

# **Study of Fingero-Imbibition Phenomena Arising In Double Phase Flow through Porous Media Using Finite Difference Method**

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**ABSTRACT:** The present paper contains the numerical study of fingero-imbibition phenomena in double phase flow through porous media. The solution is obtained for a particular case, when the saturation co-efficient is considered as constant. The solution is obtained by finite difference method, particularly known as Successive over Relaxation method.

**KEYWORDS:** Fingero-imbibition, Porous media, Double phase, Successive over Relaxation

## **I. INTRODUCTION**

Fingero-imbibition phenomenon arises due to the simultaneous occurrence of two important phenomena viz. fingering and imbibition. It is assumed that less viscous fluid is injected into porous medium saturated with resident fluid. Consider a porous medium filled with some resident fluid known as native fluid [1]. This medium is carried into contact with another fluid which preferentially wets the medium. Then there is a unstructured flow of the wetting fluid into the medium and a counter flow of the native fluid from the medium. This phenomena arising due to the difference in wetting abilities is called imbibition [2,3]. Similarly, when a fluid contained in a porous medium is displaced by

another fluid of lesser viscosity, instead of regular displacement of whole front, protuberances (fingers) may occur which shoot through the porous medium at relatively great speeds [4]. This phenomenon is called fingering or instabilities [5]. Simultaneous occurrence of phenomena of fingering and imbibition is known as finger-imbibition phenomena [6]. Such a phenomena occurs in the problem of petroleum technology. Many authors have discussed this phenomena from different point of view. Verma obtained perturbation solution of this phenomena [7]. M.N.Mehta and Verma obtained Composite expansion of finger-imbibition in double phase flow through underground porous medium [8]. S.K.Mishra and Verma applied similarity technique to obtain analytic solution.

Immiscible flow is assumed and it is also assume that injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure.

## **II. STATEMENT OF THE PROBLEM**

Consider a finite cylindrical mass of porous medium of length  $L(=1)$  saturated with native fluid (oil). It is completely surrounded by an impermeable surface except for one end of the cylinder. This face is labeled as the imbibition face ( $x=0$ ) and this end is exposed to an adjacent formation of 'injected' liquid (water). It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (water) is initiated by imbibition and the consequent displacement of native liquid (oil) produces protuberances (fingers)[9]. This arrangement describes a one -dimensional phenomenon of Fingero-Imbibition.

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## III. MATHEMATICAL FORMULATION

Assuming that the flow of two immiscible phases is governed by Darcy's law, the seepage velocity of injected fluid (water) and native fluid (oil) are,

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \quad (1)$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \quad (2)$$

Where  $k$  = Permeability of the homogeneous medium

$k_w$  = Relative permeability of injected fluid is assumed to be a functions of  $S_w$

$k_o$  = Relative permeability of native fluid, is assumed to be functions of  $S_o$ .

$P_w$  = Pressures of injected fluid

$P_o$  = Pressures of native fluid

$\mu_o, \mu_w$  = Constant viscosities

$S_w$  = Saturation of injected fluid

$S_o$  = Saturation of native fluid

Neglecting the variation in phase densities, the equation of continuity for injected fluid can be written as:

$$P \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$P \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

Where  $P$  is the porosity of the medium.

From the definition of phase saturation, it is obvious that

$$S_w + S_o = 1 \quad (5)$$

The analytic condition for imbibition phenomenon is given by

$$V_w + V_o = 0 \quad (6)$$

The capillary pressure ( $P_c$ ), arise due to the pressure discontinuity of the flowing phases across their common interface, is given by [10]

$$P_c = P_o - P_w \quad (7)$$

From equations (1), (2) and (5), we have

$$\frac{k_o}{\mu_o} \frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w} \frac{\partial P_w}{\partial x} = 0 \quad (8)$$

From (7) and (8), we have

$$\frac{k_o}{\mu_o} \left\{ \frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x} \right\} + \frac{\partial P_w}{\partial x} = 0$$

$$\therefore \frac{\partial P_w}{\partial x} = \frac{\frac{k_o}{\mu_o} k_w}{\left\{ \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} \right\}} \frac{\partial P_c}{\partial x} \quad (9)$$

From (1) and (3), we have

$$P \frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x} \left\{ \frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \right\} = 0 \quad (10)$$

From equations (9) and (10), we get

$$P \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left\{ k \frac{k_o k_w}{k_o \mu_w + k_w \mu_o} \frac{\partial P_c}{\partial x} \frac{\partial S_w}{\partial x} \right\} = 0 \quad (11)$$

Setting  $D = \frac{k_o k_w}{k_o \mu_w + k_w \mu_o}$

It is called co-efficient of saturation which is assumed to be a constant.

Capillary pressure in terms of phase saturation is defined as :

$$P_c = -\beta S_w \quad (12)$$

Equation (11) can be written as

$$P \frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x} \left[ k D \beta \frac{\partial S_w}{\partial x} \right] = 0 \quad (13)$$

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With  $S_w(x, 0) = s_0, S_w(0, t) = s_1, S_w(L, t) = 0, 0 \leq x \leq L$

$$P \frac{\partial S_w}{\partial t} - kD\beta \frac{\partial^2 S_w}{\partial x^2} = 0 \tag{14}$$

Let  $\xi = \frac{x}{L}, T = \frac{kD\beta}{PL^2} t$

$$\frac{\partial S_w}{\partial T} - \frac{\partial^2 S_w}{\partial \xi^2} = 0 \tag{15}$$

With  $S_w(\xi, 0) = s_0, S_w(0, T) = s_1, S_w(1, T) = 0, 0 \leq \xi \leq 1$

The governing equation of the problem is solved by Successive Over Relaxation (S.O.R.) method [12].

### IV. MATHEMATICAL SOLUTION

Using Crank –Nicolson method [11] for (15), we have

$$S_{w,m,n+1} = S_{w,m,n} + \frac{k}{2h^2} (S_{w,m+1,n} - 2S_{w,m,n} + S_{w,m-1,n} + S_{w,m+1,n+1} - 2S_{w,m,n+1} + S_{w,m-1,n+1})$$

Let  $r = \frac{k}{h^2}$

$$(1+r)S_{w,m,n+1} = S_{w,m,n} + \frac{r}{2} (S_{w,m+1,n} - 2S_{w,m,n} + S_{w,m-1,n} + S_{w,m+1,n+1} + S_{w,m-1,n+1})$$

$$\lambda_i = S_{w,m,n} + \frac{r}{2} (S_{w,m+1,n} - 2S_{w,m,n} + S_{w,m-1,n})$$

$$S_{w,m,n+1} = (1 - \omega)S_{w,m,n} + \omega \left[ \frac{r}{2(1+r)} (S_{w,m+1,n} + S_{w,m-1,n+1}) + \frac{\lambda_i}{(1+r)} \right]$$

Choose  $k = 0.01, h = 0.1, \omega = 1.75, s_0 = 0, s_1 = 0.5$

$$S_{w,m,n+1} = -0.75S_{w,m,n} + 1.75 \left[ 0.25(S_{w,m+1,n} + S_{w,m-1,n+1}) + \frac{\lambda_i}{2} \right] \tag{16}$$

Where  $\lambda_i = S_{w,m,n} + 0.5(S_{w,m+1,n} - 2S_{w,m,n} + S_{w,m-1,n})$

Equation (16) is obtained using S.O.R. method [12].

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

t →	T=0.01	T=0.02	T=0.03	T=0.04	T=0.05
ξ ↓	$S_w$				
0	0.5	0.5	0.5	0.5	0.5
0.1	0.21875	0.357177734	0.359340668	0.392908379	0.38163067
0.2	0.095703125	0.216827393	0.257044435	0.272927948	0.30127599
0.3	0.041870117	0.121357918	0.172900788	0.190984134	0.218968621
0.4	0.018318176	0.06468606	0.109423823	0.12684857	0.15393539
0.5	0.008014202	0.033371639	0.065860854	0.079677128	0.104849288
0.6	0.003506213	0.016818868	0.038087913	0.047730459	0.069009362
0.7	0.001533968	0.008328969	0.021335025	0.027516456	0.043860716
0.8	0.000671111	0.004068611	0.01164663	0.015342156	0.026797914
0.9	0.000293611	0.001965818	0.006181694	0.008088021	0.015203007
1	0.000128455	0.000892159	0.002895417	0.003237419	0.00776176

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## V. CONCLUDING REMARKS

From the above table, it is clear that at distance zero, the saturation is 0.5. As distance increasing, the saturation is decreasing and it tending to zero. Also from the table it is clear that initially saturation is zero and as time increasing, saturation also increasing. This solution is obtained by assuming saturation co-efficient to be constant. If it is not constant then the phenomenon is governed by another partial differential equation and other solution can be obtained.

## REFERENCES

- (1) A P Verma. "Existence and uniqueness of similarity solutions of imbibition equation", Proceedings of the Indian Academy of Sciences - Section A Part 3 Mathematical sciences, 10/1979
- (2) Verma,A.P.(1969):Imbibition in a cracked porous medium, Canadian Journal of Physics,47,22,2519-24.
- (3) Graham, J.W.and Richardson,J.G. (1959): Theory of application of imbibition phenomena in recovery of oil. Trans. A.I.M.E.,216,377.
- (4) Joshi, M. S. An analytic solution of fingero-imbibition in double phase flow through underground porous medium", International Journal of Applied Mathematics & Mechanics/09730184, 20091101
- (5) Scheidegger A.E.(1961): Dynamic similarity of instabilities of displacement fronts in porous media, Canadian Journal of Physics,39,2,1251.
- (6) Verma A.P.(1970): Fingero-imbibition in Artificial Replenishment of Ground water cracked porous medium, Water Resources Research,6,3.
- (7) Verma,A.P.(1970): Fingero-imbibition in slightly dipping porous medium, journal of Applied Physics,41(9).
- (8) M.N.Mehta ,Verma A.P.: Composite expansion of finger-imbibition in double phase flow through underground porous medium, proceeding of the Indian Academy of Sciences-section A 87(2):65-70.
- (9) A. P. Verma. "Fingero Imbibition in Artificial Replenishment of Ground Water Through Cracked Porous Medium", Water Resources Research, 1970
- (10) Vyas, N. B.; Patel, K. R.; Mehta, M. N. and Patel, T. R.. "power series solution of instability phenomena in double phase flow through porous media under magnetic field effect", International Journal of Applied Mathematics & Mechanics, 2011.
- (11) S.S.Shastry: Introductory Methods Of Numerical Analysis (4<sup>th</sup> Edition).
- (12) Jain M.K., Numerical solution of differential equations.2nd edition, Wiley Eastern, 1984.
- (13) Mehta, M. N.. "Solution by Group Invariant Method of Imbibition Phenomenon Arising in Fluid Flow Through Homogenous Porous Media", International Journal of Theoretical & Applied Mechanics/09736085, 20090301