

# Strongly $g^*$ s –Closed Sets in Topological Spaces

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**ABSTRACT:** The aim of this paper is to introduce a new class of sets namely strongly  $g^*$ s –closed sets in topological spaces and their properties. Further we discuss the strongly  $g^*$ s – continuous functions, strongly  $g^*$ s –irresolute function, strongly  $g^*$ s –homeomorphism. Moreover we define strongly  $g^*$ s –closed maps and strongly  $g^*$ s –open maps in topological spaces.

**KEYWORDS :** Strongly  $g^*$ s –closed sets, strongly  $g^*$ s – continuous functions, strongly  $g^*$ s –irresolute function, strongly  $g^*$ s –homeomorphism, strongly  $g^*$ s –closed and open maps.

## I.INTRODUCTION

In 1970, Levine [7] first considered the concept of generalized closed set (briefly,  $g$ -closed) Sets were defined and investigated. Arya and Nour [1] defined generalized semi open [briefly,  $gs$ -open] sets using semi open sets. In 1987, Bhattacharyya and Lehiri [3] introduced the class of semi-generalized closed sets ( $sg$ -closed sets). In 2000, M.K.R.S.verakumar [17] introduced the concept of  $g^*$ -closed set. Nagaveni [10], Pushpalatha [13], Maki Devi and Balachandran.P [8,9] and S.S.Benchalli and R.S.Wali [2] investigated, On  $R_w$ - closed sets. In 1993 Mahi.H.R and Balachandran.P [8] introduced strongly generalized closed sets, generalized  $\alpha$ -closed sets. In 2001, Sundharam .P and A.pushpalatha introduced, strongly generalized closed sets[4]. In 2011 G.B.Navalagi [11] introduced  $g^*$ s-closed sets in topological spaces. In this paper we introduced a new class of sets called strongly  $g^*$ s-closed sets.

## II.PREMILINARIES

### Definition:2.1

A subset of a topological space  $(X, \tau)$ ,

- (i) **generalized closed** (briefly,  $g$ -closed) [7] if  $cl(A) \subseteq U$  Whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (ii) **generalized semi closed** [3](briefly  $gs$ -closed) if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (iii) **semi-generalized closed** [3](briefly  $sg$ -closed) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi open in  $X$ .
- (iv) **generalized  $\alpha$ -closed** [8](briefly,  $g\alpha$ -closed) if  $\alpha-cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $X$ .
- (v)  **$\alpha$ -generalized closed** [8](briefly  $ag$ -closed) if  $\alpha-cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (vi) **strongly  $g$ -closed** [4] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $X$ .
- (vii)  **$g^*$ s-closed set** [11] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $gs$ -open.
- (viii)  **$T_{g^*s}$ -space** [16] if every strongly  $g^*$ s-closed set in it is closed.
- (ix)  **$gp^*$  closed** [5] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $gp$ -open.
- (x) **weekly generalized closed**(briefly,  $wg$ -closed ) [15] if  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

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## Definition :2.2

A function  $f : X \rightarrow Y$  is called ,

- (i) **strongly continuous** [6] if  $f^{-1}(V)$  is both open and closed in  $X$  for each subset  $V$  in  $Y$ .
- (ii) **perfectly continuous** [12] if  $f^{-1}(V)$  is both open and closed in  $X$  for each open set  $V$  in  $Y$ .
- (iii) **generalized continuous** [14] (g-continuous) if  $f^{-1}(V)$  is g-open in  $X$  for each open set  $V$  in  $Y$ .
- (iv) **strongly g-continuous** if [6]  $f^{-1}(V)$  is open in  $X$  for each g-open set  $V$  in  $Y$ .
- (v) **semi-generalized continuous** [14](sg-continuous) if  $f^{-1}(V)$  is sg-open in  $X$  for each open set  $V$  in  $Y$ .
- (vi) **generalized semi-continuous** [14](gs-continuous) if  $f^{-1}(V)$  is gs-open in  $X$  for each open set  $V$  in  $Y$ .

## III. STRONGLY g\*s-CLOSED SETS IN TOPOLOGICAL SPACES

### Definition: 3.1

A subset  $A$  of  $X$  is called a **strongly g\*s-closed set** if  $\text{int}(\text{scl}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is gs- open. The class of all strongly g\*s-closed sets in a topological space  $(X, \tau)$  is denoted by strongly g\*s-c $(X, \tau)$ .

### Remark: 3.2

The complement of strongly g\*s-closed set is strongly g\*s-open set.

### Theorem: 3.3

Every closed set in  $X$  is strongly g\*s-closed in  $X$  but not conversely.

#### Proof:-

Let  $A$  be a closed set in  $X$ . Let  $U$  be a gs-open set such that  $A \subseteq U$ . Since  $A$  is closed, that is  $\text{cl}(A) = A$ ,  $\text{cl}(A) \subseteq U$ . But  $\text{int}(\text{scl}(A)) \subseteq \text{cl}(A) \subseteq U$ . Therefore  $\text{int}(\text{scl}(A)) \subseteq U$ . Hence  $A$  is strongly g\*s-closed set in  $X$ .

### Example: 3.4

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{a\}\}$ . Strongly g\*s-closed sets is  $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ . Closed set is  $\{\emptyset, X, \{b, c\}\}$ . In this space the sets  $\{c\}, \{b\}$  is strongly g\*s-closed sets but not closed.

### Theorem: 3.5

Union of two strongly g\*s-closed sets is strongly g\*s-closed.

#### Proof:

Let  $A$  and  $B$  be strongly g\*s-closed sets in  $X$ . Let  $U$  be a gs-open in  $X$  such that  $A \cup B \subseteq U$ . Then  $A \subseteq U$  and  $B \subseteq U$ . Since  $A$  and  $B$  are strongly g\*s-closed set.  $\text{int}(\text{scl}(A)) \subseteq U$  and  $\text{int}(\text{scl}(B)) \subseteq U$ . Hence  $\text{int}(\text{scl}(A \cup B)) = \text{int}(\text{scl}(A)) \cup \text{int}(\text{scl}(B)) \subseteq U$ . Therefore  $A \cup B$  is strongly g\*s-closed.

### Example:3.6

Consider the topological space  $X = \{a, b, c\}$  with the topology  $\tau = \{\emptyset, X, \{c\}\}$ . Strongly g\*s-closed sets is  $\{\emptyset, X, \{b\}, \{a\}, \{a, b\}\}$ .

### Theorem: 3.7

Every strongly g\*s-closed set is gs-closed but not conversely.

#### Proof:

Let  $A$  be strongly g\*s-closed set in  $X$ . Let  $U$  be open set such that  $A \subseteq U$ . Since every open set is gs-open and  $A$  is strongly g\*s-closed. We have  $\text{int}(\text{scl}(A)) \subseteq U$ . Therefore  $A$  is gs-closed in  $X$ .

### Example: 3.8:

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{c\}\}$ . Strongly g\*s-closed sets is  $\{\emptyset, X, \{b\}, \{a\}, \{a, b\}\}$ . gs-closed is  $\{\emptyset, X, \{b\}, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ . In this space the set  $\{a, c\}, \{b, c\}$  is gs-closed but not strongly g\*s-closed.

### Theorem: 3.9

Every strongly g\*s-closed set in  $X$  is a sg-closed set in  $X$  but not conversely.

#### Proof:

Let  $A$  be strongly g\*s-closed in  $X$ . Let  $U$  be a semi-open set in  $X$  such that  $A \subseteq U$ . Since every semi open set is gs-open

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and  $A$  is strongly  $g^*s$ -closed. We have  $\text{int}(\text{scl}(A)) \subseteq U$ . Therefore  $A$  is  $sg$ -closed in  $X$ .

**Example: 3.10**

Consider the topological space  $X = \{a, b, c\}$  with the topology  $\tau = \{\emptyset, X, \{a, b\}\}$ .  $sg$ -closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ . Strongly  $g^*s$ -closed sets is  $\{\emptyset, X, \{c\}\}$ . In this space the sets  $\{a\}, \{b\}, \{a, c\}, \{b, c\}$  are  $sg$ -closed sets but not strongly  $g^*s$ -closed sets.

**Theorem: 3.11**

Every strongly  $g^*s$ -closed sets in  $X$  is a  $wg$ -closed set in  $X$  but not conversely.

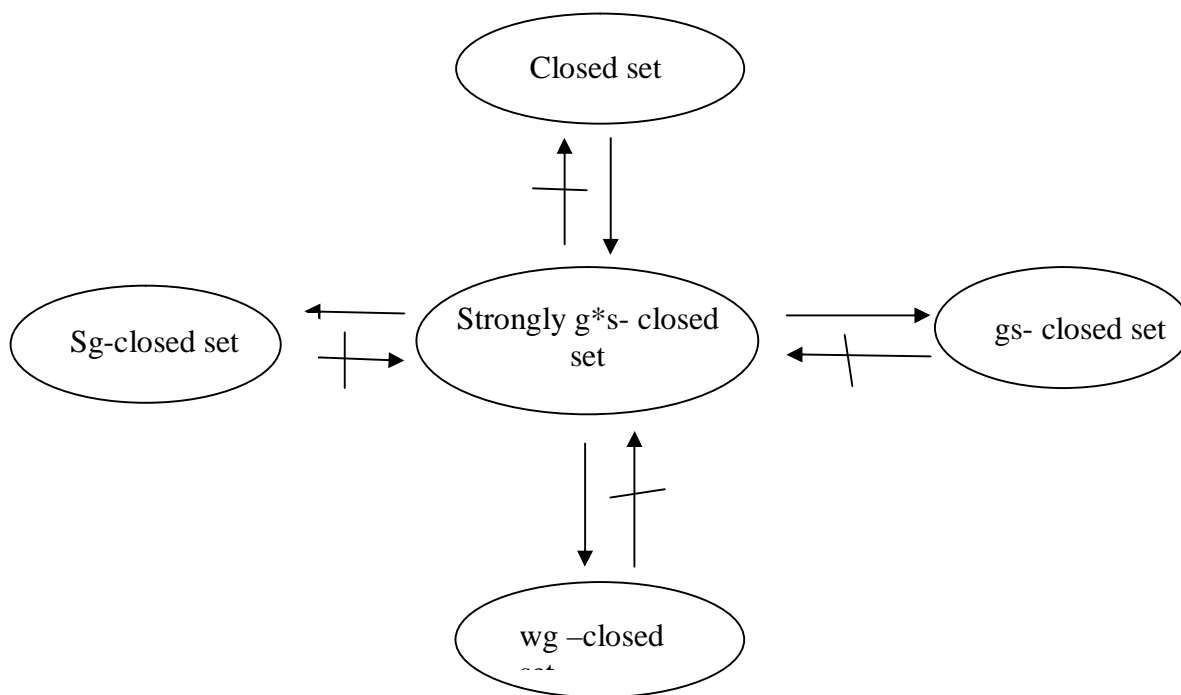
**Proof:**

Let  $A$  be a strongly  $g^*s$ -closed sets in  $X$ . Let  $U$  be a open set in  $X$ . Such that  $A \subseteq U$ . Since every open set is  $gs$ -open and  $A$  is strongly  $g^*s$ -closed. We have  $\text{int}(\text{scl}(A)) \subseteq U$ . Therefore  $A$  is  $wg$ -closed in  $X$ .

**Example: 3.12**

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{c\}, \{a, c\}\}$ . The  $wg$ -closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ . Strongly  $g^*s$  closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . In the space the set  $\{b, c\}$  is  $wg$ -closed set but not strongly  $g^*s$  closed.

From the above theorem 3.3, 3.7, 3.9, 3.11 and examples 3.4, 3.8, 3.10, 3.12. We get the following diagram.



**Remark: 3.13**

The concept of strongly  $g^*s$ -closed set is independent of the following classes of sets namely  $g$ -closed set,  $g^*$ -closed set, pre-closed set,  $g\alpha$ -closed set,  $ag$ -closed set,  $gp^*$ -closed set and strongly generalized closed sets.

**Example: 3.14**

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ .  $g$ -closed is  $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ . Strongly  $g^*s$ -closed set is  $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ . In this space, the set  $\{a, b\}$  is  $g$ -closed set but not strongly  $g^*s$ -closed set and the set  $\{c\}$  is strongly  $g^*s$  closed set but not  $g$ -closed set.

**Example: 3.15**

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Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ .  $g^*$ -closed set is  $\{\emptyset, X, \{a, c\}, \{c\}, \{b, c\}\}$ . Strongly  $g^*$ s-closed set  $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ . In this space the set  $\{a, c\}$  is  $g^*$ -closed set but not strongly  $g^*$ s-closed set and the set  $\{b\}$  is strongly  $g^*$ s-closed set but not  $g^*$ -closed set.

**Example: 3.16**

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . Pre closed set is  $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ . Strongly  $g^*$ s-closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ . In this space  $\{a\}, \{b\}$  are strongly  $g^*$ s-closed but not pre-closed sets. In topology  $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ . Pre closed set is  $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ . Strongly  $g^*$ s-closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ . In this space  $\{a\}, \{b\}$  are strongly  $g^*$ s-closed but not pre-closed sets.

**Example: 3.17**

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . In this space the set  $\{b\}$  is strongly  $g^*$ s-closed set but not  $g\alpha$ -closed set,  $ag$ -closed set and Strongly generalized closed set.  $g\alpha$ -closed set,  $ag$ -closed set and strongly generalized closed set is strongly  $g^*$ s-closed set.

**Example: 3.18**

Consider the topological space  $X = \{a, b, c\}$  with topology  $\tau = \{\emptyset, X, \{c\}, \{a, c\}\}$ .  $gp^*$ -closed set is  $\{\emptyset, X, \{b\}, \{a, b\}\}$ . Strongly  $g^*$ s-closed set is  $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . In this space the set  $\{a\}$  is strongly  $g^*$ s-closed set but not  $gp^*$ -closed set.

**Theorem: 3.19**

A subset  $A$  of  $X$  is strongly  $g^*$ s-closed set in  $X$  iff  $\text{int}(scl(A)) - A$  contains no non empty  $g$ s-closed set in  $X$ .

**Proof:**

Suppose that  $F$  is a non empty  $g$ s-closed subset of  $\text{int}(scl(A)) - A$ . Now  $F \subseteq scl(A) - A$ , Then  $F \subseteq \text{int}(scl(A)) \cap A^c$ . Therefore  $F \subseteq \text{int}(scl(A))$  and  $F \subseteq A^c$ . Since  $F^c$  is  $g$ s-open set and  $A$  is strongly  $g^*$ s-closed,  $\text{int}(scl(A)) \subseteq F^c$ . That is  $F \subseteq \text{int}(scl(A))^c$ . Since  $F^c$  is  $g$ s-open set and  $A$  is strongly  $g^*$ s-closed,  $\text{int}(scl(A)) \subseteq F^c$ . That is  $F \subseteq \text{int}(scl(A))^c$ .

Hence  $F \subseteq \text{int}(scl(A)) \cap \text{int}(scl(A))^c = \emptyset$ . That is  $F = \emptyset$ . Thus  $\text{int}(scl(A)) - A$  contains no non empty  $g$ s-closed set. Conversely, Assume that  $\text{int}(scl(A)) - A$  contains no non empty  $g$ s-closed set.

Let  $A \subseteq U$ ,  $U$  is  $g$ s-open. Suppose that  $\text{int}(scl(A))$  is not contained in  $U$ . Then  $\text{int}(scl(A)) \cap U^c$  is a non empty  $g$ s-closed set and contained in  $\text{int}(scl(A)) - A$ . Which is a contradiction. Therefore  $\text{int}(scl(A)) \subseteq U$  and hence  $A$  is strongly  $g^*$ s-closed set.

**Theorem: 3.20**

Let  $(X, \tau)$  be a compact topological space. If  $A$  is strongly  $g^*$ s-closed subset of  $X$ , then  $A$  is compact.

**Proof:**

Let  $\{U_i\}$  be a open cover of  $A$ . Since every open set is  $g$ s-open and  $A$  is strongly  $g^*$ s-closed. We get  $\text{int}(scl(A)) \subseteq \bigcup U_i$ . Since a closed subset of a compact space is compact,  $\text{int}(scl(A))$  is compact. Therefore there exist a finite subcover say  $\{U_1 \cup U_2 \cup \dots \cup U_n\}$  of  $\{U_i\}$  for  $\text{int}(scl(A))$ . So  $A \subseteq \text{int}(scl(A)) \subseteq \{U_1 \cup U_2 \cup \dots \cup U_n\}$ . Therefore  $A$  is not compact.

**Theorem: 3.21**

Let  $(X, \tau)$  be Lindelof [countably compact] and suppose that  $A$  is strongly  $g^*$ s-closed subset of  $X$ . Then  $A$  is not Lindelof [countably compact].

**Proof:**

Let  $\{U_i\}$  be a countable open cover of  $A$ . Since every open set is  $g$ s-open  $\{U_i\}$  is a countable  $g$ s-open cover of  $A$ .  $\bigcup U_i$  is  $g$ s-open. Then  $\text{int}(scl(A)) \subseteq \bigcup U_i$  because  $A$  is strongly  $g^*$ s-closed. Since a closed subset of a Lindelof space is Lindelof,  $\text{int}(scl(A))$  is not Lindelof. Therefore  $\text{int}(scl(A))$  has no countable subcover, say  $\{U_1 \cup U_2 \cup \dots \cup U_n\}$ . and it follows that  $A \subseteq \text{int}(scl(A)) \subseteq \bigcup U_i$ . Hence  $A$  is not Lindelof.

**Definition: 3.22**

A subset  $A$  of a topological space  $X$  is called strongly  $g^*$ s-open set if  $A^c$  is strongly  $g^*$ s-closed. The class of all strongly  $g^*$ s-closed sets is denoted as strongly  $g^*$ s-O  $(X, \tau)$ .

**Theorem: 3.23**

If  $A$  and  $B$  are strongly  $g^*$ s-open sets in  $X$  then  $A \cap B$  is also strongly  $g^*$ s-open set in  $X$ .

**Proof:**

Let  $A$  and  $B$  be two strongly  $g^*$ s-open sets in  $X$ . Then  $A^c$  and  $B^c$  are strongly  $g^*$ s-closed sets in  $X$ . By theorem, [Union of two strongly  $g^*$ s-closed sets is strongly  $g^*$ s-closed].  $A^c \cup B^c$  is a strongly  $g^*$ s-closed set in  $X$ . That is  $(A \cap B)^c$  is a

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strongly  $g^*$ s-closed set in  $X$ . Therefore  $(A \cap B)$  is strongly  $g^*$ s-open set in  $X$ .

**Theorem:3.24**

For each  $x \in X$ , either  $\{x\}$  is  $gs$ -closed or  $\{x\}^c$  is strongly  $g^*$ s-closed in  $X$ .

**Proof:**

If  $\{x\}$  is not  $gs$ -closed, then the only  $gs$ -open set containing  $\{x\}^c$  in  $X$ . Thus semi closure of  $\{x\}^c$  is contained in  $X$ . And hence  $\{x\}^c$  is strongly  $g^*$ s-closed in  $X$ .

## IV. STRONGLY $g^*$ s –CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

Levine [ 6 ] introduced semi continuous functions using semi open sets. The study on the properties of semi-continuous functions is further carried out by Noiri [ 12 ], crossely and Hildebrand and many others. Sundram [ 14 ] introduced the concept of generalized continuous functions includes the class of continuous functions and studies several properties related to it. In this section, we introduce the concepts of strongly  $g^*$ s –continuous functions , strongly  $g^*$ s –irresolute function, strongly  $g^*$ s –homeomorphism in Topological spaces and study their properties.

**Definition : 4. 1**

A map  $f: X \rightarrow Y$  from a topological space  $X$  into a topological space  $Y$  is called strongly  $g^*$ s –Continuous if the inverse image of every closed set in  $Y$  is strongly  $g^*$ s –closed in  $X$ .

**Theorem: 4.2**

If a map  $f: X \rightarrow Y$  is continuous, then it is strongly  $g^*$ s-continuous but not conversely.

**Proof:**

Let  $f: X \rightarrow Y$  be continuous. Let  $F$  be any closed set in  $Y$ . The inverse image  $f^{-1}(F)$  is closed in  $Y$ . Since every closed set is strongly  $g^*$ s –closed,  $f^{-1}(F)$  is strongly  $g^*$ s –closed in  $X$ . Therefore  $f$  is strongly  $g^*$ s-continuous.

**Example:4.3**

Let  $X=Y= \{a,b,c\}, \tau = \{\phi, X, \{b\}\}$  .Strongly  $g^*$ s continuous is  $\{\phi, X, \{a\}, \{c\}, \{a,c\}\}$ .  $\sigma = \{\phi, Y, \{a,b\}\}$ . Strongly  $g^*$ s continuous is  $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a,c\}, \{b,c\}\}$ . Let  $f: X \rightarrow Y$  be the identity map, Then  $f$  is not continuous. Since for the closed  $\{c\}$  in  $Y, f^{-1}(\{c\}) = \{b\}$  is not closed in  $X$ . But  $f$  is strongly  $g^*$ s-continuous. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity map. Then  $f$  is  $gs$ -continuous but not strongly  $g^*$ s-continuous. Since  $\{a,c\}$  is closed in  $Y$ . But  $f^{-1}(\{a,c\}) = \{a,c\}$  is not strongly  $g^*$ s –closed in  $X$ .

**Theorem:4.4**

If a map  $f: X \rightarrow Y$  is  $gs$ -continuous then it is strongly  $g^*$ s-continuous but not conversely.

**Proof:**

Let  $f: X \rightarrow Y$  be  $gs$ -continuous. Let  $F$  be any closed in  $Y$ . (i.e)  $F$  is  $gs$  –closed set is strongly  $g^*$ s –closed in  $Y. f^{-1}(F)$  is  $gs$  –closed in  $X$ . Therefore  $f$  is  $gs$ -continuous.

**Example:4.5**

Let  $X=Y= \{a,b,c\}, \tau = \{ \phi, X, \{a\}\}$  Strongly  $g^*$ s-closed sets is  $\{\phi, X, \{b\}, \{c\}\}, \{b,c\}\}$ .  $\sigma = \{\phi, Y, \{b\}\}$  .

**Theorem:4.6**

Let  $f: X \rightarrow Y$  be a map. Then the following statements are equivalent .

- (a)  $f$  is strongly  $g^*$ s continuous.
- (b) The inverse image of each open set in  $Y$  is strongly  $g^*$ s –open in  $x$ .

**Proof:**

(a)  $\Rightarrow$  (b)

Assume that  $f: X \rightarrow Y$  is strongly  $g^*$ s-continuous. Let  $G$  be open in  $Y$ . The  $G^c$  is closed in  $Y$ . Since  $f$  is strongly  $g^*$ s-continuous.  $f^{-1}(G^c)$  is strongly  $g^*$ s –closed in  $X$ .

But  $f^{-1}(G^c) = X - f^{-1}(G)$ . Thus  $f^{-1}(G)$  is strongly  $g^*$ s –open in  $X$ .

(b)  $\Rightarrow$  (a)

Assume that the inverse image of each open set in  $Y$  is strongly  $g^*$ s –open in  $X$ . Let  $F$  be any closed set in  $Y$ . By

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assumption is strongly  $g^*s$  –open in  $X$ . But  $f^{-1}(F^c) = X - f^{-1}(F)$ . Thus  $X - f^{-1}(F)$  is strongly  $g^*s$  –open in  $X$  and  $f^{-1}(F)$  is strongly  $g^*s$  –closed in  $X$ . Therefore  $f$  is strongly  $g^*s$  continuous. Hence (a) & (b) are equivalent.

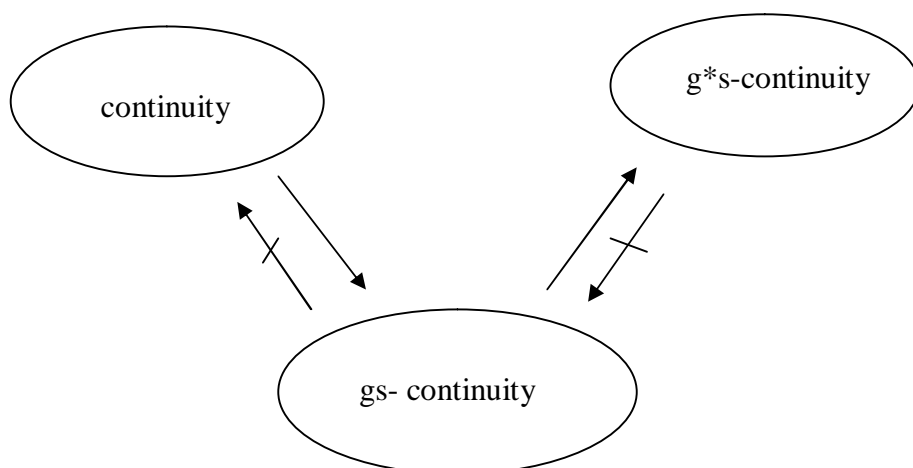
**Theorem: 4.7**

Let  $X$  and  $Z$  be any topological spaces and  $Y$  be a  $T_{g^*s}$ -spaces and  $Y$  be a  $T_{g^*s}$  -space. Then the composition  $g \circ f : X \rightarrow Y$  of the strongly  $g^*s$  –continuous map,  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  is also strongly  $g^*s$  –continuous.

**Proof:**

Let  $F$  be closed in  $Z$ . Since  $g$  is strongly  $g^*s$  –continuous,  $g^{-1}(F)$  is strongly  $g^*s$  –closed in  $Y$ . But  $Y$  is  $T_{g^*s}$ -space and so  $g^{-1}(F)$  is closed. Since  $f$  is strongly  $g^*s$  –continuous,  $f^{-1}(g^{-1}(F))$  is strongly  $g^*s$  –closed in  $X$ . But  $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ . Therefore  $g \circ f$  is strongly  $g^*s$  –continuous.

We illustrate the relations between various generalizations of continuous functions in the following diagram:



None of the implications in the above diagram can be reversed.

## V. STRONGLY $g^*s$ –CLOSED MAPS AND $g^*s$ -OPEN MAPS IN TOPOLOGICAL SPACES

In this section, we introduce the concepts of strongly  $g^*s$  –closed maps and strongly  $g^*s$ -open maps in Topological spaces.

**Definition : 5.1**

A map  $f: X \rightarrow Y$  is called strongly  $g^*s$  –closed map if for each closed set  $F$  in  $X$ ,  $f(F)$  is a strongly  $g^*s$  –closed in  $Y$ .

**Theorem: 5.2**

If  $f: X \rightarrow Y$  is a closed map then it is strongly  $g^*s$  –closed but not conversely.

**Proof:**

Since every closed set is strongly  $g^*s$  –closed, the result follows.

**Example: 5.3**

Consider the topological spaces  $X=Y= \{a,b,c\}$  with topologies  $\tau = \{ \phi, X, \{a\} \}$  and  $\sigma = \{ \phi, Y, \{b,c\} \}$ . Here  $\tau = \{ \phi, X, \{a\} \}$ . Strongly  $g^*s$ -closed sets is  $\{ \phi, X, \{b\}, \{c\}, \{b, c\} \}$ .  $\sigma = \{ \phi, Y, \{b,c\} \}$ . Let  $f$  be the identity map from  $X$  onto  $Y$ . Then  $f$  is  $gs$  –continuous but not a closed map. Since the closed set  $\{b,c\}$  in  $(X, \tau)$ ,  $f(\{b,c\}) = \{b,c\}$  is not closed set in  $Y$ .

**Definition :5.4**

A map  $f: X \rightarrow Y$  is called a  $gs$  –open map if  $f(U)$  is strongly  $g^*s$  –open in  $Y$  for every open set  $U$  in  $X$ .

**Theorem: 5.5**

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If  $f: X \rightarrow Y$  is an open map then it is strongly  $g^*$ s –open but not conversely.

**Proof:**

Let  $f: X \rightarrow Y$  be an open map. Let  $U$  be any open set in  $X$ . Then  $f(U)$  is an open set in  $Y$ . Then  $f(U)$  is strongly  $g^*$ s –open. Since every open set is strongly  $g^*$ s –open. Therefore  $f$  is strongly  $g^*$ s –open.

**Example: 5.6**

Let  $X=Y=\{a, b, c\}$  with topologies  $\tau = \{\phi, X, \{a, b\}\}$  and  $\sigma = \{\phi, Y, \{b\}\}$ . First we consider the topology  $\tau = \{\phi, X, \{a, b\}\}$  Strongly  $g^*$ s continuous  $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ .  $\sigma = \{\phi, Y, \{b\}\}$ . Then the identity function  $f: X \rightarrow Y$  is strongly  $g^*$ s –open but not open. Since for the open set  $\{a, b\}$  in  $(X, \tau), f(\{a, b\}) = \{a, b\}$  is strongly  $g^*$ s –open but not open in  $(Y, \sigma)$ . Therefore  $f$  is not an open map.

**Theorem: 5.7**

A map  $f: X \rightarrow Y$  is strongly  $g^*$ s –closed if and only if for each subset  $S$  of  $Y$  and for each open set containing  $f^{-1}(s)$  there is a strongly  $g^*$ s –open set  $V$  of  $Y$  such that  $S \subseteq V$  and  $f^{-1}(V) \subseteq U$ .

**Proof:**

Suppose  $f$  is strongly  $g^*$ s –closed. Let  $S$  be a sub set of  $Y$  and  $U$  is an open set of  $X$  such that  $f^{-1}(V) \subseteq U$ . Conversely, suppose that  $F$  is a closed set in  $X$ . Then  $f^{-1}(Y-f(F)) = X-F$  and  $X-F$  is open. By hypothesis, there is a strongly  $g^*$ s –open set  $V$  of  $Y$ . such that  $f^{-1}(F) \subseteq U$  and  $f^{-1}(V) \subseteq X-F$ . Therefore  $Y-V \subseteq f(F) \subseteq f(X-U) \subseteq Y-V$ . which implies  $f(F) = Y-V$ . Since  $Y-V$  is strongly  $g^*$ s –closed,  $f(F)$  is strongly  $g^*$ s –closed and thus  $f$  is strongly  $g^*$ s –closed map.

**Theorem: 5.8**

If  $f: X \rightarrow Y$  is closed and  $h: Y \rightarrow Z$  is strongly  $g^*$ s –closed then  $h \circ f: X \rightarrow Z$  is strongly  $g^*$ s –closed.

**Proof:**

Let  $f: X \rightarrow Y$  is a closed map and  $h: Y \rightarrow Z$  is a strongly  $g^*$ s –closed map. Let  $V$  be any closed set in  $X$ . Since  $f: X \rightarrow Y$  is closed,  $f(V)$  is closed in  $Y$  and Since  $h: Y \rightarrow Z$  is strongly  $g^*$ s –closed,  $h(f(V))$  is a strongly  $g^*$ s –closed set in  $Z$ . Therefore  $h \circ f: X \rightarrow Z$  is a strongly  $g^*$ s –closed.

**Theorem: 5.9**

If  $f: X \rightarrow Y$  is a continuous, strongly  $g^*$ s –closed map from a normal space  $X$  onto a space  $Y$ , then  $Y$  is normal.

**Proof:**

Let  $A, B$  be disjoint closed set of  $Y$ . Then  $f^{-1}(A), f^{-1}(B)$  are disjoint closed sets of  $X$ . Since  $X$  is normal, then are disjoint open set  $U, V$  in  $X$ . such that  $f^{-1}(A) \subseteq U$  and  $f^{-1}(B) \subseteq V$ . Since  $f$  is strongly  $g^*$ s –closed set  $G, H$  in  $Y$  Such that  $A \subseteq G, B \subseteq H$ , and  $f^{-1}(G) \subseteq U$  and  $f^{-1}(H) \subseteq V$ . Since  $U, V$  are disjoint  $\text{int}(G)$  and  $\text{int}(H)$  are disjoint open sets. Since  $G$  is strongly  $g^*$ s –open  $A$  is closed and  $A \subseteq G$ . which implies  $\text{int}(G)$ . Similarly  $B \subseteq \text{int}(H)$ . Hence  $Y$  is normal.

## VI. CONCLUSION

Through the above findings, this paper has attempted to compare strongly  $g^*$ s –closed sets with other closed sets in topological spaces. An attempt of this paper is to state that the definitions and results that shown in this paper, will result in obtaining several characterizations and enable to study various properties as well. In further study the various mappings may be introduced.

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