

On Topological $sb\hat{g}$ – quotient mappings

K.Bala Deepa Arasi¹, S.Navaneetha Krishnan² and S.Pious Missier³

¹ Assistant Professor of Mathematics, A.P.C.Mahalaxmi College for Women, Thoothukudi, TN, India

^{2,3} Associate Professor of Mathematics, V.O. Chidambaram College, Thoothukudi, TN, India

ABSTRACT: In this paper, we define new class of mappings namely $sb\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce some new mappings namely strongly $sb\hat{g}$ -open map, strongly $sb\hat{g}$ -quotient map, $sb\hat{g}^*$ -quotient map and strongly $sb\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

KEYWORDS: $sb\hat{g}$ – closed set, $sb\hat{g}$ -continuous function, $sb\hat{g}$ -irresolute function, $sb\hat{g}$ -quotient mapping, strongly $sb\hat{g}$ -open map, strongly $sb\hat{g}$ -quotient map, $sb\hat{g}^*$ -quotient map, strongly $sb\hat{g}^*$ -quotient map.

I. INTRODUCTION

In 1963, N.Levine[8] introduced semi-open sets in Topology and studied their properties. Andrijevic[1] introduced one such new version called b-open sets in 1996. N.Levine[9] introduced the concept of generalized closed sets and studied their properties in 1970. By considering the concept of g-closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. M.K.R.S.Veerakumar[13] defined \hat{g} -closed sets in 2003. Also, R.Subasree and M.Mariasingam[10] introduced $b\hat{g}$ -closed sets in 2013. We introduced $sb\hat{g}$ -closed sets[2] and studied their properties in 2015.

In this paper, we define new class of mappings namely $sb\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce some new mappings namely strongly $sb\hat{g}$ -open map, strongly $sb\hat{g}$ -quotient map, $sb\hat{g}^*$ -quotient map and strongly $sb\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

II. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces in which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ) , $Cl(A)$, $Int(A)$ and A^c denote the closure of A , interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X, τ) is called a

- (i) semi-open set[8] if $A \subseteq Cl(Int(A))$.
- (ii) α -open set[7] if $A \subseteq Int(Cl(Int(A)))$.
- (iii) b-open set[1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

Definition 2.2: A subset A of a topological space (X, τ) is called a

- (i) $b\hat{g}$ -closed set[10] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X .
- (ii) $sb\hat{g}$ -closed set[2] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $b\hat{g}$ -open in X .

The complements of the above mentioned closed sets and their respective open sets.

Remark 2.3: The family of $sb\hat{g}$ -closed sets and $sb\hat{g}$ -open sets are denoted by $sb\hat{g}-C(X)$ and $sb\hat{g}-O(X)$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Definition 2.4: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a

- (i) continuous[6] if $f^{-1}(V)$ is closed in X for every closed set V in Y .
- (ii) α -continuous[6] if $f^{-1}(V)$ is α -closed in X for every closed set V in Y .
- (iii) $sb\hat{g}$ -continuous[11] if $f^{-1}(V)$ is $sb\hat{g}$ -closed in X for every closed set V in Y .
- (iv) $sb\hat{g}$ -continuous[3] if $f^{-1}(V)$ is $sb\hat{g}$ -closed in X for every closed set V in Y .
- (v) $sb\hat{g}$ – irresolute map[3] if $f^{-1}(V)$ is $sb\hat{g}$ -closed in (X, τ) for every $sb\hat{g}$ -closed set V in (Y, σ) .
- (vi) $sb\hat{g}$ - open map[3] if $f(V)$ is $sb\hat{g}$ – open in (Y, σ) for every open set V in (X, τ) .
- (vii) Contra $sb\hat{g}$ -continuous map[4] if $f^{-1}(V)$ is $sb\hat{g}$ -closed in (X, τ) for every open set V in (Y, σ) .
- (viii) $sb\hat{g}$ – homeomorphism[3] if f is both $sb\hat{g}$ – continuous map and $sb\hat{g}$ – open map.
- (ix) Strongly $sb\hat{g}$ -continuous[3] if $f^{-1}(V)$ is closed in (X, τ) for every $sb\hat{g}$ -closed set V in (Y, σ) .
- (x) Quotient[12] if f is continuous and $f^{-1}(V)$ is open in X implies V is open in Y .
- (xi) α -quotient[12] if f is α -continuous and $f^{-1}(V)$ is open in X implies V is α -open in Y .
- (xii) $sb\hat{g}$ -quotient[12] if f is $sb\hat{g}$ -continuous and $f^{-1}(V)$ is open in X implies V is $sb\hat{g}$ -open in Y .

Definition 2.5: A space X is called a $T_{sb\hat{g}}$ -space[2] if every $sb\hat{g}$ -closed set in it is closed.

III. $sb\hat{g}$ – QUOTIENT MAP

Definition 3.1: A surjective map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a $sb\hat{g}$ -quotient map if f is $sb\hat{g}$ -continuous and $f^{-1}(V)$ is open in (X, τ) implies V is $sb\hat{g}$ -open in (Y, σ) .

Example 3.2: Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \{p\}, \{p, q\}, \{p, r\}\}$

Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = r, f(c) = q, f(d) = r$.

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$

Clearly, f is a $sb\hat{g}$ -quotient map.

Definition 3.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a strongly $sb\hat{g}$ -open map if $f(V)$ is $sb\hat{g}$ -open in Y for each $sb\hat{g}$ -open set V in X .

Example 3.4: Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r, s\}$ with topologies

$\tau = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$ and $\sigma = \{Y, \{p\}, \{p, r\}, \{p, q, s\}\}$

Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = s, f(b) = p, f(c) = r, f(d) = q$.

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}\}$

Clearly, f is a strongly $sb\hat{g}$ -open map.

Theorem 3.5: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is surjective $sb\hat{g}$ -continuous and $sb\hat{g}$ -open map then f is $sb\hat{g}$ -quotient map.

Proof: Let $f^{-1}(V)$ be open in (X, τ) . Since f is $sb\hat{g}$ -open map, $f(f^{-1}(V))$ is $sb\hat{g}$ -open in (Y, σ) . Since f is surjective, V is $sb\hat{g}$ -open in (Y, σ) . Hence, f is $sb\hat{g}$ -quotient map.

Theorem 3.6: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is surjective $sb\hat{g}$ -continuous and strongly $sb\hat{g}$ -open map then f is $sb\hat{g}$ -quotient map.

Proof: Let $f^{-1}(V)$ be open in (X, τ) . By Proposition 3.4 in [2], $f^{-1}(V)$ is $sb\hat{g}$ -open in (X, τ) . Since f is strongly $sb\hat{g}$ -open, $f(f^{-1}(V))$ is $sb\hat{g}$ -open in (Y, σ) . Since f is surjective, V is $sb\hat{g}$ -open in (Y, σ) . Hence, f is $sb\hat{g}$ -quotient map.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Corollary 3.7: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is surjective $sb\hat{g}$ -continuous and f^{-1} is $sb\hat{g}$ -irresolute then f is $sb\hat{g}$ -quotient map.

Proof: The proof follows from the fact that f^{-1} is $sb\hat{g}$ -irresolute and f is strongly $sb\hat{g}$ -open is equivalent.

Theorem 3.8: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is $sb\hat{g}$ -homeomorphism then f is $sb\hat{g}$ -quotient map.

Proof: Since f is $sb\hat{g}$ -homeomorphism, f is bijective and f is $sb\hat{g}$ -continuous. Let $f^{-1}(V)$ be open in (X,τ) . Since f^{-1} is $sb\hat{g}$ -continuous, $f(f^{-1}(V)) = V$ is $sb\hat{g}$ -open in (Y,σ) . Hence, f is $sb\hat{g}$ -quotient map.

Theorem 3.9: Every quotient map is $sb\hat{g}$ -quotient map

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a quotient map. By proposition 3.5(a) in [3] and proposition 3.4 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

Example 3.10: Let $X=\{a,b,c,d\}$ and $Y = \{p,q,r,s\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \{q\}, \{p,q\}, \{q,r,s\}\}$

Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = q, f(b) = p, f(c) = r, f(d) = s$.

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{q\}, \{p,q\}, \{q,r\}, \{q,s\}, \{p,q,r\}, \{p,q,s\}, \{q,r,s\}\}$

Clearly, f is $sb\hat{g}$ -quotient map. Since the inverse image of open sets $\{p,q\}, \{q,r,s\}$ in Y are $\{a,b\}, \{a,c,d\}$ which are not open in X , f is not continuous. Therefore, f is not a quotient map.

Theorem 3.11: Every α -quotient map is $sb\hat{g}$ -quotient map

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a α -quotient map. By proposition 3.5(b) in [3] and proposition 3.7 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

Example 3.12: Let $X=\{a,b,c\}$ and $Y = \{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = p, f(b) = q, f(c) = r$

$\alpha\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$

$\alpha\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Clearly, f is $sb\hat{g}$ -quotient map. Since the inverse image of open set $\{p,r\}$ in Y is $\{a,c\}$ which is not α -open in X , f is not α -continuous. Therefore, f is not α -quotient map.

Theorem 3.13: Every $sb\hat{g}$ -quotient map is $b\hat{g}$ -quotient map

Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a $sb\hat{g}$ -quotient map. By proposition 3.7(f) in [3] and proposition 3.23 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

Example 3.14: Let $X=\{a,b,c\}$ and $Y = \{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a,c\}\}$ and $\sigma = \{Y, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = p, f(b) = r, f(c) = q$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a,c\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Clearly, f is $\text{sb}\hat{g}$ -quotient map. Since the inverse image of open sets $\{p\}, \{q\}$ and $\{p,r\}$ in Y are $\{a\}, \{c\}$ and $\{a,b\}$ which are not $\text{sb}\hat{g}$ -open in X , f is not $\text{sb}\hat{g}$ -continuous. Therefore, f is not $\text{sb}\hat{g}$ -quotient map.

IV. STRONG FORM OF $\text{sb}\hat{g}$ – QUOTIENT MAP

Definition 4.1: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective map. Then f is called strongly $\text{sb}\hat{g}$ -quotient map provided a set V of (Y, σ) is open in Y if and only if $f^{-1}(V)$ is $\text{sb}\hat{g}$ -open in X .

Example 4.2: Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r\}$ with topologies
 $\tau = \{X, \Phi, \{d\}, \{a, b, c\}\}$ and $\sigma = \{Y, \Phi, \{r\}, \{p, q\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = f(b) = p, f(c) = q, f(d) = r$.
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{d\}, \{a, b, c\}\}$
 Clearly, f is strongly $\text{sb}\hat{g}$ -quotient map.

Theorem 4.3:

- (i) Every strongly $\text{sb}\hat{g}$ -quotient map is $\text{sb}\hat{g}$ -open map.
- (ii) Every strongly $\text{sb}\hat{g}$ -quotient map is strongly $\text{sb}\hat{g}$ -open map.

Proof:

- (i) Let f be a strongly $\text{sb}\hat{g}$ -quotient map and V be any open set in X . By Theorem 3.4 in [2], V is a $\text{sb}\hat{g}$ -open set in (X, τ) . That is, $f^{-1}(f(V))$ is $\text{sb}\hat{g}$ -open in (X, τ) . Since f is strongly $\text{sb}\hat{g}$ -quotient, $f(V)$ is open in Y and hence $\text{sb}\hat{g}$ -open in Y . Therefore, f is a $\text{sb}\hat{g}$ -open map.
- (ii) Let f be a strongly $\text{sb}\hat{g}$ -quotient map and V be any $\text{sb}\hat{g}$ -open set in X . That is $f^{-1}(f(V))$ is $\text{sb}\hat{g}$ -open in (X, τ) . Since f is strongly $\text{sb}\hat{g}$ -quotient, $f(V)$ is open in Y and hence $\text{sb}\hat{g}$ -open in Y . Hence, f is a strongly $\text{sb}\hat{g}$ -open map.

The following example shows that the converse of the above theorem need not be true.

Example 4.4:

- (i) Let $X = Y = \{a, b, c, d\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b, c\}, \{a, b, c\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = d, f(d) = c$.
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$
 $\text{sb}\hat{g}\text{-O}(Y) = \{Y, \Phi, \{a\}, \{b, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}\}$
 Clearly, f is a $\text{sb}\hat{g}$ -open map. Since the inverse image of an open set $\{b, c\}$ in Y is $\{b, d\}$ which is not $\text{sb}\hat{g}$ -open set in X , f is not $\text{sb}\hat{g}$ -continuous. Therefore, f is not strongly $\text{sb}\hat{g}$ -quotient map.
- (ii) Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \Phi, \{p\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = r$.
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}\}$
 $\text{sb}\hat{g}\text{-O}(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$
 Clearly, f is a strongly $\text{sb}\hat{g}$ -open map. Since the inverse image of $\text{sb}\hat{g}$ -open set $\{p, q\}$ and $\{p, r\}$ in Y are $\{a, b\}$ and $\{a, c\}$ which are not open in Y , f is not strongly $\text{sb}\hat{g}$ -quotient map.

Definition 4.5: A surjective map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be a $\text{sb}\hat{g}^*$ -quotient map, if f is $\text{sb}\hat{g}$ -irresolute and $f^{-1}(V)$ is $\text{sb}\hat{g}$ -open in X implies V is open in Y .

Example 4.6: Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r\}$ with topologies
 $\tau = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = r, f(b) = p, f(c) = q, f(d) = p$.
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$
Clearly, f is $sb\hat{g}^*$ -quotient map.

Theorem 4.7:

- (i) Every $sb\hat{g}^*$ -quotient map is $sb\hat{g}$ -irresolute.
- (ii) Every $sb\hat{g}^*$ -quotient map is strongly $sb\hat{g}$ -open map.

Proof:

- (i) Proof follows from definitions 4.5 and proposition 3.4 in [2].
- (ii) Let f be a $sb\hat{g}^*$ -quotient map. Let V be any $sb\hat{g}$ -open set in X . Then $f^{-1}(f(V))$ is $sb\hat{g}$ -open in (X, τ) . Since f is $sb\hat{g}^*$ -quotient, $f(V)$ is open in Y and hence from proposition 3.4 in [2], $f(V)$ is $sb\hat{g}$ -open in Y . Hence, f is a strongly $sb\hat{g}$ -open map.

The following example shows that the converse of the above theorem need not be true.

Example 4.8:

- (i) Let $X = \{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{p,r\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = f(d) = r$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,r\}\}$
 Clearly, f is $sb\hat{g}$ -irresolute map. Since the inverse image of $sb\hat{g}$ -open sets $\{a,b\}$ in X is $\{p,q\}$ which is not open in Y , f is not $sb\hat{g}^*$ -quotient map.
- (ii) Let $X=Y = \{a,b,c,d\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{b,c,d\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c, f(d) = d$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b,c,d\}\}$
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$
 Clearly, f is a strongly $sb\hat{g}$ -open map. Since the inverse image of the $sb\hat{g}$ -open sets $\{a,d\}, \{b,c\}$ and $\{a,b,c\}$ in Y are $\{a,d\}, \{b,c\}$ and $\{a,b,c\}$ which are not $sb\hat{g}$ -open in X , f is not $sb\hat{g}^*$ -quotient map.

Definition 4.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective map. Then f is called strongly $sb\hat{g}^*$ -quotient if a set V is $sb\hat{g}$ -open in Y if and only if $f^{-1}(V)$ is $sb\hat{g}$ -open in X .

Example 4.10: Let $X = \{a,b,c\}$ and $Y = \{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{p,q\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = r$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$
 Clearly, f is strongly $sb\hat{g}^*$ -quotient map.

Theorem 4.11: Every $sb\hat{g}^*$ -quotient map is strongly $sb\hat{g}^*$ -quotient map.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be $sb\hat{g}^*$ -quotient map. Let V be a $sb\hat{g}$ -open set in Y . Since f is $sb\hat{g}^*$ -irresolute, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Conversely, Let $f^{-1}(V)$ be $sb\hat{g}$ -open in X . Since f is $sb\hat{g}^*$ -quotient, V is open in Y . By Theorem 3.4 in [2], V is $sb\hat{g}$ -open in Y . Therefore, f is strongly $sb\hat{g}^*$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

Example 4.12: Let $X = \{a,b,c\}$ and $Y = \{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{p,q\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = r$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$$

Clearly, f is a strongly $sb\hat{g}^*$ -quotient map. Since $f^{-1}(\{p, r\}) = \{a, c\}$ which is $sb\hat{g}$ -open in X but $\{p,r\}$ is not open in Y , f is not $sb\hat{g}^*$ -quotient map.

V. COMPARISONS

Theorem 5.1: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective map. Then f is strongly $sb\hat{g}$ -quotient map if and only if f is $sb\hat{g}^*$ -quotient.

Proof: Let V be any $sb\hat{g}$ -open set in Y . As f is surjective, $f(f^{-1}(V))$ is $sb\hat{g}$ -open in Y . By Theorem 4.3(ii), f is strongly $sb\hat{g}$ -open map. Hence, $f(f^{-1}(V))$ is $sb\hat{g}$ -open in Y implies $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Thus, f is a $sb\hat{g}$ -irresolute map. Now if $f^{-1}(V)$ is $sb\hat{g}$ -open in X and since f is strongly $sb\hat{g}$ -quotient map, V is open in Y . This means that $f^{-1}(V)$ is $sb\hat{g}$ -open in X implies V is open in Y . Hence, f is $sb\hat{g}^*$ -quotient map.

Conversely, Let V be any open set in Y . By Theorem 3.4 in [2], V is $sb\hat{g}$ -open in Y . Since f is $sb\hat{g}$ -irresolute, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . This means that V is open in Y implies $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Now if $f^{-1}(V)$ is $sb\hat{g}$ -open in X and since f is $sb\hat{g}^*$ -quotient map, V is open set in Y . This means that $f^{-1}(V)$ is $sb\hat{g}$ -open in X implies V is open in Y . Hence, f is strongly $sb\hat{g}$ -quotient map.

Theorem 5.2:

- (i) Every strongly $sb\hat{g}$ -quotient map is $sb\hat{g}$ -quotient map.
- (ii) Every $sb\hat{g}^*$ -quotient map is $sb\hat{g}$ -quotient map.

Proof:

- (i) Let V be any open set in Y . Since f is strongly $sb\hat{g}$ -quotient map, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . This means that f is $sb\hat{g}$ -continuous. Now let $f^{-1}(V)$ be any open set in X . By proposition 3.4 in [2], $f^{-1}(V)$ is $sb\hat{g}$ -open in X and since f is strongly $sb\hat{g}$ -quotient map, V is an open set in Y . Therefore, V is $sb\hat{g}$ -open in Y . This means that $f^{-1}(V)$ is open in X implies V is $sb\hat{g}$ -open in Y . Hence, f is $sb\hat{g}$ -quotient map.
- (ii) Let V be any open set in Y . By proposition 3.4 in [2], V is $sb\hat{g}$ -open in Y . Since f is $sb\hat{g}^*$ -quotient map, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . This means that for every open set V in Y , $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Now let $f^{-1}(V)$ be any open set in X . Again by proposition 3.4 in [2], $f^{-1}(V)$ is $sb\hat{g}$ -open in X and since f is $sb\hat{g}^*$ -quotient map, V is open in Y . Therefore, V is $sb\hat{g}$ -open in Y . This means that $f^{-1}(V)$ is open in X implies V is $sb\hat{g}$ -open in Y . Hence, f is $sb\hat{g}$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

Example 5.3:

- (i) Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = r$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$
 Clearly, f is $sb\hat{g}$ -quotient map. Since $f^{-1}(\{q, r\}) = \{b, c\}$ which is $sb\hat{g}$ -open in X , but $\{q, r\}$ is not open in Y , f is not strongly $sb\hat{g}$ -quotient map.
- (ii) Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r, s\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$ and $\sigma = \{Y, \Phi, \{q\}, \{p, q\}, \{q, r, s\}\}$
 Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = q, f(b) = p, f(c) = r, f(d) = s$.
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{q\}, \{p, q\}, \{q, r\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}\}$
 Clearly, f is $sb\hat{g}$ -quotient map. Since $f^{-1}(\{q, r\}) = \{a, c\}$ which is $sb\hat{g}$ -open in X , but $\{q, r\}$ is not open in Y , f is not strongly $sb\hat{g}$ -quotient map.

Theorem 5.4: Every strongly $sb\hat{g}^*$ -quotient map is $sb\hat{g}$ -quotient.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be strongly $sb\hat{g}^*$ -quotient map. Let V be an open set in Y . By Theorem 3.4 in [2], V is $sb\hat{g}$ -open set in Y . Since f is strongly $sb\hat{g}^*$ -quotient, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Hence, f is $sb\hat{g}$ -continuous. Let $f^{-1}(V)$ be an open set in X . Then, $f^{-1}(V)$ is $sb\hat{g}$ -open in X . Since f is strongly $sb\hat{g}^*$ -quotient map, V is $sb\hat{g}$ -open in Y . Therefore, f is $sb\hat{g}$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

Example 5.5: Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p, f(b) = q, f(c) = r$.

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

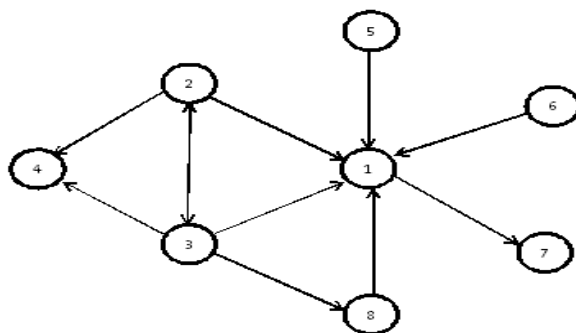
$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

Clearly, f is $sb\hat{g}$ -quotient. Since $f^{-1}(\{q, r\}) = \{b, c\}$ which is $sb\hat{g}$ -open in X , but $\{q, r\}$ is not $sb\hat{g}$ -open in Y , f is not strongly $sb\hat{g}^*$ -quotient map.

Theorem 5.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be strongly $sb\hat{g}^*$ -quotient map and Y is $T_{sb\hat{g}}$ -space. Then f is $sb\hat{g}^*$ -quotient map.

Proof: Let U be $sb\hat{g}$ -open in Y . Since f is strongly $sb\hat{g}^*$ -quotient map, $f^{-1}(U)$ is $sb\hat{g}$ -open in X . Hence, f is $sb\hat{g}$ -irresolute function. Let $f^{-1}(U)$ be $sb\hat{g}$ -open in X . Since f is strongly $sb\hat{g}^*$ -quotient map, U is $sb\hat{g}$ -open in Y . Since Y is $T_{sb\hat{g}}$ -space, U is open in Y . Hence, f is $sb\hat{g}^*$ -quotient map.

Remark 5.7: From the above discussions and known results, we have the following diagram. In this diagram, $A \rightarrow B$ means A implies B but not conversely. $A \leftrightarrow B$ means A and B are independent of each other.



- | | | |
|-----------------------------------|---|--------------------------------|
| 1. $sb\hat{g}$ -quotient map | 2. Strongly $sb\hat{g}$ -quotient map | 3. $sb\hat{g}^*$ -quotient map |
| 4. Strongly $sb\hat{g}$ -open map | 5. Quotient map | 6. α -quotient map. |
| 7. $b\hat{g}$ -quotient map | 8. Strongly $sb\hat{g}^*$ -quotient map | |

VI. APPLICATIONS OF $sb\hat{g}$ -QUOTIENT MAPS

Theorem 6.1: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an open surjective, $sb\hat{g}$ -irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $sb\hat{g}$ -quotient map. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $sb\hat{g}$ -quotient map.

Proof: Let V be any open set in (Z, η) . Since g is a $sb\hat{g}$ -quotient map, $g^{-1}(V)$ is $sb\hat{g}$ -open in Y and since f is $sb\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $sb\hat{g}$ -open set in (X, τ) . This means that for every open set V in (Z, η) , $(g \circ f)^{-1}(V)$ is $sb\hat{g}$ -open in (X, τ) . Hence, $g \circ f$ is a $sb\hat{g}$ -continuous map.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Now assume that $(gof)^{-1}(V)$ is open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is open in X . Since f is an open map, $f[f^{-1}(g^{-1}(V))]$ is open in Y . It follows that $g^{-1}(V)$ is open in Y , because f is surjective. Since g is a sbg-quotient map, V is sbg-open in Z . Hence, $gof: (X, \tau) \rightarrow (Z, \eta)$ is a sbg-quotient map.

Theorem 6.2: Let $h: (X, \tau) \rightarrow (Y, \sigma)$ is a sbg-quotient map and $g: (X, \tau) \rightarrow (Z, \eta)$ is a continuous map that is constant on each set $h^{-1}(y)$ for $y \in Y$, then g induces a sbg-Continuous map $f: (Y, \sigma) \rightarrow (Z, \eta)$ such that $f \circ h = g$.

Proof: To prove: f is sbg-continuous. Since g is constant on $h^{-1}(y)$ for each $y \in Y$, the set $g[h^{-1}(y)]$ is a one point set in (Z, η) . If $f(y)$ denote this point, then it is clear that f is well-defined and for each $x \in X$, $f(h(x)) = g(x)$. We claim that f is sbg-continuous. For, if we let V be any open set in (Z, η) then $g^{-1}(V)$ is an open set as g is continuous. But $g^{-1}(V) = h^{-1}(f^{-1}(V))$ is open in (X, τ) . Since h is a sbg-quotient map, $f^{-1}(V)$ is a sbg-open set. Hence, f is sbg-Continuous.

Theorem 6.3: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a strongly sbg-open, surjective and sbg-irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a strongly sbg-quotient map. Then, $gof: (X, \tau) \rightarrow (Z, \eta)$ is a strongly sbg-quotient map.

Proof: To Prove: $gof: (X, \tau) \rightarrow (Z, \eta)$ is a strongly sbg-quotient map. Let V be any open set in (Z, η) . Since g is a strongly sbg-quotient map, $g^{-1}(V)$ is sbg-open set in Y and since f is sbg-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is a sbg-open set in (X, τ) . This means that for every open set V in (Z, η) , $(gof)^{-1}(V)$ is sbg-open in (X, τ) . Hence, gof is a sbg-continuous map.

Now assume that $(gof)^{-1}(V)$ is sbg-open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is sbg-open in X . Since f is strongly sbg-open, $f[f^{-1}(g^{-1}(V))]$ is sbg-open in Y . Since f is surjective, it follows that, $g^{-1}(V)$ is sbg-open in Y . This gives that, V is an open set in Y , since g is strongly sbg-quotient. This implies that, if $(gof)^{-1}(V)$ is sbg-open in X implies V is open in Y . Hence, gof is strongly sbg-quotient map.

Theorem 6.4: Let $h: (X, \tau) \rightarrow (Y, \sigma)$ be a sbg-quotient map where (X, τ) and (Y, σ) are T_{sbg} -spaces. Then $f: (Y, \sigma) \rightarrow (Z, \eta)$ is strongly sbg-continuous if and only if the composite map $f \circ h: (X, \tau) \rightarrow (Z, \eta)$ is strongly sbg-continuous.

Proof: Let V be any open set in (Z, η) . Then $f^{-1}(V)$ is open in Y , since f is strongly sbg-continuous. Now, $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is sbg-open in X , since h is a sbg-quotient map. Since X is a T_{sbg} -space, $h^{-1}(f^{-1}(V))$ is open in X . This implies that for every sbg-open set V in Z , $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is open in X . Thus, $f \circ h$ is strongly sbg-continuous.

Conversely, let $f \circ h$ is strongly sbg-continuous, then for every sbg-open set V in Z , $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is open in X . Since h is a sbg-quotient map, $f^{-1}(V)$ is sbg-open in Y . Since Y is a T_{sbg} -space, $f^{-1}(V)$ is open in Y . Hence, f is a strongly sbg-continuous map.

Theorem 6.5: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective, strongly sbg-open and sbg-irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a sbg*-quotient map. Then $gof: (X, \tau) \rightarrow (Z, \eta)$ is sbg*-quotient map.

Proof: Let V be a sbg-open set in (Z, η) . Since g is a sbg*-quotient map, $g^{-1}(V)$ is sbg-open in Y . Since f is sbg-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is a sbg-open set in (X, τ) . This means that for every sbg-open set V in (Z, η) , $(gof)^{-1}(V)$ is sbg-open in (X, τ) . Hence, gof is a sbg-irresolute map.

Now, suppose that $(gof)^{-1}(V)$ is sbg-open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is sbg-open in X . Since f is strongly sbg-open, $f[f^{-1}(g^{-1}(V))]$ is sbg-open in Y . Since f is surjective, it follows that, $g^{-1}(V)$ is sbg-open in Y . This gives that V is an open set in Y , since g is sbg*-quotient. This implies that, if $(gof)^{-1}(V)$ is sbg-open in X implies V is open in Y . Hence, gof is a sbg*-quotient map.

Theorem 6.6: The composition of two sbg*-quotient maps is also sbg*-quotient.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two sbg*-quotient maps. To Prove their composition $gof: (X, \tau) \rightarrow (Z, \eta)$ is a sbg*-quotient map. Let V be a sbg-open set in (Z, η) . Since g is sbg*-quotient, $g^{-1}(V)$ is

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$sb\hat{g}$ -open in Y . Since f is $sb\hat{g}^*$ -quotient, $f^{-1}(g^{-1}(V))$ is $sb\hat{g}$ -open in X . That is, $(gof)^{-1}(V)$ is $sb\hat{g}$ -open in X . Hence, gof is $sb\hat{g}$ -irresolute. Let $(gof)^{-1}(V)$ is $sb\hat{g}$ -open in X . That is $f^{-1}(g^{-1}(V))$ is $sb\hat{g}$ -open in X . Since f is $sb\hat{g}^*$ -quotient, $g^{-1}(V)$ is open in Y , then $g^{-1}(V)$ is $sb\hat{g}$ -open in Y . Since g is $sb\hat{g}^*$ -quotient, V is open in Z . This implies that, $(gof)^{-1}(V)$ is $sb\hat{g}$ -open in X implies V is open in Z . Hence, gof is $sb\hat{g}^*$ -quotient map.

Theorem 6.7: The composition of two strongly $sb\hat{g}$ -open map is also strongly $sb\hat{g}$ -open map.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two strongly $sb\hat{g}$ -open maps. To Prove their composition $gof: (X, \tau) \rightarrow (Z, \eta)$ is a strongly $sb\hat{g}$ -open map. Let U be $sb\hat{g}$ -open set in X . Since f is strongly $sb\hat{g}$ -open map, $f(U)$ is $sb\hat{g}$ -open set in Y . Since g is strongly $sb\hat{g}$ -open map, $g(f(U))$ is $sb\hat{g}$ -open set in Z . Therefore, gof is strongly $sb\hat{g}$ -open map.

Theorem 6.8: The composition of two strongly $sb\hat{g}$ -quotient map is also strongly $sb\hat{g}$ -quotient map.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two strongly $sb\hat{g}$ -quotient maps. By theorem 5.1, f and g are $sb\hat{g}^*$ -quotient map. By theorem 6.6, gof is $sb\hat{g}^*$ -quotient map. By the converse part of theorem 5.1, gof is strongly $sb\hat{g}$ -quotient map.

Theorem 6.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a strongly $sb\hat{g}$ -quotient map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be $sb\hat{g}^*$ -quotient map and Y be $T_{sb\hat{g}}$ -space. Then $gof: (X, \tau) \rightarrow (Z, \eta)$ is $sb\hat{g}^*$ -quotient.

Proof: Let V be a $sb\hat{g}$ -open set in Z . Since g is $sb\hat{g}^*$ -quotient map, $g^{-1}(V)$ is $sb\hat{g}$ -open in Y . Since Y is $T_{sb\hat{g}}$ -space, $g^{-1}(V)$ is open in Y . Since f is strongly $sb\hat{g}$ -quotient map, $f^{-1}(g^{-1}(V))$ is $sb\hat{g}$ -open in X . That is, $(gof)^{-1}(V)$ is $sb\hat{g}$ -open in X . Hence, $gof: (X, \tau) \rightarrow (Z, \eta)$ is $sb\hat{g}$ -irresolute. Let $(gof)^{-1}(V)$ be $sb\hat{g}$ -open in X . That is, $f^{-1}(g^{-1}(V))$ is $sb\hat{g}$ -open in X . Since f is strongly $sb\hat{g}$ -quotient map, $g^{-1}(V)$ is open in Y . By Proposition 3.4 in [2], $g^{-1}(V)$ is $sb\hat{g}$ -open in Y . Since g is $sb\hat{g}^*$ -quotient map, V is open in Z . Hence, gof is $sb\hat{g}^*$ -quotient.

VII. CONCLUSION

The properties of $sb\hat{g}$ -quotient map, $sb\hat{g}^*$ -quotient map and strong form of $sb\hat{g}$ -quotient map have been studied. Further the relation between strong and weak form of $sb\hat{g}$ -quotient map have been investigated. Based on composite function, several theorems have been proved by using $sb\hat{g}$ -quotient map.

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