

Comparison of the Lateral Surface Area of a Cube with Fascinating Numbers

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ABSTRACT: In this paper, we scrutinize the relation between the lateral surface area of a cube and some fascinating numbers by applying Bhramaguptha lemma.

KEYWORDS: Binary quadratic equation, Integral Solutions, Pell Equation

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Notations Used: $S_n = 6n(n-1)+1$ be a star number of rank n

$CH_n = \frac{1}{2}[7n^2 - 7n + 2]$ be a centred heptagonal number of rank n

$CHD_n = \frac{1}{2}[17n^2 - 17n + 2]$ be a centred heptadecagonal number of rank n

I. INTRODUCTION

The theory of numbers offers a rich variety of fascinating problems [1-4]. The equation of the form $y^2 = Dx^2 + 1$ where D is a given positive square free integer is known as Pell equation and is one of the oldest Diophantine equation that has interested mathematicians all over the world have integer solutions. In this context, one may refer [5-9]. In [10-14], the relations between Pythagorean triangles and polygonal numbers are analysed. In this communication, we present the relation between the lateral surface area of a cube and some fascinating numbers. Also, we find some interesting relations among the ranks of the numbers and the edge of a cube.

II. METHOD OF ANALYSIS

This paper consists of three sections. In each section, the relation between the lateral surface area of a cube and some fascinating numbers such as star number, centred heptagonal number and centred heptadecagonal number are determined by applying Bhramaguptha lemma which is explained below.

III. RELATION BETWEEN THE LATERAL SURFACE AREA OF A CUBE AND STAR NUMBER

Assume that the lateral surface area of the cube of side a unit is equal to four times star number of rank n . The mathematical statement of our assumption is

$$4a^2 = 4S_n \tag{1}$$

which reduces to $y^2 = 6x^2 - 2$ (2)

where $y = 2a, x = 2n - 1$ (3)

The initial solution to (2) is given by

$$x_0 = 1, y_0 = 2$$

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To find the general solution to (2), consider the corresponding Pellian equation

$$y^2 = 6x^2 + 1 \tag{4}$$

whose initial solution is given by

$$\tilde{x}_0 = 2, \tilde{y}_0 = 5$$

Therefore, the general solution to (4) are represented by

$$\tilde{y}_s = \frac{1}{2} f_s, \tilde{x}_s = \frac{1}{2\sqrt{6}} g_s$$

where

$$f_s = (5 + 2\sqrt{6})^{s+1} + (5 - 2\sqrt{6})^{s+1}$$

$$g_s = (5 + 2\sqrt{6})^{s+1} - (5 - 2\sqrt{6})^{s+1}, s = -1, 0, 1, 2, 3, \dots$$

By applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, the non-trivial integer solutions to (1) are pointed out by

$$x_{s+1} = \frac{1}{2} f_s + \frac{1}{\sqrt{6}} g_s \tag{5}$$

$$y_{s+1} = f_s + \frac{3}{\sqrt{6}} g_s \tag{6}$$

Comparing (3) and (5), we obtain

$$n_{s+1} = \frac{1}{4} f_s + \frac{1}{2\sqrt{6}} g_s + 1 \tag{7}$$

Equating (3) and (6), we get

$$a_{s+1} = \frac{1}{2} f_s + \frac{3}{2\sqrt{6}} g_s \tag{8}$$

Using (7) and (8), the recurrence relations for the values of the ranks of star number and the sides of the cube are represented by

$$n_{s+1} + n_{s+3} - 10n_{s+2} = -4, n_0 = 1, n_1 = 5$$

$$a_{s+1} + a_{s+3} - 10a_{s+2} = 0, a_0 = 1, a_1 = 11$$

Some numerical examples satisfying our assumption are tabulated below.

TABLE 1: NUMERICAL EXAMPLES

s	n_s	a_s	LHS of (1)	RHS of (1)
0	5	11	484	484
1	45	109	47524	47524
2	441	1079	4656964	4656964
3	4361	10681	456335044	456335044
4	43165	105731	$4.471617744 \times 10^{10}$	$4.471617744 \times 10^{10}$
5	427285	1046629	$4.381729054 \times 10^{10}$	$4.381729054 \times 10^{10}$

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A few interesting properties between the ranks of the star number and the edge of the cube satisfying (1) are given below:

1. $6(12n_{2s+2} - 4a_{2s+2} - 4)$ is a nasty number
2. $12n_{3s+3} - 4a_{3s+3} + 36n_{s+1} - 12a_{s+1} - 24$ is a cubic integer
3. $12n_{2s+2} - 4a_{2s+2} - 4$ is a perfect square
4. $n_{s+2} = 5n_{s+1} + 2a_{s+1} - 2$
5. $n_{s+3} = 49n_{s+1} + 20a_{s+1} - 24$
6. $a_{s+2} = 12n_{s+1} + 5a_{s+1} - 6$
7. $a_{s+3} = 120n_{s+1} + 49a_{s+1} - 60$

IV. REMARKS

1. By considering linear combination among the ranks of the star number and the edge of the cube satisfying (1), one may generate different hyperbolas. A few examples are given in table 2 below:

TABLE 2: HYPERBOLAS

S.No	X, Y	Hyperbola
1	$12n_{s+1} - 4a_{s+1} - 6, 4n_{s+1} - 2a_{s+1} - 2$	$X^2 - 6Y^2 = 4$
2	$12n_{s+1} - 4a_{s+1} - 6, 18n_{s+1} - 2n_{s+2} - 8$	$4X^2 - 6Y^2 = 16$
3	$12n_{s+1} - 4a_{s+1} - 6, 178n_{s+1} - 2n_{s+1} - 88$	$200X^2 - 3Y^2 = 800$
4	$218n_{s+1} - 2n_{s+1} - 108, 4n_{s+1} - 2a_{s+1} - 2$	$X^2 - 600Y^2 = 400$
5	$218n_{s+1} - 2n_{s+1} - 108, 178n_{s+1} - 2n_{s+1} - 88$	$2X^2 - 3Y^2 = 800$
6	$12n_{s+1} - 4a_{s+1} - 6, 22a_{s+1} - 2a_{s+2}$	$9X^2 - 6Y^2 = 36$

2. By considering linear combination among the ranks of the star number and the edge of the cube satisfying (1), one may obtain different parabolas. A few examples are given in table 3 below.

TABLE 3: PARABOLAS

S.No	X, Y	Parabola
1	$12n_{2s+2} - 4a_{2s+2} - 4, 4n_{s+1} - 2a_{s+1} - 2$	$6Y^2 = X - 4$
2	$12n_{2s+2} - 4a_{2s+2} - 4, 18n_{s+1} - 2n_{s+2} - 8$	$6Y^2 = 4X - 16$
3	$12n_{2s+2} - 4a_{2s+2} - 4, 178n_{s+1} - 2n_{s+3} - 88$	$6Y^2 = 400X - 1600$
4	$12n_{2s+2} - 4a_{2s+2} - 4, 22a_{s+1} - 2a_{s+2}$	$6Y^2 = 9X - 36$
5	$12n_{2s+2} - 4a_{2s+2} - 4, 218a_{s+1} - 2a_{s+3}$	$Y^2 = 600X - 2400$
6	$12n_{2s+2} - 4a_{2s+2} - 4, 218a_{s+2} - 22a_{s+3}$	$Y^2 = 600X - 2400$

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V. RELATION BETWEEN THE LATERAL SURFACE AREA OF A CUBE AND CENTRED HEPTAGONAL NUMBER

Assume that the lateral surface area of the cube of edge a unit is equal to sixteen times centred heptagonal number of rank n .

The mathematical statement of our assumption is

$$4a^2 = 16CH_n \tag{9}$$

which reduces to $y^2 = 14x^2 + 2$ (10)

where $y = 2a, x = 2n - 1$ (11)

The initial solution to (10) is given by

$$x_0 = 1, y_0 = 4$$

Consider the corresponding Pellian equation to (10) as

$$y^2 = 14x^2 + 1 \tag{12}$$

whose initial solution is given by

$$\tilde{x}_0 = 4, \tilde{y}_0 = 15$$

Therefore, the general solution to (12) are represented by

$$\tilde{y}_s = \frac{1}{2} f_s, \tilde{x}_s = \frac{1}{2\sqrt{14}} g_s$$

where

$$f_s = (15 + 4\sqrt{14})^{s+1} + (15 - 4\sqrt{14})^{s+1}$$

$$g_s = (15 + 4\sqrt{14})^{s+1} - (15 - 4\sqrt{14})^{s+1}, s = -1, 0, 1, 2, 3, \dots$$

By applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, the non trivial integer solutions of (10) are given by

$$x_{s+1} = \frac{1}{2} f_s + \frac{2}{\sqrt{14}} g_s \tag{13}$$

$$y_{s+1} = 2f_s + \frac{7}{\sqrt{14}} g_s \tag{14}$$

Equating (11) and (13), we obtain

$$n_{s+1} = \frac{1}{4} f_s + \frac{2}{\sqrt{14}} g_s + 1 \tag{15}$$

Comparing (11) and (14), we get

$$a_{s+1} = f_s + \frac{7}{2\sqrt{14}} g_s \tag{16}$$

Using (15) and (16), the recurrence relations for the values of the ranks of centred heptagonal number and the sides of the cube are given by

$$n_{s+1} + n_{s+3} - 30n_{s+2} = -14, n_0 = 1, n_1 = 16$$

$$a_{s+1} + a_{s+3} - 30a_{s+2} = 0, a_0 = 2, a_1 = 58$$

Some numerical examples satisfying our assumption are given in the following table 4.

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TABLE 4: NUMERICAL EXAMPLES

s	n_s	a_s	LHS of (9)	RHS of (9)
1	16	58	13456	13456
2	465	1738	12082576	12082576
3	13920	52082	$1.85013889 \times 10^{10}$	$1.85013889 \times 10^{10}$
4	417121	1560722	$9.743412645 \times 10^{12}$	$9.743412645 \times 10^{12}$
5	12499696	46769578	$8.749573705 \times 10^{15}$	$8.749573705 \times 10^{15}$
6	374573745	1401526618	$7.857107443 \times 10^{18}$	$7.857107443 \times 10^{18}$

A few interesting properties between the ranks of centred heptagonal number and the sides of the cube satisfying (9) are given below:

1. $6(-28n_{2s+2} + 8a_{2s+2} + 16)$ is a nasty number
2. $8a_{3s+3} - 28n_{3s+3} - 84n_{s+1} - 24a_{s+1} + 56$ is a cubical integer
3. $8a_{2s+2} - 28n_{2s+2} + 16$ is a perfect square
4. $2n_{s+2} = 30n_{s+1} + 8a_{s+1} - 15$
5. $2n_{s+3} = 898n_{s+1} + 240a_{s+1} - 448$
6. $a_{s+2} = 56n_{s+1} + 15a_{s+1} - 28$
7. $a_{s+3} = 1680n_{s+1} + 449a_{s+1} - 840$

VI. REMARKS

1. By considering linear combination among the ranks of centred heptagonal number and the sides of the cube satisfying (9), one may obtain different hyperbolas. A few examples are given in table 5 below:

TABLE 5: HYPERBOLAS

S.No	X, Y	Hyperbola
1	$28n_{s+1} - 8a_{s+1} - 14, 8n_{s+1} - 2a_{s+1} - 4$	$X^2 - 14Y^2 = 4$
2	$28n_{s+1} - 8a_{s+1} - 14, 62n_{s+1} - 2n_{s+2} - 30$	$8X^2 - 7Y^2 = 32$
3	$28n_{s+1} - 8a_{s+1} - 14, 1858n_{s+1} - 2n_{s+3} - 928$	$7200X^2 - 7Y^2 = 28800$
4	$3476n_{s+2} - 116n_{s+3} - 1680, 8n_{s+1} - 2a_{s+1} - 4$	$X^2 - 56Y^2 = 16$
5	$1738n_{s+1} - 2n_{s+3} - 868, 8n_{s+1} - 2a_{s+1} - 4$	$X^2 - 126Y^2 = 3600$
6	$3476n_{s+2} - 116n_{s+3} - 1680, 1858n_{s+2} - 62n_{s+3} - 898$	$2X^2 - 7Y^2 = 32$
7	$1738n_{s+1} - 2n_{s+3} - 868, 1858n_{s+1} - 2n_{s+3} - 928$	$8X^2 - 7Y^2 = 28800$
8	$28n_{s+1} - 8a_{s+1} - 14, 3476n_{s+2} - 116a_{s+3}$	$X^2 - 56Y^2 = 4$
9	$28n_{s+1} - 8a_{s+1} - 14, 1738a_{s+1} - 2a_{s+3}$	$4581367X^2 - 2Y^2 = 18325460$

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2. By considering linear combination among the ranks of centred heptagonal number and the sides of the cube satisfying (9), one may obtain different parabolas. A few examples are given in table 6 below.

TABLE 6: PARABOLAS

S.No	X, Y	Parabola
1	$28n_{2s+2} - 8a_{2s+2} - 16,8n_{s+1} - 2a_{s+1} - 4$	$14Y^2 = X - 4$
2	$28n_{2s+2} - 8a_{2s+2} - 16,62n_{s+1} - 2n_{s+2} - 30$	$7Y^2 = 8X - 32$
3	$28n_{2s+2} - 8a_{2s+2} - 16,1858n_{s+2} - 62n_{s+3} - 898$	$7Y^2 = 7200X - 4$
4	$28n_{2s+2} - 8a_{2s+2} - 16,3476a_{s+2} - 116a_{s+3}$	$56Y^2 = X - 4$
5	$28n_{2s+2} - 8a_{2s+2} - 16,1738a_{s+1} - 2a_{s+3}$	$14Y^2 = 32069569X - 128278276$

VII. RELATION BETWEEN THE LATERAL SURFACE AREA OF A CUBE AND CENTRED HEPTADECAGONAL NUMBER

Assume that the lateral surface area of the cube of side a unit is equal to eight times centred heptadecagonal number of rank n .

The mathematical statement of our assumption is

$$4a^2 = 8CHD_n \tag{17}$$

which can be written as

$$y^2 = 17x^2 - 9 \tag{18}$$

where $y = 2a, x = 2n - 1$

$$\tag{19}$$

The initial solution to (18) is given by

$$x_0 = 3, y_0 = 12$$

To find the general solutions to (18), consider the corresponding Pellian equation

$$y^2 = 17x^2 + 1 \tag{20}$$

whose initial solution is given by

$$\tilde{x}_0 = 8, \tilde{y}_0 = 33$$

Therefore, the general solution to (20) are represented by

$$\tilde{y}_s = \frac{1}{2} f_s, \tilde{x}_s = \frac{1}{2\sqrt{17}} g_s$$

where

$$f_s = (33 + 8\sqrt{17})^{s+1} + (33 - 8\sqrt{17})^{s+1}$$

$$g_s = (33 + 8\sqrt{17})^{s+1} - (33 - 8\sqrt{17})^{s+1}, s = -1, 0, 1, 2, 3, \dots$$

By applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$, the integral solutions of (18) are given by

$$x_{s+1} = \frac{3}{2} f_s + \frac{6}{\sqrt{17}} g_s \tag{21}$$

$$y_{s+1} = 6f_s + \frac{51}{2\sqrt{17}} g_s \tag{22}$$

Equating (19) and (21), we obtain

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$$n_{s+1} = \frac{3}{4}f_s + \frac{3}{\sqrt{17}}g_s + 1 \tag{23}$$

Comparing (19) and (22), we get

$$a_{s+1} = 3f_s + \frac{51}{4\sqrt{17}}g_s \tag{24}$$

Using (23) and (24), the recurrence relations for the values of the ranks of centred heptadecagonal number and the sides of the cube are given by

$$n_{s+1} + n_{s+3} - 66n_{s+2} = -32, n_0 = 2, n_1 = 98$$

$$a_{s+1} + a_{s+3} - 66a_{s+2} = 0, a_0 = 6, a_1 = 402$$

Some numerical examples satisfying (17) are given in the following table7.

TABLE 7: NUMERICAL EXAMPLES

s	n _s	a _s	LHS of (17)	RHS of (17)
1	98	402	646416	646416
2	6434	26526	2814514704	2814514704
3	424514	1750314	1.225439639×10 ¹³	1.225439639×10 ¹³
4	28011458	115494198	5.335563908×10 ¹⁶	5.335563908×10 ¹⁶
5	1848331682	7620866754	2.323104403×10 ²⁰	2.323104403×10 ²⁰

A few interesting properties between the ranks of centred heptadecagonal number and the sides of the cube satisfying (17) are given below:

1. $(2/3)(204n_{2s+2} - 48a_{2s+2} - 84)$ is a nasty number
2. $(1/9)(204n_{3s+3} - 48a_{3s+3} + 612n_{s+1} - 144a_{s+1} - 408)$ is a cubic integer
3. $(1/9)(204n_{2s+2} - 48a_{2s+2} - 84)$ is a perfect square
4. $n_{s+2} = 33n_{s+1} + 8a_{s+1} - 16$
5. $n_{s+3} = 2177n_{s+1} + 528a_{s+1} - 1088$
6. $a_{s+2} = 136n_{s+1} + 33a_{s+1} - 66$
7. $a_{s+3} = 8976n_{s+1} - 2177a_{s+1} - 4488$

VIII. REMARKS

1. By considering linear combination among the solutions of (17), one may obtain different hyperbolas. A few examples are given in table 8 below:

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TABLE 8: HYPERBOLAS

S.No	X, Y	Hyperbola
1	$204n_{s+1} - 48a_{s+1} - 102, 16n_{s+1} - 4a_{s+1} - 8$	$X^2 - 153Y^2 = 324$
2	$204n_{s+1} - 48a_{s+1} - 102, 130n_{s+1} - 2n_{s+2} - 64$	$48X^2 - 459Y^2 = 15552$
3	$204n_{s+1} - 48a_{s+1} - 102, 8578n_{s+1} - 2n_{s+3} - 4288$	$7744X^2 - 17Y^2 = 2509056$
4	$204n_{s+1} - 48a_{s+1} - 102, 25734n_{s+2} - 390n_{s+3} - 12672$	$16X^2 - 17Y^2 = 5184$
5	$134n_{s+1} - 2n_{s+2} - 66, 16n_{s+1} - 4a_{s+1} - 8$	$X^2 - 17Y^2 = 36$
6	$8842n_{s+1} - 2n_{s+3} - 4420, 16n_{s+1} - 4a_{s+1} - 8$	$X^2 - 74052Y^2 = 156816$
7	$53052n_{s+2} - 804n_{s+3} - 26124, 16n_{s+1} - 4a_{s+1} - 8$	$X^2 - 612Y^2 = 1296$
8	$134n_{s+1} - 2n_{s+2} - 66, 130n_{s+1} - 2n_{s+2} - 64$	$16X^2 - 17Y^2 = 576$
9	$8842n_{s+1} - 2n_{s+3} - 4420, 8578n_{s+1} - 2n_{s+3} - 4288$	$16X^2 - 17Y^2 = 2509056$
10	$53052n_{s+2} - 804n_{s+3} - 26124, 25734n_{s+2} - 390n_{s+3} - 12672$	$4X^2 - 17Y^2 = 5184$

2. By considering linear combination among the solutions of (17), one may obtain different parabolas. A few examples are given in table 9 below.

TABLE 9: PARABOLAS

S.No	X, Y	Parabola
1	$204n_{2s+2} - 48a_{2s+2} - 84, 16n_{s+1} - 4a_{s+1} - 8$	$17Y^2 = X - 4$
2	$204n_{2s+2} - 48a_{2s+2} - 84, 130n_{s+1} - 2n_{s+2} - 64$	$17Y^2 = 16X - 576$
3	$204n_{2s+2} - 48a_{2s+2} - 84, 25734n_{s+2} - 390n_{s+3} - 12672$	$17Y^2 = 4X - 144$
4	$204n_{2s+2} - 48a_{2s+2} - 84, 8578n_{s+1} - 2n_{s+3} - 4288$	$17Y^2 = 88X - 3168$
5	$204n_{2s+2} - 48a_{2s+2} - 84, 134a_{s+1} - 2a_{s+2}$	$17Y^2 = 289X - 10404$
6	$204n_{2s+2} - 48a_{2s+2} - 84, 53052a_{s+2} - 804a_{s+3}$	$17Y^2 = 10404X - 374544$
7	$204n_{2s+2} - 48a_{2s+2} - 84, 8842a_{s+1} - 2a_{s+3}$	$17Y^2 = 1258884X - 45319824$

IX. CONCLUSION

In this paper, we have presented the relation between the lateral surface area of a cube and some fascinating numbers like star number, centred heptagonal number and centred heptadecagonal number. In such a way that, one can evaluate various relations for other three dimensional and four dimensional figurate numbers.

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