

Variance of Time to Recruitment for a Two Grade Manpower System with Two Types of Decisions for Correlated Wastage

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ABSTRACT: In this paper a two graded organization is considered in which depletion of manpower occurs due to its policy decisions. Assuming that the wastages (loss in man-hours) are exchangeable and constantly correlated exponential random variables and policy decisions are classified into two types in terms of their intensity of attrition, three mathematical models are constructed according to different forms of the mandatory threshold for wastages in the organization. Explicit analytical expressions for the system characteristics namely mean and variance of time to recruitment are obtained for all the models using an univariate CUM policy of recruitment based on shock model approach, when the inter-policy decision times form (i) a geometric process and (ii) an order statistics. The influence of the nodal parameters on these system characteristics are studied and relevant conclusions are presented.

KEYWORDS: Two grade manpower system, Two types of policy decisions, Correlated wastage, Geometric Process, Order statistics, Univariate CUM policy of recruitment, Variance of time to recruitment.

I. INTRODUCTION

Exit of personnel is a common phenomenon in any marketing organization. In [1] and [2] several stochastic models for a manpower system with grades are discussed using Markovian and renewal theoretic approach. In [5] the authors have initiated the study on problem of time to recruitment for a single grade manpower system and obtained the variance of time to recruitment when the inter-decision times and the wastages form a sequence of independent and identically distributed exponential random variables, using shock model approach. In [3] the author has studied the work in [5] for correlated wastages. In [4] and [6] the authors have studied the work in [5] for a two grade manpower system according as the inter-decision times with same attrition form an ordinary renewal process or an order statistics. The present paper studies the work in [4] and [6] when (i) the inter-decision times which are classified into two types according to their intensity (high or low) of attrition, form a geometric process or an order statistics. In the present paper, three mathematical models are constructed which differ from each other in the context of permitting or not permitting transfer of personnel between two grades and providing a better allowable loss of manpower in the organization. More specifically, in Model-1, the breakdown threshold is minimum of the thresholds for the loss of manpower in the two grades. In Model-2, the breakdown threshold is the maximum of the thresholds for the grades. In Model-3, the breakdown threshold is the sum of the thresholds for the grades. This paper is organized as follows: In sections II, III and IV Models 1, 2 and 3 are respectively described and analytical expressions for mean and variance of time to recruitment are derived. The analytical results are numerically illustrated by assuming specific distributions and the influence of nodal parameters on the system characteristics are reported.

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II. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-1

Consider an organization having two grades in which decisions are taken at random epochs in $[0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated wastage in manpower (measured as loss in man-hours) to the organization, if a person quits and it is linear and cumulative. Let X_i be the wastage due to the i^{th} decision epoch, $i=1, 2, 3, \dots$. It is assumed that X_i 's are exchangeable and constantly correlated exponential random variables with distribution function $G(\cdot)$. Let ' ρ ' be the correlation between

X_i and X_j where $i \neq j$ and $\phi(n, x) = \int_0^x e^{-z} z^{n-1} dz$. Let $b = c(1-\rho)$ where ' c ' is the mean of X_i , $i=1, 2, 3, \dots$. Let

S_n be the cumulative wastage in man-hour, in the first ' n ' decisions. Let $U_i, i = 1, 2, 3, \dots$ be the time between $i-1^{\text{th}}$ and i^{th} decisions. The best distribution when the inter-decision times have high or low intensity of attrition is the hyper exponential distribution. Let $U_i, i = 1, 2, 3, \dots, k$ are independent and identically distributed hyper exponential random variables with distribution (density) function $F(\cdot)(f(\cdot))$, and high(low) attrition rate $\lambda_h(\lambda_l)$ and $p(q)$ be the proportion of decisions having high (low) attrition rate. Let $F_k(t)$ ($f_k(t)$) be the distribution(probability density) function of $\sum_{i=1}^k U_i$.

The time to recruitment is denoted by T and its cumulative distribution function, probability density function, mean and variance are denoted by $L(\cdot)$, $\ell(\cdot)$, $E[T]$ and $V[T]$ respectively. Let $\sigma^*(\cdot)$ be the Laplace transform of $\sigma(\cdot)$. Let Y be the breakdown threshold for the cumulative wastage in the organization. Let $Y_A(Y_B)$ be the threshold level for the cumulative wastage in grade A(B) is a exponential random variable with mean $1/\alpha_A$ ($1/\alpha_B$). The loss of man-hours process and the inter-decision time process are statistically independent. The univariate policy of recruitment is **Recruitment is done as and when the total wastages in the organization exceeds the breakdown threshold Y** . Let $V_k(t)$ be the probability that there are exactly k -decision epochs in $(0, t]$. Since the number of decisions made in $(0, t]$ form a renewal process, we note that $V_k(t) = F_k(t) - F_{k+1}(t)$, where $F_0(t) = 1$.

Main results

By definition, $S_{N(t)}$ is the total loss of man-hours in the $N(t)$ decisions taken in $(0, t]$. Therefore

$$P(T > t) = P(S_{N(t)} < Y) \tag{1}$$

and using laws of probability and on simplification we get

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) \int_0^{\infty} G_k(y) h(y) dy \tag{2}$$

Since $Y = \min(Y_A, Y_B)$ it can be shown that

$$h(y) = (\alpha_A + \alpha_B) e^{-(\alpha_A + \alpha_B)y} \tag{3}$$

From (2) and (3) we get

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] [W_{(\alpha_A + \alpha_B, k)}] \tag{4}$$

$$\text{where } W_{(\tau, k)} = (1 + b\tau) \left(1 + \frac{k\rho b\tau}{(1-\rho)(1+b\tau)} \right)^{-1} \tag{5}$$

Since $L(t) = 1 - P(T > t)$, from (5) and by taking Laplace transforms on both sides we get

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$$\ell^*(s) = -\sum_{k=0}^{\infty} [f_k^*(s) - f_{k+1}^*(s)] [W_{(\alpha_A + \alpha_B, k)}] \tag{6}$$

It is known that

$$E[T] = -\left. \frac{d(\ell^*(s))}{ds} \right|_{s=0}, \quad E[T^2] = \left. \frac{d^2(\ell^*(s))}{ds^2} \right|_{s=0} \quad \text{and} \quad V[T] = E[T^2] - (E[T])^2. \tag{7}$$

Case(i): The inter-decision times $U_i, i=1,2,3..$ form a geometric process with parameter 'a'.

Since $\{U_i\}$ is a geometric process it is known that

$$f_k^*(s) = \prod_{n=1}^k f^*\left(\frac{s}{a^{n-1}}\right) \tag{8}$$

From (6), (7) and (8) we get

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k [W_{(\alpha_A + \alpha_B, k)}] \tag{9}$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2}\right)^k [W_{(\alpha_A + \alpha_B, k)}] - (E[U])^2 \left[\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}}\right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}}\right)^2 \right] [W_{(\alpha_A + \alpha_B, k)}] \right] \tag{10}$$

By the hypothesis that $U=U_i, i=1,2,3..$ follows hyper exponential distribution, it can be shown that

$$E[U] = \frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \quad \text{and} \quad E[U^2] = 2 \left(\frac{p\lambda_l^2 + q\lambda_h^2}{\lambda_h^2\lambda_l^2} \right). \tag{11}$$

Case(ii): The inter-decision time $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ be the order statistics selected from the sample U_1, U_2, \dots, U_m

with respective density function $f_{u(1)}, f_{u(2)}, \dots, f_{u(m)}$.

From (6) and (7) it is shown that

$$E[T] = E[U] \sum_{k=0}^{\infty} [W_{(\alpha_A + \alpha_B, k)}] \tag{12}$$

and

$$E[T^2] = \sum_{k=0}^{\infty} (2k(E[U])^2 + E[U^2]) [W_{(\alpha_A + \alpha_B, k)}] \tag{13}$$

Using the theory of order statistics it can be shown that

$$E[U] = \begin{cases} \sum_{r_1=0}^m \frac{m c_{r_1} p^{r_1} q^{m-r_1}}{(\lambda_h - \lambda_l) r_1 + \lambda_l m}, & \text{if } f(t) = f_{u(1)}(t) \\ \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} \frac{(-1)^{m-r_1} m c_{r_1} (m-r_1) c_{r_2} p^{r_2} q^{m-r_1-r_2}}{(\lambda_h + \lambda_l) r_2 + \lambda_l r_1 - m\lambda_l}, & \text{if } f(t) = f_{u(m)}(t) \end{cases} \tag{14}$$

and

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$$E[U^2] = \begin{cases} 2 \sum_{r_1=0}^m \frac{m c_{r_1} p^{r_1} q^{m-k-r_1}}{((\lambda_h - \lambda_l)r_1 + \lambda_l m)^2}, & \text{if } f(t) = f_{u(1)}(t) \\ 2 \sum_{r_1=0}^k \sum_{r_2=0}^{k-r_1} \frac{(-1)^{k-r_1} k c_{r_1} (k-r_1) c_{r_2} p^{r_2} q^{k-r_1-r_2}}{((\lambda_h + \lambda_l)r_2 + \lambda_l r_1 - k \lambda_l)^2}, & \text{if } f(t) = f_{u(k)}(t) \end{cases} \quad (15)$$

III. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-2

For this model $Y = \max(Y_A, Y_B)$. All the other assumptions and notations are as in Model-I. In this model it can be shown that

$$h(y) = (\alpha_A) e^{-(\alpha_A)y} + (\alpha_B) e^{-(\alpha_B)y} - (\alpha_A + \alpha_B) e^{-(\alpha_A + \alpha_B)y} \quad (16)$$

Proceeding as in Model-I we get

$$\ell^*(s) = - \sum_{k=0}^{\infty} [f_k^*(s) - f_{k+1}^*(s)] [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] \quad (17)$$

Case(i) : The inter-decision times $U_i, i=1,2,3..$ form a geometric process with parameter 'a'. Proceeding as in Model-1 it is found that

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] \quad (18)$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2}\right)^k [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] - (E[U])^2 \left[\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}}\right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}}\right)^2 \right] [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] \right] \quad (19)$$

where $E[U]$ and $V[U]$ are given by (11).

Case (ii) : Consider the order statistics $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ as described in case(ii) of Model-1

In this case it is shown that

$$E[T] = E[U] \sum_{k=0}^{\infty} [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] \quad (20)$$

and

$$E[T^2] = \sum_{k=0}^{\infty} (2k(E[U])^2 + E[U^2]) [W_{(\alpha_A, k)} + W_{(\alpha_B, k)} - W_{(\alpha_A + \alpha_B, k)}] \quad (21)$$

where $E[U]$ and $E[U^2]$ are given by (14) and (15).

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IV. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-3

For this model $Y = Y_A + Y_B$. All the other assumptions and notations are as in Model-I. In this Model it can be

$$\text{shown that } h(y) = \left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) e^{-(\alpha_A)y} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) e^{-(\alpha_B)y}. \quad (22)$$

Case(i) : The inter-decision times $U_i, i=1,2,3,\dots$ form a geometric process with parameter 'a'. Proceeding as in Model-1 it is found that

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a} \right)^k \left[\left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) W_{(\alpha_B, k)} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) W_{(\alpha_A, k)} \right] \quad (23)$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2} \right)^k \left[\left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) W_{(\alpha_B, k)} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) W_{(\alpha_A, k)} \right] - \\ (E[U])^2 \left(\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}} \right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}} \right)^2 \right] \right) \left[\left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) W_{(\alpha_B, k)} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) W_{(\alpha_A, k)} \right] \quad (24)$$

where $E[U]$ and $V[U]$ are given by (11).

Case (ii) : Consider the order statistics $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ as described in case(ii) of Model-1.

In this case it is found that

$$E[T] = E[U] \sum_{k=0}^{\infty} \left[\left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) W_{(\alpha_B, k)} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) W_{(\alpha_A, k)} \right] \quad (25)$$

and

$$E[T^2] = \sum_{k=0}^{\infty} \left(2k(E[U])^2 + E[U^2] \right) \left[\left(\frac{\alpha_B}{\alpha_B - \alpha_A} \right) W_{(\alpha_B, k)} - \left(\frac{\alpha_A}{\alpha_B - \alpha_A} \right) W_{(\alpha_A, k)} \right] \quad (26)$$

where $E[U]$ and $E[U^2]$ are given by (14) and (15).

Note :

Let Y_{A1} be the normal exponential threshold for the cumulative wastage in grade A and $1/\alpha_{A1}$ ($\alpha_{A1} > 0$) be its mean. Let Y_{A2} be the exponential threshold for frequent breaks taken by the existing workers in grade A and $1/\alpha_{A2}$ ($\alpha_{A2} > 0$) be its mean. Similarly, let Y_{B1} and Y_{B2} be the normal exponential threshold for wastages and exponential threshold for frequent breaks of existing workers in grade B with means $1/\alpha_{B1}, 1/\alpha_{B2}$ ($\alpha_{B1}, \alpha_{B2} > 0$) respectively. Let H_A , and H_B be the cumulative distribution function of Y_A , and Y_B respectively. It is assumed that Y_{A1}, Y_{A2}, Y_{B1} and Y_{B2} are statistically independent. Here the thresholds Y_A and Y_B for the grades A and B respectively are taken as $Y_A = Y_{A1} + Y_{A2}$ and $Y_B = Y_{B1} + Y_{B2}$. Since $Y_A = Y_{A1} + Y_{A2}$ and $Y_B = Y_{B1} + Y_{B2}$ it can be shown that

$$H_A(x) = 1 - \left(\frac{\alpha_{A2}}{\alpha_{A2} - \alpha_{A1}} \right) e^{-\alpha_{A1}x} + \left(\frac{\alpha_{A1}}{\alpha_{A2} - \alpha_{A1}} \right) e^{-\alpha_{A2}x} \quad (27)$$

and

$$H_B(x) = 1 - \left(\frac{\alpha_{B2}}{\alpha_{B2} - \alpha_{B1}} \right) e^{-\alpha_{B1}x} + \left(\frac{\alpha_{B1}}{\alpha_{B2} - \alpha_{B1}} \right) e^{-\alpha_{B2}x} \quad (28)$$

Model-1

Since $Y = \min(Y_A, Y_B)$, from (28) and (29) we get

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$$h(y) = \gamma_1(\alpha_{A1} + \alpha_{B1})e^{-(\alpha_{A1} + \alpha_{B1})y} + \gamma_2(\alpha_{A2} + \alpha_{B2})e^{-(\alpha_{A2} + \alpha_{B2})y} - \gamma_3(\alpha_{A1} + \alpha_{B2})e^{-(\alpha_{A1} + \alpha_{B2})y} - \gamma_4(\alpha_{A2} + \alpha_{B1})e^{-(\alpha_{A2} + \alpha_{B1})y} \quad (29)$$

where

$$\gamma_1 = \frac{\alpha_{A2}\alpha_{B2}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B2} - \alpha_{B1})}, \gamma_2 = \frac{\alpha_{A1}\alpha_{B1}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B2} - \alpha_{B1})}, \gamma_3 = \frac{\alpha_{A2}\alpha_{B1}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B2} - \alpha_{B1})}$$

and $\gamma_4 = \frac{\alpha_{A1}\alpha_{B2}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B2} - \alpha_{B1})}$. (30)

Case(i) : The inter-decision times $U_i, i=1,2,3,\dots$ form a geometric process with parameter 'a'.

In this case the mean and variance are given by

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k \left[\gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} + \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} - \gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} - \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} \right] \quad (31)$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2}\right)^k \left[\gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} + \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} - \gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} - \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} \right] - (E[U])^2$$

$$\left(\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}} \right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}} \right)^2 \right] \right) \left[\gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} + \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} - \gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} - \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} \right] \quad (32)$$

where $E[U]$ and $V[U]$ are given by (11).

Case (ii) : Consider the order statistics $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ as described in case(ii) of Model-1.

In this case the mean and variance are given by

$$E[T] = E[U] \sum_{k=0}^{\infty} \left[\gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} + \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} - \gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} - \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} \right] \quad (33)$$

and

$$E[T^2] = \sum_{k=0}^{\infty} \left(2k(E[U])^2 + E[U^2] \right) \left[\gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} + \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} - \gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} - \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} \right] \quad (34)$$

where $E[U]$ and $E[U^2]$ are given by (14) and (15).

Model-II

Since $Y = \max(Y_A, Y_B)$ from (28) and (29) we get

$$h(y) = \gamma_3(\alpha_{A1} + \alpha_{B2})e^{-(\alpha_{A1} + \alpha_{B2})y} + \gamma_4(\alpha_{A2} + \alpha_{B1})e^{-(\alpha_{A2} + \alpha_{B1})y} + \gamma_5(\alpha_{A1})e^{-(\alpha_{A1})y} + \gamma_6(\alpha_{B1})e^{-(\alpha_{B1})y} - \gamma_7(\alpha_{A2})e^{-(\alpha_{A2})y} - \gamma_8(\alpha_{B2})e^{-(\alpha_{B2})y} - \gamma_1(\alpha_{A1} + \alpha_{B1})e^{-(\alpha_{A1} + \alpha_{B1})y} - \gamma_2(\alpha_{A2} + \alpha_{B2})e^{-(\alpha_{A2} + \alpha_{B2})y}. \quad (35)$$

$$\gamma_5 = \frac{\alpha_{A2}}{(\alpha_{A2} - \alpha_{A1})}, \gamma_6 = \frac{\alpha_{A1}}{(\alpha_{A2} - \alpha_{A1})}, \gamma_7 = \frac{\alpha_{B2}}{(\alpha_{B2} - \alpha_{B1})} \text{ and } \gamma_8 = \frac{\alpha_{B1}}{(\alpha_{B2} - \alpha_{B1})}. \quad (36)$$

where

Case(i) : The inter-decision times $U_i, i=1,2,3,\dots$ form a geometric process with parameter 'a'.

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k \left[\gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} + \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} + \gamma_5 W_{(\alpha_{A1}, k)} + \gamma_6 W_{(\alpha_{B1}, k)} - \gamma_7 W_{(\alpha_{A2}, k)} - \gamma_8 W_{(\alpha_{B2}, k)} - \gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} - \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} \right] \quad (37)$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2}\right)^k \left[\gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} + \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} + \gamma_5 W_{(\alpha_{A1}, k)} + \gamma_6 W_{(\alpha_{B1}, k)} - \right. \\ \left. \gamma_7 W_{(\alpha_{A2}, k)} - \gamma_8 W_{(\alpha_{B2}, k)} - \gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} - \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} \right] - \\ (E[U])^2 \left[\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}} \right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}} \right)^2 \right] \left[\gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} + \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} + \gamma_5 W_{(\alpha_{A1}, k)} + \gamma_6 W_{(\alpha_{B1}, k)} - \right. \right. \\ \left. \left. \gamma_7 W_{(\alpha_{A2}, k)} - \gamma_8 W_{(\alpha_{B2}, k)} - \gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} - \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} \right] \right] \quad (38)$$

where $E[U]$ and $V[U]$ are given by (11).

Case (ii) : Consider the order statistics $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ as described in case(ii) of Model-1.

In this case, the mean and variance of time to recruitment are given by

$$E[T] = E[U] \sum_{k=0}^{\infty} \left[\gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} + \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} + \gamma_5 W_{(\alpha_{A1}, k)} + \gamma_6 W_{(\alpha_{B1}, k)} - \right. \\ \left. \gamma_7 W_{(\alpha_{A2}, k)} - \gamma_8 W_{(\alpha_{B2}, k)} - \gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} - \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} \right] \quad (39)$$

and

$$E[T^2] = \sum_{k=0}^{\infty} \left(2k(E[U])^2 + E[U^2] \right) \left[\gamma_3 W_{(\alpha_{A1} + \alpha_{B2}, k)} + \gamma_4 W_{(\alpha_{A2} + \alpha_{B1}, k)} + \gamma_5 W_{(\alpha_{A1}, k)} + \gamma_6 W_{(\alpha_{B1}, k)} - \right. \\ \left. \gamma_7 W_{(\alpha_{A2}, k)} - \gamma_8 W_{(\alpha_{B2}, k)} - \gamma_1 W_{(\alpha_{A1} + \alpha_{B1}, k)} - \gamma_2 W_{(\alpha_{A2} + \alpha_{B2}, k)} \right] \quad (40)$$

where $E[U]$ and $E[U^2]$ are given by (14) and (15).

Model-III

Since $Y = Y_A + Y_B$, from (28) and (29) we get

$$h(y) = \gamma_9 (\alpha_{A1}) e^{-(\alpha_{A1})y} + \gamma_{10} (\alpha_{B1}) e^{-(\alpha_{B1})y} - \gamma_{11} (\alpha_{A2}) e^{-(\alpha_{A2})y} - \gamma_{12} (\alpha_{B2}) e^{-(\alpha_{B2})y}. \quad (41)$$

where

$$\gamma_9 = \frac{\alpha_{A2} \alpha_{B1} \alpha_{B2}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B1} - \alpha_{A1})(\alpha_{B2} - \alpha_{A1})}, \gamma_{10} = \frac{\alpha_{A1} \alpha_{A2} \alpha_{B2}}{(\alpha_{B2} - \alpha_{B1})(\alpha_{B1} - \alpha_{A1})(\alpha_{B1} - \alpha_{A2})}, \\ \gamma_{11} = \frac{\alpha_{A1} \alpha_{B1} \alpha_{B2}}{(\alpha_{A2} - \alpha_{A1})(\alpha_{B1} - \alpha_{A2})(\alpha_{B2} - \alpha_{A2})} \text{ and } \gamma_{12} = \frac{\alpha_{A1} \alpha_{A2} \alpha_{B1}}{(\alpha_{B2} - \alpha_{B1})(\alpha_{B2} - \alpha_{A1})(\alpha_{B2} - \alpha_{A2})} \quad (42)$$

Case(i) : The inter-decision times $U_i, i=1,2,3,\dots$ form a geometric process with parameter 'a'.

In this case, the mean and variance of time to recruitment are given by

$$E[T] = E[U] \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k \left[\gamma_9 W_{(\alpha_{A1}, k)} + \gamma_{10} W_{(\alpha_{B1}, k)} - \gamma_{11} W_{(\alpha_{A2}, k)} - \gamma_{12} W_{(\alpha_{B2}, k)} \right] \quad (43)$$

and

$$E[T^2] = V[U] \sum_{k=0}^{\infty} \left(\frac{1}{a^2}\right)^k \left[\gamma_9 W_{(\alpha_{A1}, k)} + \gamma_{10} W_{(\alpha_{B1}, k)} - \gamma_{11} W_{(\alpha_{A2}, k)} - \gamma_{12} W_{(\alpha_{B2}, k)} \right] - \\ (E[U])^2 \left[\sum_{k=0}^{\infty} \left[\left(\sum_{i=1}^k \frac{1}{a^{i-1}} \right)^2 - \left(\sum_{i=1}^{k+1} \frac{1}{a^{i-1}} \right)^2 \right] \left[\gamma_9 W_{(\alpha_{A1}, k)} + \gamma_{10} W_{(\alpha_{B1}, k)} - \gamma_{11} W_{(\alpha_{A2}, k)} - \gamma_{12} W_{(\alpha_{B2}, k)} \right] \right] \quad (44)$$

Where $E[U]$ and $V[U]$ are given by (11).

Case (ii) : Consider the order statistics $U_{(1)}, U_{(2)}, \dots, U_{(m)}$ as described in case(ii) of Model-1.

In this case, the mean and variance of time to recruitment are given by

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$$E[T] = E[U] \sum_{k=0}^{\infty} [\gamma_9 W_{(\alpha_{A1}, k)} + \gamma_{10} W_{(\alpha_{B1}, k)} - \gamma_{11} W_{(\alpha_{A2}, k)} - \gamma_{12} W_{(\alpha_{B2}, k)}] \quad (45)$$

and

$$E[T^2] = \sum_{k=0}^{\infty} (2k(E[U])^2 + E[U^2]) [\gamma_9 W_{(\alpha_{A1}, k)} + \gamma_{10} W_{(\alpha_{B1}, k)} - \gamma_{11} W_{(\alpha_{A2}, k)} - \gamma_{12} W_{(\alpha_{B2}, k)}] \quad (46)$$

where $E[U]$ and $E[U^2]$ are given by (14) and (15).

V. NUMERICAL ILLUSTRATION

The influence of parameters on the performance measures namely the mean and variance of the time for recruitment is studied numerically. In the following table these performance measures are calculated by varying the parameter ‘ ρ ’ and the other parameters $\alpha_{A1}=0.1$, $\alpha_{A2}=0.2$, $\alpha_{B1}=0.3$, $\alpha_{B2}=0.4$, $p=0.4$, $\lambda_h=0.3$ and $\lambda_l=0.2$.

Table : Effect of ‘ ρ ’ on the performance measures $E[T]$ and $V[T]$

ρ	Case(i)				Case(ii)	
	$f(t) = f_{U(1)}(t)$		$f(t) = f_{U(m)}(t)$			
	E[T]	V[T]	E[T]	V[T]	E[T]	V[T]
MODEL -1						
-0.6	0.6706	0.21455	14.7397	38.6722	4.0382	11.80426
-0.3	0.6334	0.209397	13.9225	33.2705	3.8701	11.28273
0.3	0.5294	0.188413	11.6371	21.3412	3.3776	9.734001
0.6	0.4452	0.163272	9.786	15.6169	2.9573	8.309645
MODEL -2						
-0.6	0.7989	0.211597	17.5607	67.3373	4.6217	13.06908
-0.3	0.7794	0.214113	17.1308	61.9708	4.5343	12.88459
0.3	0.6962	0.215368	15.3029	43.7146	4.1640	11.92752
0.6	0.6032	0.204739	13.2588	29.1119	3.7368	10.68055
MODEL -3						
-0.6	0.8482	0.1990	18.6427	83.8940	4.8480	13.4147
-0.3	0.8382	0.2009	18.4243	80.8459	4.8045	13.3338
0.3	0.7812	0.2084	17.1717	65.1476	4.5549	12.8000
0.6	0.6975	0.2114	15.3309	45.9037	4.1774	11.9093

From the above table it is found that

$E(T)$ and $V(T)$ decreases in all the three models as ‘ ρ ’ increases for the cases(i) and (ii).

Conclusion : Since the time to recruitment is more elongated in Model-3 than the first two Models, Model-3 is preferable from the organization point of view.

REFERENCES:

- [1] Barthlomew,D.J., and Forbes,A.F., “Statistical techniques for man power planning”, JohnWiley&Sons, 1979.



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International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

- [2] Grinold,R.C., and Marshall,K.J.,”Man Power Planning”, NorthHolland, Newyork , 1977.
- [3] Muthaiyan,A., “A study on stochastic models in manpower planning”, Ph.D. thesis, Bharathidasan University, Thiruchirappalli, 2010.
- [4] Parameswari,K., Sridharan,J., and Srinivasan,A., “Time to recruitment in a two grade manpower system having correlated wastage based on order statistics”, Archimedes Journal of Mathematics, Vol.3(2), pp.171-196, 2013.
- [5] Sathiyamoorthi,R., and Elangovan,R., “Shock model approach to determine the expected time for recruitment”, Journal of Decision and Mathematika Sciences, Vol.3(1-3), pp.67-78, 1998.
- [6] Sridharan,J., Parameswari, K., and Srinivasan,A., “A stochastic model on time to recruitment in a two grade manpower system having correlated wastages”, Bessel Journal of Mathematics, Vol.3, pp.209-224, 2013.