

# Rational Contractive Condition in Multiplicative Metric Space and Common Fixed Point Theorem

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**ABSTRACT:** The aim of this paper is to prove fixed point theorem using a rational contractive condition in the setup of complete multiplicative metric spaces.

**AMS Subject Classification:** 47H10, 54H25

**KEYWORDS:** rational inequalities, multiplicative metric spaces

## I. INTRODUCTION AND PRELIMINARIES

In 2008, Bashirov et al. [12] introduced the notion of multiplicative metric spaces, and studied the concept of multiplicative calculus and proved the elementary theorem of multiplicative calculus. In 2012, Florack and Assen [13] had shown the use of the conception of multiplicative calculus in biomedical image analysis. In 2011, Bashirov et al. [14] exploit the adaptability of multiplicative calculus over the Newtonian calculus. Furthermore, Bashirov et al. [12] illustrated the usefulness of multiplicative calculus with some provocative applications. In 2012, Özavşar and Çevikel [15] investigate multiplicative metric spaces by remarking its topological properties, and introduced concept of multiplicative contraction mapping and proved some fixed point theorems of multiplicative contraction mappings on multiplicative spaces.

Recently, He et al. [16] proved common fixed point theorems for four self-mappings in multiplicative metric space. Very recently, Abbas et ai. [17] Proved some common fixed point results of quasi-weak commutative mappings on a closed ball in the framework of multiplicative metric space. Kang et al. [18] introduced the notions of compatible mappings and its variants in multiplicative metric spaces, and proved some common fixed point theorems for these mappings.

This paper deals with some common fixed point theorems which are established using multiplicative contraction in multiplicative metric space with rational inequities. The letters  $\mathbb{R}$ ,  $\mathbb{R}_+$  and  $\mathbb{N}$  denote the set of all real numbers, the set of all positivereal numbers and the set of all natural numbers, respectively. Some useful definitions are as follows:

**Definition 1.1.** ([8]) Let  $X$  be a nonempty set. Multiplicative metric is a mapping  $d : X \times X \rightarrow \mathbb{R}_+$  satisfying the following conditions:

$$d(x, y) \geq 1 \text{ for all } x, y \in X \text{ and } d(x, y) = 1 \text{ if and only if } x = y, \quad (1.1)$$

$$d(x, y) = d(y, x) \text{ for all } x, y \in X, \quad (1.2)$$

$$d(x, z) \leq d(x, y) \cdot d(y, z) \text{ for all } x, y, z \in X \text{ (multiplicative triangle inequality)}. \quad (1.3)$$

# International Journal of Innovative Research in Science, Engineering and Technology

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Note that  $\mathbb{R}_+$  is a complete multiplicative metric space with respect to the multiplicative metric.

**Example.1.2. ([8])** Let  $\mathbb{R}_+^n$  be the collection of all n-tuples of positive real numbers. Let  $d^*: \mathbb{R}_+^n \times \mathbb{R}_+^n \rightarrow \mathbb{R}$  be defined as follows

$$d^*(x, y) = \left| \frac{x_1}{y_1} \right|^* \cdot \left| \frac{x_2}{y_2} \right|^* \cdot \left| \frac{x_3}{y_3} \right|^* \cdot \dots \cdot \left| \frac{x_n}{y_n} \right|^*,$$

where  $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}_+^n$  and  $|\cdot|^*: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is defined as follows

$$|a|^* = \begin{cases} a & \text{if } a \geq 1 \\ \frac{1}{a} & \text{if } a < 1 \end{cases}$$

All conditions of multiplicative metric are satisfied.

Note that this multiplicative metric is a generalization of the metric on  $\mathbb{R}_+$ .

**Definition 1.3. ([8])** (Multiplicative convergence): Let  $(X, d)$  be a multiplicative metric space,  $\{x_n\}$  be a sequence in  $X$  and  $x \in X$ . If for every multiplicative open ball  $B_\epsilon(x)$ , there exists a natural number  $N$  such that  $n \geq N \Rightarrow x_n \in B_\epsilon(x)$ , then the sequence  $\{x_n\}$  is said to be multiplicative convergent to  $x$ , denoted by  $x_n \rightarrow x (n \rightarrow \infty)$ .

**Lemma 1.4. ([8])** Let  $(X, d)$  be a multiplicative metric space,  $\{x_n\}$  be a sequence in  $X$  and  $x \in X$ . Then  $x_n \rightarrow x (n \rightarrow \infty)$  if and only if  $d(x_n, x) \rightarrow 1 (n \rightarrow \infty)$ .

**Lemma 1.5. ([8])** Let  $(X, d)$  be a multiplicative metric space and  $\{x_n\}$  be a sequence in  $X$ . Then  $\{x_n\}$  is a multiplicative Cauchy sequence if and only if  $d(x_n, x_m) \rightarrow 1 (m, n \rightarrow \infty)$ .

Now. We prove the common fixed point theorem for two maps. This result shows the participation of rational functions to set the common fixed point between the self-mappings.

## II. MAIN RESULTS

**Theorem 2.1.** Let  $S$  and  $T$  be mappings of a complete multiplicative metric space  $(X, d)$  into itself satisfying the conditions

$$SX \subset X, TX \subset X$$

$$(2.1) \quad d(Sx, Ty) \leq \left\{ \max \left\{ \left( \frac{d(x, Sx)[d(y, Sx) + d(y, Ty)]}{1 + d(Sx, Ty)}, \frac{d(y, Sx)d(x, Ty) + d(x, y)d(Sx, y)}{d(Sx, Ty) + d(Sx, y)}, \frac{d(x, Sx)d(y, Sx) + d(x, y)d(Sx, Ty)}{d(y, Ty) + d(y, Sx)}, \frac{d(y, Ty)d(x, Ty) + d(x, Ty)d(y, Sx)}{d(y, Ty) + d(y, Sx)} \right) \right\}^\lambda \right. \quad (2.2)$$

for all  $x, y \in X$ , where  $\lambda \in (0, 1/2)$ .

Then  $S$  and  $T$  have a unique common fixed point.

**Proof.** Let  $x_0$  be an arbitrary in  $X$ . Since  $S(X) \subseteq X$  and  $T(X) \subseteq X$ , we construct the sequence  $\{x_n\}$  in  $X$ , such that

$$x_{2n+1} = Sx_{2n} \quad \text{and} \quad x_{2n+2} = Tx_{2n+1} \quad \forall n \geq 0 \quad (2.3)$$

from the definition of  $\{x_n\}$  and using (2.2), we have

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

$$\begin{aligned}
 d(x_{2n+1}, x_{2n+2}) &\leq d(Sx_{2n}, Tx_{2n+1}) \\
 &\leq \left\{ \max \left\{ \begin{aligned} &\frac{d(x_{2n}, Sx_{2n})[d(x_{2n+1}, Sx_{2n}) + d(x_{2n+1}, Tx_{2n+1})]}{1 + d(Sx_{2n}, Tx_{2n+1})}, \\ &\frac{d(x_{2n+1}, Sx_{2n})d(x_{2n}, Tx_{2n+1}) + d(x_{2n}, x_{2n+1})d(Sx_{2n}, x_{2n+1})}{d(Sx_{2n}, Tx_{2n+1}) + d(Sx_{2n}, x_{2n+1})}, \\ &\frac{d(x_{2n}, Sx_{2n})d(x_{2n+1}, Sx_{2n}) + d(x_{2n}, x_{2n+1})d(Sx_{2n}, Tx_{2n+1})}{d(x_{2n+1}, Tx_{2n+1}) + d(x_{2n+1}, Sx_{2n})}, \\ &\frac{d(x_{2n+1}, Tx_{2n+1})d(x_{2n}, Tx_{2n+1}) + d(x_{2n}, Tx_{2n+1})d(x_{2n+1}, Sx_{2n})}{d(x_{2n+1}, Tx_{2n+1}) + d(x_{2n+1}, Sx_{2n})} \end{aligned} \right\} \right\}^\lambda \\
 &\leq \left\{ \max \left\{ \begin{aligned} &\frac{d(x_{2n}, x_{2n+1})[d(x_{2n+1}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]}{1 + d(x_{2n+1}, x_{2n+2})}, \\ &\frac{d(x_{2n+1}, x_{2n+1})d(x_{2n}, x_{2n+2}) + d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+1})}{d(x_{2n+1}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})}, \\ &\frac{d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+1}) + d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})}{d(x_{2n+1}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})} \end{aligned} \right\} \right\}^\lambda \quad [\text{using (2.3)}] \\
 &\leq \left\{ \max \left\{ \begin{aligned} &\frac{d(x_{2n}, x_{2n+1})[1 + d(x_{2n+1}, x_{2n+2})]}{1 + d(x_{2n+1}, x_{2n+2})}, \\ &\frac{d(x_{2n}, x_{2n+2}) + d(x_{2n}, x_{2n+1})}{d(x_{2n+1}, x_{2n+2}) + 1}, \\ &\frac{d(x_{2n+1}, x_{2n+2}) + 1}{d(x_{2n}, x_{2n+1}) + d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})} \end{aligned} \right\} \right\}^\lambda \quad (\text{using (1.1)}) \\
 &\leq \left\{ \max \left\{ \begin{aligned} &d(x_{2n}, x_{2n+1}), \\ &d(x_{2n}, x_{2n+1}), \\ &d(x_{2n}, x_{2n+2}) \end{aligned} \right\} \right\}^\lambda \quad [\text{since, } d(x_{2n}, x_{2n+2}) \leq \\
 &\quad d(x_{2n}, x_{2n+1}) \cdot d(x_{2n+1}, x_{2n+2})] \\
 &\quad d(x_{2n+1}, x_{2n+2}) \leq d^\lambda(x_{2n}, x_{2n+2})
 \end{aligned}$$

$$\begin{aligned}
 d(x_{2n+1}, x_{2n+2}) &\leq d^\lambda(x_{2n}, x_{2n+1}) \cdot d^\lambda(x_{2n+1}, x_{2n+2}) \\
 d^{1-\lambda}(x_{2n+1}, x_{2n+2}) &\leq d^\lambda(x_{2n}, x_{2n+1}) \\
 d(x_{2n+1}, x_{2n+2}) &\leq d^{\frac{\lambda}{1-\lambda}}(x_{2n}, x_{2n+1}) \\
 (2.4)
 \end{aligned}$$

Similarly, we obtain

$$\begin{aligned}
 d(x_{2n}, x_{2n+1}) &= d(Tx_{2n-1}, Sx_{2n}) [\text{using (2.3)}] \\
 &= d(Sx_{2n}, Tx_{2n-1}) \quad [\text{using (1.2)}]
 \end{aligned}$$

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$\leq \left\{ \max \left\{ \begin{aligned} & \frac{d(x_{2n}, Sx_{2n})[d(x_{2n-1}, Sx_{2n}) + d(x_{2n-1}, Tx_{2n-1})]}{1 + d(Sx_{2n}, Tx_{2n-1})}, \\ & \frac{d(x_{2n-1}, Sx_{2n})d(x_{2n}, Tx_{2n-1}) + d(x_{2n}, x_{2n-1})d(Sx_{2n}, x_{2n-1})}{d(Sx_{2n}, Tx_{2n-1}) + d(Sx_{2n}, x_{2n-1})}, \\ & \frac{d(x_{2n}, Sx_{2n})d(x_{2n-1}, Sx_{2n}) + d(x_{2n}, x_{2n-1})d(Sx_{2n}, Tx_{2n-1})}{d(x_{2n-1}, Tx_{2n-1}) + d(x_{2n-1}, Sx_{2n})}, \\ & \frac{d(x_{2n-1}, Tx_{2n-1})d(x_{2n}, Tx_{2n-1}) + d(x_{2n}, Tx_{2n-1})d(x_{2n-1}, Sx_{2n})}{d(x_{2n-1}, Tx_{2n-1}) + d(x_{2n-1}, Sx_{2n})} \end{aligned} \right\} \right\}^\lambda$$

[using (2.2)]

$$\leq \left\{ \max \left\{ \begin{aligned} & \frac{d(x_{2n}, x_{2n+1})[d(x_{2n-1}, x_{2n+1}) + d(x_{2n-1}, x_{2n})]}{1 + d(x_{2n+1}, x_{2n})}, \\ & \frac{d(x_{2n-1}, x_{2n+1})d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n-1})d(x_{2n+1}, x_{2n-1})}{d(x_{2n+1}, x_{2n}) + d(x_{2n+1}, x_{2n-1})}, \\ & \frac{d(x_{2n}, x_{2n+1})d(x_{2n-1}, x_{2n+1}) + d(x_{2n}, x_{2n-1})d(x_{2n+1}, x_{2n})}{d(x_{2n-1}, x_{2n}) + d(x_{2n-1}, x_{2n+1})}, \\ & \frac{d(x_{2n-1}, x_{2n})d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n-1})d(x_{2n-1}, x_{2n})}{d(x_{2n-1}, x_{2n}) + d(x_{2n-1}, x_{2n+1})} \end{aligned} \right\} \right\}^\lambda \quad \text{[using (1.1) and (2.3)]}$$

$$\leq \left\{ \max \left\{ d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n}), d(x_{2n}, x_{2n-1}), \frac{1}{1} \right\} \right\}^\lambda \quad \text{[using (1.1), (1.2) and (1.3)]}$$

$$d(x_{2n}, x_{2n+1}) \leq d^\lambda(x_{2n-1}, x_{2n+1})$$

$$d(x_{2n}, x_{2n+1}) \leq d^\lambda(x_{2n-1}, x_{2n}) \cdot d^\lambda(x_{2n}, x_{2n+1}).$$

$$d(x_{2n}, x_{2n+1}) \leq d^{\frac{\lambda}{1-\lambda}}(x_{2n-1}, x_{2n}) \quad \text{[using (2.5)]}$$

$$d(x_{2n+1}, x_{2n+2}) \leq d^h(x_{2n}, x_{2n+1}), \text{ for all } n.$$

using (2.4) and (2.5), we have

$$d(x_n, x_{n+1}) \leq d^h(x_{n-1}, x_n) \tag{2.6}$$

hence

$$d(x_n, x_{n+1}) \leq d^{h^1}(x_{n-1}, x_n) \leq d^{h^2}(x_{n-1}, x_n) \leq d^{h^3}(x_{n-1}, x_n) \leq \dots \leq d^{h^n}(x_{n-1}, x_n) \text{ for all } n \text{ [using (2.6)]}$$

for all  $n \geq 2$ . let  $m, n \in \mathbb{N}$  such that  $m \geq n$ . Then we get

$$d(x_m, x_n) \leq d(x_m, x_{m-1}) \cdot d(x_{m-1}, x_{m-2}) \cdot d(x_{m-2}, x_{m-3}) \dots d(x_{n+1}, x_n) \text{ [using (1.3)]}$$

$$d(x_m, x_n) \leq d^{h^{m-1}}(x_1, x_0) \cdot d^{h^{m-2}}(x_1, x_0) \cdot d^{h^{m-3}}(x_1, x_0) \dots d^{h^n}(x_1, x_0)$$

$$d(x_m, x_n) \leq d^{\frac{h^n}{1-h}}(x_1, x_0).$$

Letting limit as  $m, n \rightarrow \infty$ , we have

$d(x_m, x_n) \rightarrow 1$ . Therefore  $\{x_n\}$  is a multiplicative Cauchy sequence.

By the completeness of  $X$ .

There exist  $z$  belongs to  $X$  such that  $x_n$  approaches to  $z$  as  $n$  approaches to infinity.

Moreover, we will show that  $Sz = z$ , if possible  $Sz \neq z$ . By the notion of a complete multiplicative metric space.

## International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$\begin{aligned}
 d(z, Sz) &\leq d(z, x_{2n+2}) \cdot d(x_{2n+2}, Sz). \\
 &= d(z, x_{2n+2}) \cdot d(Sz, x_{2n+2}) \\
 &= d(z, x_{2n+2}) \cdot d(Sz, Tx_{2n+1}) \\
 &\leq d(z, x_{2n+2}) \cdot \left\{ \max \left\{ \begin{aligned} &\frac{d(z, Sz)[d(x_{2n+1}, Sz) + d(x_{2n+1}, Tx_{2n+1})]}{1 + d(Sz, Tx_{2n+1})}, \\ &\frac{d(x_{2n+1}, Sz)d(z, Tx_{2n+1}) + d(z, x_{2n+1})d(Sz, x_{2n+1})}{d(Sz, Tx_{2n+1}) + d(Sz, x_{2n+1})}, \\ &\frac{d(z, Sz)d(x_{2n+1}, Sz) + d(z, x_{2n+1})d(Sz, Tx_{2n+1})}{d(x_{2n+1}, Tx_{2n+1}) + d(x_{2n+1}, Sz)}, \\ &\frac{d(x_{2n+1}, Tx_{2n+1})d(z, Tx_{2n+1}) + d(z, Tx_{2n+1})d(x_{2n+1}, Sz)}{d(x_{2n+1}, Tx_{2n+1}) + d(x_{2n+1}, Sz)} \end{aligned} \right\} \right\}^\lambda
 \end{aligned}$$

Taking  $n \rightarrow \infty$ , we have

$$\begin{aligned}
 &\leq d(z, z) \cdot \left\{ \max \left\{ \begin{aligned} &\frac{d(z, Sz)[d(z, Sz) + d(z, z)]}{1 + d(Sz, z)}, \frac{d(z, Sz)d(z, z) + d(z, z)d(Sz, z)}{d(Sz, z) + d(Sz, z)}, \\ &\frac{d(z, Sz)d(z, Sz) + d(z, z)d(Sz, z)}{d(z, z) + d(z, Sz)}, \\ &\frac{d(z, z)d(z, z) + d(z, z)d(z, Sz)}{d(z, z) + d(z, Sz)} \end{aligned} \right\} \right\}^\lambda \quad [\text{since, } Tx_{2n+1} = x_{2n+2}] \\
 &\leq \left\{ \max \left\{ \begin{aligned} &d(z, Sz), 1, \\ &1 \end{aligned} \right\} \right\}^\lambda
 \end{aligned}$$

$$d(z, Sz) \leq d^\lambda(z, Sz)$$

Which is a contradiction, since  $\lambda \in (0, 1/2)$

which implies that  $d(z, Sz) = 1$  i.e.,

$$Sz = z \tag{2.7}$$

Now we will show that  $Tz$  is equal to  $z$ , if possible let us suppose  $Tz$  and  $z$  are unequal, then we have

$$\begin{aligned}
 d(z, Tz) &\leq d(z, x_{2n+1}) \cdot d(x_{2n+1}, Tz). \\
 &= d(z, x_{2n+1}) \cdot d(Sx_{2n}, Tz) \leq d(z, x_{2n+1}) \cdot \left\{ \max \left\{ \begin{aligned} &\frac{d(x_{2n}, Sx_{2n})[d(z, Sx_{2n}) + d(z, Tz)]}{1 + d(Sx_{2n}, Tz)}, \\ &\frac{d(z, Sx_{2n})d(x_{2n}, Tz) + d(x_{2n}, z)d(Sx_{2n}, z)}{d(Sx_{2n}, Tz) + d(Sx_{2n}, z)}, \\ &\frac{d(z, Sx_{2n})d(z, Sx_{2n}) + d(x_{2n}, z)d(Sx_{2n}, Tz)}{d(z, Tz) + d(z, Sx_{2n})}, \\ &\frac{d(z, Tz)d(x_{2n}, Tz) + d(x_{2n}, Tz)d(z, Sx_{2n})}{d(z, Tz) + d(z, Sx_{2n})} \end{aligned} \right\} \right\}^\lambda
 \end{aligned}$$

Taking  $n \rightarrow \infty$ , we have

$$\begin{aligned}
 &\leq d(z, z) \cdot \left\{ \max \left\{ \begin{aligned} &\frac{d(z, z)[d(z, z) + d(z, Tz)]}{1 + d(z, Tz)}, \frac{d(z, z)d(z, Tz) + d(z, z)d(z, z)}{d(z, Tz) + d(z, z)}, \\ &\frac{d(z, z)d(z, z) + d(z, z)d(z, Tz)}{d(z, Tz) + d(z, z)}, \\ &\frac{d(z, Tz)d(z, Tz) + d(z, Tz)d(z, z)}{d(z, Tz) + d(z, z)} \end{aligned} \right\} \right\}^\lambda \quad [\text{using (2.7)}] \\
 &\leq 1 \cdot \left\{ \max \left\{ \begin{aligned} &1, 1, \\ &1, \\ &d(z, Tz) \end{aligned} \right\} \right\}^\lambda \quad [\text{using (1.1)}] \\
 &d(z, Tz) \leq d^\lambda(z, Tz)
 \end{aligned}$$

# International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 6, June 2016

Which is a contradiction, since  $\lambda \in (0,1/2)$

Hence, we have

$$Tz = z \tag{2.8}$$

Using (2.7) and (2.8), we have

$$Sz = Tz = z \tag{2.9}$$

So, z is a fixed point of S and T.

### UNIQUENESS

Let z and w are two common fixed points, we have to prove that  $z = w$ , if possible  $z \neq w$ , then we have

$$d(z,w)=d(Sz,Tw)$$

$$\begin{aligned} &\leq \left\{ \max \left\{ \begin{aligned} &\frac{d(z,Sz)[d(w,Sz)+d(w,Tw)]}{1+d(Sz,Tw)}, \frac{d(w,Sz)d(z,Tw)+d(z,w)d(Sz,w)}{d(Sz,Tw)+d(Sz,w)}, \\ &\frac{d(z,Sz)d(w,Sz)+d(z,w)d(Sz,Tw)}{d(w,Tw)+d(w,Sz)}, \\ &\frac{d(w,Tw)d(z,Tw)+d(z,Tw)d(w,Sz)}{d(w,Tw)+d(w,Sz)} \end{aligned} \right\} \right\}^\lambda \\ &\leq \left\{ \max \left\{ \begin{aligned} &\frac{d(z,z)[d(w,z)+d(w,w)]}{1+d(z,w)}, \frac{d(w,z)d(z,w)+d(z,w)d(z,w)}{d(z,w)+d(z,w)}, \\ &\frac{d(z,z)d(w,z)+d(z,w)d(z,w)}{d(w,w)+d(w,z)}, \\ &\frac{d(w,w)d(z,w)+d(z,w)d(w,z)}{d(w,w)+d(w,z)} \end{aligned} \right\} \right\}^\lambda \quad [\text{using (2.9)}] \\ &\leq \left\{ \max \left\{ \begin{aligned} &1, d(z,w), \\ &d(w,z), \\ &d(w,z) \end{aligned} \right\} \right\}^\lambda \end{aligned}$$

$$d(z,w) \leq d^\lambda(z,w)$$

Which is a contradiction, since  $\lambda \in (0,1/2)$

which implies that  $d(z,w) = 1$  i.e,  $z = w$  which proves the uniqueness of fixed point.

**Corollary :** If S is a self-mapping defined on a complete multivalued metric space  $(X,d)$  satisfying

$SX \subset X$  and

$$(2.10) d(Sx,Sy) \leq \left\{ \max \left\{ \begin{aligned} &\frac{d(x,Sx)[d(y,Sx)+d(y,Sy)]}{1+d(Sx,Sy)}, \frac{d(y,Sx)d(x,Sy)+d(x,y)d(Sx,y)}{d(Sx,Sy)+d(Sx,y)}, \\ &\frac{d(x,Sx)d(y,Sx)+d(x,y)d(Sx,Sy)}{d(y,Sy)+d(y,Sx)}, \\ &\frac{d(y,Sy)d(x,Sy)+d(x,Sy)d(y,Sx)}{d(y,Sy)+d(y,Sx)} \end{aligned} \right\} \right\}^\lambda$$

(2.11)

for all  $x, y \in X$ , where  $\lambda \in (0,1/2)$  Then S has a unique fixed point.

**Proof:** we can prove the result by applying previous theorem setting  $T=S$ .

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Vol. 5, Issue 6, June 2016

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