomposition. Qiang Zhu, Yansong Wang, and Gongqi Shen [2011] have been studied the instantaneous frequency component in a transient signal using different linear and bilinear TFRs namely, the short-time Fourier transform (STFT), wavelet transform (WT), Wigner-Ville distribution (WVD), pseudo-WVD and (PWVD) and the Hilbert-Huang transform (HHT). Upegui-Botero et al. [2012] has been compared the Cohen's classes (WVD, CWD, RID and AOK) of time frequency distribution on the horizontal component of seismic records of the Puerto Rico Strong Motion Program. S. Yilmaz et al. [2013] had been compared time frequency analysis methods namely, STFT (linear) and WVD (quadratic) for the determination of maximum and minimum amplitude value in 2012 Georgia earthquake records and concluded that STFT gave better value than the WVD.

Comparative Table:

Method	Application	Limitation
Fourier Transform	Used only for the analysis and reconstruction stationary signal.	No time varying spectral information. Not suitability for the analysis and synthesis non-stationary signals.
Linear Time-Frequency Transform	Suitable for analysis and reconstruction of non-stationary signals. Discovering the pattern of frequency changes which often recognises the nature of the signal. Used to suppress the noise in noise-corrupted signals. Having Poorer Time-frequency resolution.	Affected by window effect i.e. window of type and window length.
Quadratic Time-Frequency Distribution	Compute the energy distribution in signals in time-frequency domain which classify and detect features of the signal. Comparatively good resolution as compare to linear TFRs.	Not invertible and suffered from the cross-term interference effects.

V. A CASE STUDY OF LINEAR TIME-FREQUENCY ANALYSIS AND RESULT INTERPRETATION

In this case study, the seismogram recorded at Vinakkhal station (epicentral distance 40 km), one of the stations from the twelve stations array surrounding the Tehri Dam in Uttarakhand is used to examine time, frequency and time-frequency analysis. This array is established and supervised by Department of Earthquake Engineering IIT Roorkee. The event was recorded on 25-08-2009 at the original time 19 hr 22min 23.86sec of magnitude 2.08. The frequency sampling rate of seismogram's recording is 100sps. Vertical component of this seismogram of 60 sec. duration is analysed by STFT (Hanning Window) and GT linear time-frequency transform and compared. Furthermore, Gabor

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Expansion technique is used to reconstruct the interested frequency band filtered signal by making some modifications in the time- frequency domain of the signal being analysed.

The vertical component of time history of duration 60 sec recorded at station is shown in fig3 (a) and corresponding FT of this signal is shown in fig 3(b). Fig 4 (a) and (b) shows the Time-frequency representation with STFT coefficients and GT coefficients respectively. The window length (512 in samples) and frequency bins (512 in samples) parameters are same for both the methods. The results are obtained using the advanced signal processing toolkit developed by National Instruments LABVIEW and discussed below. Fig4 (a) and Fig4 (b) exhibit at least four frequency components. These components are difficult to find using traditional FT analysis (Fig 3b). These frequencies are annotated with f1, f2, f3 and f4 in both cases and marked at different time i.e. f1=7.5Hz at 9.0 sec, f2=7.0 Hz at 8.5 sec, f3=6.5Hz at 9.6 sec and f4=6.0Hz at 0.5 sec [Fig4 (a) and Fig4 (b)].

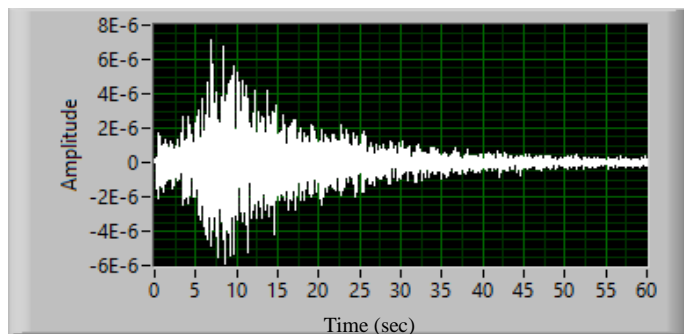
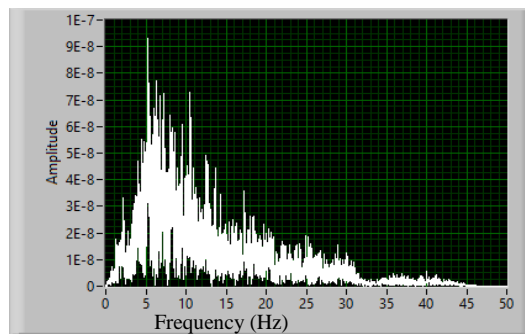
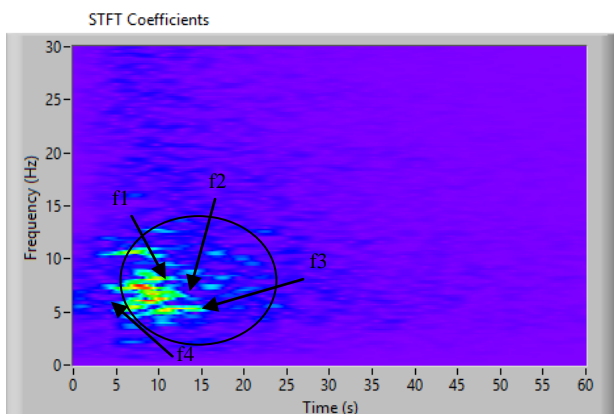


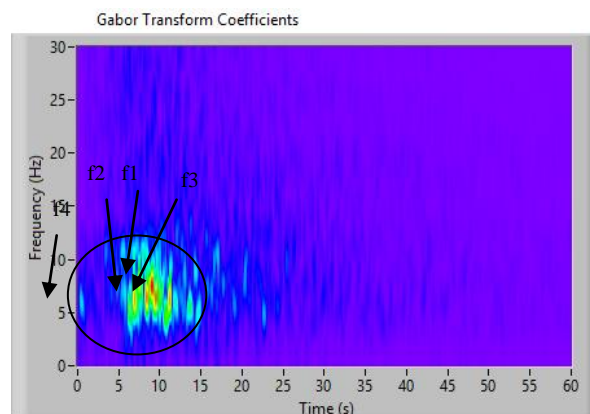
Fig3. (a) Noisy signal time history (sec.)



(b) FT of the noise signal



(a)



(b)

Fig4. Time-frequency representation using, (a) STFT coefficients and (b) GT coefficients, with 99.8% overlapping.

The maximum energy released on this station is found approximately at 9.0 sec. in time domain analysis, whereas frequency domain shows the corresponding frequency range 7.0-7.5Hz. It is clearly seen in Fig4 (a) and Fig4 (b) that the frequency components in time-frequency map increase with time upto 9.0 sec and then decreases continuously. It is also observed that a low frequency anomaly (small spectral amplitude), spread along the entire signal length beyond 10 Hz. This variation used to characterises the various phases involved in the recorded seismogram.

Fig5 shows the filtered frequency band time-history of the signal being analysed and fig6 (a) and fig6 (b) represent corresponding GT and STFT time-frequency representation respectively. Moreover, it is visualised that resolution of time-frequency representation obtained using GT is better as compared to the STFT representation.

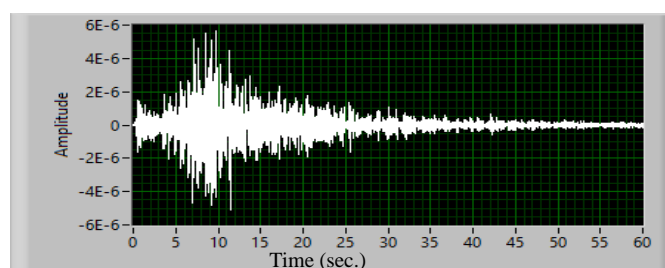


Fig 5. Noise suppressed signal obtained using Gabor expansion.

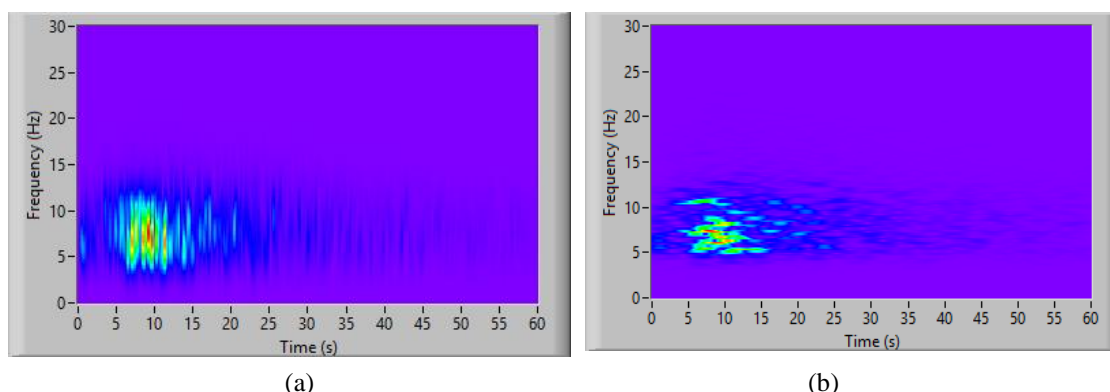


Fig6. Filtered time-frequency representation using, (a) GT coefficients and (b) STFT coefficients, with 99.8% overlapping.

VI. CONCLUSION

Since, time-domain and frequency-domain analyses are not adequate for analysis and processing of non-stationary signals. Thus, the time-frequency representations are powerful tools in this area. This paper is reviewing both linear and quadratic time-frequency representations. The linear time-frequency representations (STFT, GT, Gabor expansion and WT) and the quadratic representation (Wigner-Ville distribution) are discussed broadly.

Based on the comparative study of linear and quadratic representations techniques, it is concluded that each technique has its strengths and drawbacks.

The linear time-frequency technique viz., STFT and GT is applied on the time history recorded of the Vinakhhhal station to find the variation in frequency components with time.

The Gabor expansion is applied to generate a time history of interested frequency components after filtering the unwanted frequency components through time-frequency filter. Based on time-frequency representation of time history of the Vinakhhhal station, it is found that the resolution of GT is better than STFT method.

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