

Six Maps with a Common Fixed Point Satisfying Weak Compatible and Commuting Mapping in complex valued metric space Using Rational Inequalities

Kamal Kumar¹, Nisha Sharma^{2,*}, Rajeev Jha³, Ritu Sharma⁴, Sheetal Sharma⁵

Department of Mathematics, Pt. JLN Govt. College Faridabad, Sunrise University Alwar, India¹

Department of Mathematics, Pt. JLN Govt. College Faridabad, Faridabad, Haryana, India²

Department of Mathematics, Teerthankar Mahaveer University, Moradabad, U.P, India³

Department of Mathematics, Pt. JLN Govt. College Faridabad, India⁴

Assistant Professor, Department of computer Science and Engineering, Amity university sector-125, Noida, India⁵

*Corresponding Author

ABSTRACT. Azam et al. (2011), introduce the notion of complex valued metric spaces and obtained common fixed point result for mappings in the context of complex valued metric spaces. In this paper we prove common fixed point theorems for six maps in complex valued metric space satisfying commuting and weakly compatible mapping with different type of inequality, our result generalizes the result of Tiwari et al.[8]

KEYWORDS: commuting mapping, weakly compatible maps, common fixed points, complete complex valued metric space

MSC: 46T99, 47H10, 54H25, 54C30

I. INTRODUCTION

Banach contraction principle was the starting point for many researchers during last few decades in the field of nonlinear analysis. The concept of complex valued metric space which is a generalization of the classical metric space was recently introduced by Azam, Fisher and Khan [1]. The study of metric space expressed the most common important role to many fields both in pure and applied science [3]. Abounding researchers extended the notion of a metric space such as vector valued metric space of Perov [2], a cone metric spaces of Huang and Zhang [6], a modular metric spaces of Chistyakov [10], etc. For the sake of completeness we recollect some basic definitions and fundamental results on the topic in the literature. We mainly follow the notions and notations of Azam et al. [1].

Let \mathbb{C} be the set of all complex numbers, for $z_1, z_2 \in \mathbb{C}$, define a partial order \preceq on \mathbb{C} as follows;

$z_1 \preceq z_2$ iff $\text{Re}(z_1) \leq \text{Re}(z_2)$, $\text{Im}(z_1) \leq \text{Im}(z_2)$

it follows that

$z_1 \preceq z_2$, if one of the following conditions is satisfied:

- i. $\text{Re}(z_1) = \text{Re}(z_2)$, $\text{Im}(z_1) < \text{Im}(z_2)$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

- ii. $\text{Re}(z_1) < \text{Re}(z_2), \text{Im}(z_1) = \text{Im}(z_2)$
- iii. $\text{Re}(z_1) < \text{Re}(z_2), \text{Im}(z_1) < \text{Im}(z_2)$
- iv. $\text{Re}(z_1) = \text{Re}(z_2), \text{Im}(z_1) = \text{Im}(z_2)$

In particular, we will write $z_1 \preceq z_2$ if $z_1 \neq z_2$ and one the (i), (ii), (iii) conditions is satisfied and we will write $z_1 \prec z_2$. Note that

$$0 \preceq z_1 \preceq z_2 \Rightarrow |z_1| < |z_2|,$$

$$z_1 \preceq z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3$$

Definition 1.2.[1] Let X be a nonempty set. A mapping $d: X \times X \rightarrow \mathbb{C}$ is called a complex valued metric on X if the following conditions are satisfied:

$$(1.1) 0 \preceq d(x,y) \text{ for all } x,y \in X \text{ and } d(x,y)=0 \Leftrightarrow x=y.$$

$$(1.2) d(x,y)=d(y,x) \text{ for all } x,y \in X$$

$$(1.3) d(x,y) \preceq d(x,z)+d(z,y) \text{ for all } x,y,z \in X.$$

In this case, we say that (X,d) is a complex valued metric space.

Definition 1.3.[3] Let \mathbb{C} be a complex valued metric space,

- We say that a sequence $\{x_n\}$ is said to be a Cauchy sequence X , if for every $c \in \mathbb{C}$, with $0 \prec c$ there is $n_0 \in \mathbb{N}$ such that for all $n > n_0$ such that $d(x_n, x_m) \prec c$.
- We say that a sequence $\{x_n\}$ converges to an element $x \in X$. If for every $c \in \mathbb{C}$, with $0 \prec c$ there exist an integer $n_0 \in \mathbb{N}$ such that for all $n > n_0$ such that $d(x_n, x) \prec c$ and we write $x_n \rightarrow x$.
- We say that (X,d) is complete if every Cauchy sequence in X converges to a point in X .

Lemma 1.4.[3] Any sequence $\{x_n\}$ in complex valued metric space (X,d) , converges to x if and only if $|d(x_n, x)| \rightarrow 0$ as $n \rightarrow \infty$

Lemma 1.5.[3] Any sequence $\{x_n\}$ in complex valued metric space (X,d) is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \rightarrow 0$ as $n \rightarrow \infty$, where $m \in \mathbb{N}$

Definition 1.6.[9] Let S and T be self-maps of metric space (X,d) , then S, T are said to be weakly commuting if $d(STx, TSx) \preceq d(Sx, Tx)$, for all $x \in X$

Definition 1.7.[4] Two self-maps S, T of a non-empty set X are said to be weakly compatible is $STx = TSx$ whenever $Sx = Tx$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

II. MAIN RESULTS

In this section, we prove some common fixed point theorems for rational type contraction conditions. Our main result runs as follows

Theorem 2.1. Let (X, d) be a complete complex valued metric space and P, Q, R, S, T, U be self-maps of X satisfying the following condition

(2.1) $TU(X) \subseteq P(X)$ and $RS(X) \subseteq Q(X)$ and

$$(2.2) \quad d(RSx, TUy) \lesssim \left(ad(Px, Qy) \frac{[d(Px, RSx) + d(TUy, Qy)]}{d(RSx, TUy) + d(Qy, Px)} + bd(TUy, RSx) \frac{[d(TUy, Qy) + d(Px, RSx)]}{d(Px, Qy) + d(TUy, RSx)} \right. \\ \left. + cd(Qy, RSx) \frac{[d(Px, TUy)]}{d(RSx, TUy) + d(RSx, Px)} + ed(Qy, RSx) \frac{[d(TUy, Px)]}{d(Px, Qy) + d(TUy, RSx)} \right)$$

for all $x, y \in X$ where $a, b, c, e \geq 0$ and $a + b + c + e < 1$. Assume that pairs (TU, Q) , (RS, P) are weakly compatible, and the pairs (T, U) , (T, Q) , (R, S) , (R, P) and (S, P) are commuting pairs of maps. Then T, U, R, S, Q and P have a unique common fixed point in X .

Proof: Let $x_0 \in X$ by (2.1) we can define inductively a sequence $\{y_n\}$ in X such that $y_{2n} = RSx_{2n} = Qx_{2n+1}$ and $TUx_{2n+1} = Px_{2n+2} = y_{2n+1}$ for all $n=0, 1, 2, 3 \dots$

By (2.2), we have

$$d(y_{2n}, y_{2n+1}) = d(RSx_{2n}, TUx_{2n+1})$$

$$\approx \left(\begin{aligned} & ad(Px_{2n}, Qx_{2n+1}) \frac{[d(Px_{2n}, RSx_{2n}) + d(TUx_{2n+1}, Qx_{2n+1})]}{d(RSx_{2n}, TUx_{2n+1}) + d(Qx_{2n+1}, Px_{2n})} \\ & + bd(TUx_{2n+1}, RSx_{2n}) \frac{[d(TUx_{2n+1}, Qx_{2n+1}) + d(Px_{2n}, RSx_{2n})]}{d(Px_{2n}, Qx_{2n+1}) + d(TUx_{2n+1}, RSx_{2n})} \\ & + cd(Qx_{2n+1}, RSx_{2n}) \frac{[d(Px_{2n}, TUx_{2n+1})]}{d(RSx_{2n}, TUx_{2n+1}) + d(RSx_{2n}, Px_{2n})} \\ & + ed(Qx_{2n+1}, RSx_{2n}) \frac{[d(TUx_{2n+1}, Px_{2n})]}{d(Px_{2n}, Qx_{2n+1}) + d(TUx_{2n+1}, RSx_{2n})} \end{aligned} \right)$$

$$\approx \left(\begin{aligned} & ad(y_{2n-1}, y_{2n}) \frac{[d(y_{2n-1}, y_{2n}) + d(y_{2n+1}, y_{2n})]}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n-1})} \\ & + bd(y_{2n+1}, y_{2n}) \frac{[d(y_{2n+1}, y_{2n}) + d(y_{2n-1}, y_{2n})]}{d(y_{2n-1}, y_{2n}) + d(y_{2n+1}, y_{2n})} \\ & + cd(y_{2n}, y_{2n}) \frac{[d(y_{2n-1}, y_{2n+1})]}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n-1})} \\ & + ed(y_{2n}, y_{2n}) \frac{[d(y_{2n+1}, y_{2n-1})]}{d(y_{2n-1}, y_{2n}) + d(y_{2n+1}, y_{2n})} \end{aligned} \right)$$

$$d(y_{2n}, y_{2n+1}) \lesssim ad(y_{2n-1}, y_{2n}) + bd(y_{2n}, y_{2n+1})$$

$$(1 - b)d(y_{2n}, y_{2n+1}) \lesssim ad(y_{2n-1}, y_{2n})$$

Therefore,

$$(2.3) \quad d(y_{2n}, y_{2n+1}) \lesssim \frac{a}{1-b} d(y_{2n-1}, y_{2n})$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Similarly we have,

$$d(y_{2n+1}, y_{2n+2}) = d(TUX_{2n+1}, RSX_{2n+2})$$

$$= d(RSX_{2n+2}, TUX_{2n+1})$$

$$\approx \left(\begin{aligned} & ad(PX_{2n+2}, QX_{2n+1}) \frac{[d(PX_{2n+2}, RSX_{2n+2}) + d(TUX_{2n+1}, QX_{2n+1})]}{d(RSX_{2n+2}, TUX_{2n+1}) + d(QX_{2n+1}, PX_{2n+2})} \\ & + bd(TUX_{2n+1}, RSX_{2n+2}) \frac{[d(TUX_{2n+1}, QX_{2n+1}) + d(PX_{2n+2}, RSX_{2n+2})]}{d(PX_{2n+2}, QX_{2n+1}) + d(TUX_{2n+1}, RSX_{2n+2})} \\ & + cd(QX_{2n+1}, RSX_{2n+2}) \frac{[d(PX_{2n+2}, TUX_{2n+1})]}{d(RSX_{2n+2}, TUX_{2n+1}) + d(RSX_{2n+2}, PX_{2n+2})} \\ & + ed(QX_{2n+1}, RSX_{2n+2}) \frac{[d(TUX_{2n+1}, PX_{2n+2})]}{d(PX_{2n+2}, QX_{2n+1}) + d(TUX_{2n+1}, RSX_{2n+2})} \end{aligned} \right)$$

$$\approx \left(\begin{aligned} & ad(y_{2n+1}, y_{2n}) \frac{[d(y_{2n+1}, y_{2n+2}) + d(y_{2n+1}, y_{2n})]}{d(y_{2n+2}, y_{2n+1}) + d(y_{2n}, y_{2n+1})} \\ & + bd(y_{2n+1}, y_{2n+2}) \frac{[d(y_{2n+1}, y_{2n}) + d(y_{2n+1}, y_{2n+2})]}{d(y_{2n+1}, y_{2n}) + d(y_{2n+1}, y_{2n+2})} \\ & + cd(y_{2n}, y_{2n+2}) \frac{[d(y_{2n+1}, y_{2n+1})]}{d(y_{2n+2}, y_{2n+1}) + d(y_{2n+2}, y_{2n+1})} \\ & + ed(y_{2n}, y_{2n+2}) \frac{[d(y_{2n+1}, y_{2n+1})]}{d(y_{2n+1}, y_{2n}) + d(y_{2n+1}, y_{2n+2})} \end{aligned} \right)$$

$$d(y_{2n+1}, y_{2n+2}) \approx ad(y_{2n+1}, y_{2n}) + bd(y_{2n+1}, y_{2n+2})$$

Therefore,

$$(2.4) d(y_{2n+1}, y_{2n+2}) \approx \frac{a}{1-b} d(y_{2n}, y_{2n+1})$$

$$(2.5) d(y_{2n}, y_{2n+1}) \approx k d(y_{2n-1}, y_{2n}), \text{ where } k = \frac{a}{1-b}$$

Using (2.3), (2.4) and (2.5), we have

$$d(y_n, y_{n+1}) \approx k d(y_{n-1}, y_n)$$

$$\approx k^2 d(y_{n-2}, y_{n-1})$$

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$$\approx k^n d(y_0, y_1) \text{ for all } n.$$

$$(2.6) d(y_n, y_{n+1}) \approx k^n d(y_0, y_1) \text{ for all } n.$$

therefore,

$$d(y_n, y_m) \approx d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{m-1}, y_m)$$

$$\approx (k^n + k^{n+1} + \dots + k^{m-1}) d(y_1, y_0)$$

$$\approx \frac{k^n}{1-k} d(y_1, y_0), \quad (\text{using 2.6})$$

$$d(y_n, y_m) \approx \frac{k^n}{1-k} d(y_1, y_0)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Which implies that, $d(y_n, y_m) \rightarrow 0$ as $(n, m \rightarrow \infty)$. Hence $\{y_n\}$ is a Cauchy sequence, by the completeness of X , there exist $z \in X$ such that,

$$(2.7) \quad \lim_{n \rightarrow \infty} RSx_{2n} = \lim_{n \rightarrow \infty} Qx_{2n+1} = \lim_{n \rightarrow \infty} TUX_{2n+1} = \lim_{n \rightarrow \infty} Px_{2n+2} = z$$

Since, $TU(X) \subseteq P(X)$ there exist $u \in X$ such that $z = Pu$,

We claim that $RSu = z$, if possible $RSu \neq z$, then by using (2.2), we have

$$\begin{aligned} d(RSu, z) &= d(RSu, TUX_{2n-1}) + d(TUX_{2n-1}, z) \\ &\lesssim \left(\begin{aligned} &ad(Pu, Qx_{2n-1}) \frac{[d(Pu, RSu) + d(TUX_{2n-1}, Qx_{2n-1})]}{d(RSu, TUX_{2n-1}) + d(Qx_{2n-1}, Pu)} \\ &+ bd(TUX_{2n-1}, RSu) \frac{[d(TUX_{2n-1}, Qx_{2n-1}) + d(Pu, RSu)]}{d(Pu, Qx_{2n-1}) + d(TUX_{2n-1}, RSu)} \\ &+ cd(Qx_{2n-1}, RSu) \frac{[d(Pu, TUX_{2n-1})]}{d(RSu, TUX_{2n-1}) + d(RSu, Pu)} \\ &+ ed(Qx_{2n-1}, RSu) \frac{[d(TUX_{2n-1}, Pu)]}{d(Pu, Qx_{2n-1}) + d(TUX_{2n-1}, RSu)} \end{aligned} \right) + d(TUX_{2n-1}, z) \\ d(RSu, z) &\lesssim \left(\begin{aligned} &ad(z, z) \frac{[d(z, RSu) + d(z, z)]}{d(RSu, z) + d(z, z)} + bd(z, RSu) \frac{[d(z, z) + d(z, RSu)]}{d(z, z) + d(z, RSu)} \\ &+ cd(z, RSu) \frac{[d(z, z)]}{d(RSu, z) + d(RSu, z)} + ed(z, RSu) \frac{[d(z, z)]}{d(z, z) + d(z, RSu)} \end{aligned} \right) + d(z, z) \text{ [using (2.7)]} \end{aligned}$$

taking the limit as $n \rightarrow \infty$, we have

$d(RSu, z) \lesssim bd(z, RSu)$, which is a contradiction. Therefore,

$$(2.8) \quad RSu = Pu = z$$

Since, $RS(X) \subseteq Q(X)$ there exist $v \in X$ such that $z = Qv$.

We claim that $TUv = z$, if possible $TUv \neq z$, then by (2.2) and (2.8), we have

$$\begin{aligned} d(z, TUv) &= d(RSu, TUv) \\ &\lesssim \left(\begin{aligned} &ad(Pu, Qv) \frac{[d(Pu, RSu) + d(TUv, Qv)]}{d(RSu, TUv) + d(Qv, Pu)} + bd(TUv, RSu) \frac{[d(TUv, Qv) + d(Pu, RSu)]}{d(Pu, Qv) + d(TUv, RSu)} \\ &+ cd(Qv, RSu) \frac{[d(Pu, TUv)]}{d(RSu, TUv) + d(RSu, Pu)} + ed(Qv, RSu) \frac{[d(TUv, Pu)]}{d(Pu, Qv) + d(TUv, RSu)} \end{aligned} \right) \\ &\lesssim \left(\begin{aligned} &ad(z, z) \frac{[d(z, z) + d(TUv, z)]}{d(z, TUv) + d(z, z)} + bd(TUv, z) \frac{[d(TUv, z) + d(z, z)]}{d(z, z) + d(TUv, z)} \\ &+ cd(z, z) \frac{[d(z, TUv)]}{d(z, TUv) + d(z, z)} + ed(z, z) \frac{[d(TUv, z)]}{d(z, z) + d(TUv, z)} \end{aligned} \right) \text{ [using (2.8)]} \end{aligned}$$

$$d(z, TUv) \lesssim (bd(TUv, z))$$

$d(z, TUv) \lesssim bd(TUv, z)$, which is a contradiction.

therefore, $TUv = Qv = z$ so,

$$(2.9) \quad Pu = TUv = Qv = z$$

Similarly, Q and TU are weakly compatible maps, we have $TUz = Qz$.

Now we claim that z is a fixed point of TU . If $TUz \neq z$, then by (2.2), we have

$$d(z, TUz) = d(RSu, TUz)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$$d(z, TUz) \lesssim \left(\begin{aligned} & ad(Pu, Qz) \frac{[d(Pu, RSu) + d(TUz, Qz)]}{d(RSu, TUz) + d(Qz, Pu)} + bd(TUz, RSu) \frac{[d(TUz, Qz) + d(Pu, RSu)]}{d(Pu, Qz) + d(TUz, RSu)} \\ & + cd(Qz, RSu) \frac{[d(Pu, TUz)]}{d(RSu, TUz) + d(RSu, Pu)} + ed(Qz, RSu) \frac{[d(TUz, Pu)]}{d(Pu, Qz) + d(TUz, RSu)} \end{aligned} \right)$$

$$d(z, TUz) \lesssim \left(\begin{aligned} & ad(z, TUz) \frac{[d(z,z)+d(TUz,TUz)]}{d(z,TUz)+d(TUz,z)} + bd(TUz, z) \frac{[d(TUz,TUz)+d(z,z)]}{d(z,TUz)+d(TUz,z)} \\ & cd(TUz, z) \frac{[d(z,TUz)]}{d(z,TUz)+d(z,z)} + ed(TUz, z) \frac{[d(TUz,z)]}{d(z,TUz)+d(TUz,z)} \end{aligned} \right) \text{ [using (2.8)]}$$

$$d(z, TUz) \lesssim cd(TUz, z) + \frac{e}{2}d(TUz, z)$$

$d(z, TUz) \lesssim (c + \frac{e}{2})d(TUz, z)$, which is a contradiction.

Therefore, $TUz=z$, hence $Qz=z$. So, we have

$$(2.10) \quad TUz=Qz=z.$$

Similarly, P and RS are weakly compatible maps, we have $RSz=Pz$.

Now we claim that z is a fixed point of RS. If $RSz \neq z$, then by (2.2), we have

$$d(RSz, z) = d(RSz, TUz)$$

$$\lesssim \left(\begin{aligned} & ad(Pz, Qz) \frac{[d(Pz, RSz) + d(TUz, Qz)]}{d(RSz, TUz) + d(Qz, Pz)} + bd(TUz, RSz) \frac{[d(TUz, Qz) + d(Pz, RSz)]}{d(Pz, Qz) + d(TUz, RSz)} \\ & + cd(Qz, RSz) \frac{[d(Pz, TUz)]}{d(RSz, TUz) + d(RSz, Pz)} + ed(Qz, RSz) \frac{[d(TUz, Pz)]}{d(Pz, Qz) + d(TUz, RSz)} \end{aligned} \right)$$

we have,

$$d(RSz, z) \lesssim \left(\begin{aligned} & ad(RSz, z) \frac{[d(RSz,RSz)+d(z,z)]}{d(RSz,z)+d(z,RSz)} + bd(RSz, z) \frac{[d(z,z)+d(RSz,RSz)]}{d(RSz,z)+d(z,RSz)} \\ & + cd(z, RSz) \frac{[d(RSz,z)]}{d(RSz,z)+d(RSz,RSz)} + ed(z, RSz) \frac{[d(z,RSz)]}{d(RSz,z)+d(z,RSz)} \end{aligned} \right) \text{ [using (2.10)]}$$

$$d(RSz, z) \lesssim \left(cd(z, RSz) + \frac{e}{2}d(z, RSz) \right)$$

$d(RSz, z) \lesssim (c + \frac{e}{2})d(z, RSz)$, which is a contradiction.

Therefore, $RSz=z$. So, we have

$$(2.11) \quad RSz=Pz=z.$$

So, z is a common fixed point of TU, Q, P and RS.

By commuting condition of pairs

$$Tz=T(TUz)=T(UTz)=TU(Tz)$$

$$Tz=T(Pz)=P(Tz) \text{ and } Uz=U(TUz)=(UT)(Uz)=(TU)(Uz)$$

$Uz=U(Pz)=P(Uz)$ which follows that, Tz and Uz are common fixed points of TU and P, Then

$$(2.12) \quad Tz=z=Uz=Pz=TUz$$

Similarly, By commuting property,

$$Rz=R(RSz)=R(SRz)=RS(Rz)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$Rz=R(Qz)=Q(Rz)$ and $Sz=S(RSz)=(SR)(Sz)=(RS)(Sz)$

$(Sz)=S(Qz)=Q(Sz)$ which follows that, Rz and Sz are common fixed points of RS and Q , Then

$$(2.13) \quad Rz=z=Sz=Qz=RSz$$

Therefore z is a common fixed point of T, U, R, S, P and Q .

Uniqueness

Let w be other common fixed point of T, U, R, P, S and Q . if possible $w \neq z$. Then by (2.2), we have

$$d(z,w)=d(RSz,TUw)$$

$$\begin{aligned} & \lesssim \left(ad(Pz, Qw) \frac{[d(Pz, RSz) + d(TUw, Qw)]}{d(RSz, TUw) + d(Qw, Pz)} + bd(TUw, RSz) \frac{[d(TUw, Qw) + d(Pz, RSz)]}{d(Pz, Qw) + d(TUw, RSz)} \right) \\ & \quad + cd(Qw, RSz) \frac{[d(Pz, TUw)]}{d(RSz, TUw) + d(RSz, Pz)} + ed(Qw, RSz) \frac{[d(TUw, Pz)]}{d(Pz, Qw) + d(TUw, RSz)} \\ & = \left(ad(z, w) \frac{[d(z, z) + d(w, w)]}{d(z, w) + d(w, z)} + bd(w, z) \frac{[d(w, w) + d(z, z)]}{d(z, w) + d(w, z)} \right) \\ & \quad + cd(w, z) \frac{[d(z, w)]}{d(z, w) + d(z, z)} + ed(w, z) \frac{[d(w, z)]}{d(z, w) + d(w, z)} \\ & = \left(c + \frac{e}{2} \right) d(w, z) \end{aligned}$$

$$d(z,w) \lesssim \left(c + \frac{e}{2} \right) d(w, z), \text{ a contradiction.}$$

So, $z=w$.

Hence, T, U, R, S, Q and P have a unique common fixed point in X .

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International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

BIOGRAPHY



Kamal Kumar is an assistant professor in Department of Mathematics at Pt. JLN Govt. P.G college, Faridabad, India. He has been teaching for 10 years. He has published 8 research papers in international journals. His research area includes fixed Point theory, analysis and Topology.



Nisha Sharma is an assistant professor in Department of Mathematics at Pt. JLN Govt. P.G college, Faridabad, India. She has been teaching for 2 years. She has published 10 research papers in international journals. Her research interest includes fixed Point theory, analysis and Topology.



Rajeev Jha is presently working in Department of mathematics, College of Engineering, TeerthankarUniverdsity, Maradabad, India. He has 19 years of experience in teaching and research. He has published 160 research papers in national/ International Journals and attended 50 national/International conferences in India and Abroad.



Ritu Sharma is an assistant professor in Department of Mathematics at Pt. JLN Govt. P.G college, Faridabad, India. She has been teaching for 5 years. Her research interest includes fixed Point theory and analysis.



Sheetal Joshi is presently working inAmity University, Noida. She has 7 years of experience in teaching and research. He has published 7 research papers in national/ International Journals.