

# Neighbourhood Prime Labeling On Some Graphs

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**ABSTRACT:** A bijective function  $f: V(G) \rightarrow \{1, 2, 3, \dots, n\}$  is said to be a neighbourhood prime labeling, if for every vertex  $v \in V(G)$  with  $deg(v) > 1$ ,  $gcd\{f(u) : u \in N(v)\} = 1$ . A graph which admits neighbourhood prime labeling is called a neighbourhood prime graph. In this paper, the neighbourhood prime labeling of some star related graphs, Petersen graph  $P(n, m)$ ,  $H$ -graphs,  $(m, n)$  – kite graphs, Jelly fish  $J(m, n)$  have been discussed.  
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**KEYWORDS:** Neighbourhood prime labeling, Neighbourhood prime graph

## I. INTRODUCTION

Roger Entringer initiated the notion of prime labeling and was introduced in 1980's by Tout et al [6] which paved way for many researches in this area. Motivated by the study of prime labeling, S.K.Patel and N.Shrimali in [3] introduced the notion of neighbourhood prime labeling in 2015, in which they have established the sufficient condition for a graph to admit neighbourhood prime labeling and proved that paths, cycles, helm, closed helm and flower have neighbourhood prime labeling. In the next paper the same authors investigate neighbourhood prime labeling for union of two cycles, union of two wheels and union of a finite number of paths. Also C.Ananthavalli and K.Nagarajan [1] investigate neighbourhood prime labeling for some special graphs like friendship graph, gear, ladder, triangular book and coconut tree.

T. Tharmaraj and P. Sarasija [5] prove that the subdivision of the edges of stars of bistar  $B_{m,n}$ , the graph obtained by the subdivision of the edges of bistar  $B_{m,n}$ , are square graceful graph.

R.Varatharajan, S. Navanaeethakrishnan and K. Nagarajan [8] prove that the graph  $\langle K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} \rangle$  is divisor cordial. N. Ramya, K. Rangarajan, R. Sattanathan [4] proved that that  $H$ -graph which is a 3-regular graph satisfy prime labeling. Through out the paper for general terms West [9] is referred. In this paper, the neighbourhood prime labeling of some star related graphs, Petersen graph  $P(n, m)$ ,  $H$ -graphs  $(m, n)$  – kite graphs, Jelly fish  $J(m, n)$  have been discussed.

**Definition1.1:** For a vertex  $v$  in  $G$ , the neighbourhood of  $v$  is the set of all vertices in  $G$  which are adjacent to  $v$  and is denoted by  $N(v)$ .

**Definition1.2:** Let  $G = (V(G), E(G))$  be a graph with  $n$  vertices. A bijective function  $f: V(G) \rightarrow \{1, 2, 3, \dots, n\}$  is said to be a neighbourhood prime labeling, if for every vertex  $v \in V(G)$  with  $deg(v) > 1$ ,  $gcd\{f(u) : u \in N(v)\} = 1$ . A graph which admits neighbourhood prime labeling is called a neighbourhood prime graph.

**Remark 1.3:** If in a graph  $G$ , every vertex is of degree at most 1, then such a graph is neighbourhood prime vacuously.

The basic principle followed in this labeling is Euclid's division algorithm which is based on the principle that gcd of two numbers does not change if the smaller number is subtracted from the larger number, which is described by Burton [2] as if  $m = qn + r$ , then  $gcd(u_m, u_n) = gcd(u_r, u_n) = u_{gcd(m,n)}$ .

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## II. NEIGHBOURHOOD PRIME LABELLING ON SOME STAR RELATED GRAPHS

**Definition 2.1:** Bistar is a graph obtained from a path  $P_2$  by joining the root of stars  $S_m$  and  $S_n$  at the terminal vertices of  $P_2$ . It is denoted by  $B_{m,n}$

**Theorem 2.2:** The Bistar  $B_{m,n}$  is neighbourhood prime graph

**Proof:**

The Bistar  $B_{m,n}$  has  $m + n + 2$  vertices and  $m + n + 1$  edges.

Define the bijective function  $f: V(G) \rightarrow \{1, 2, \dots, m + n + 2\}$  as follows

$$f(u) = 1, f(v) = 2$$

$$f(u_i) = i + 2; 1 \leq i \leq m$$

$$f(v_i) = i + 2 + m; 1 \leq i \leq n$$

The vertices with degree greater one are  $u$  and  $v$  alone.

$$N(u) \supset \{v, u_i\} \text{ with } 1 \leq i \leq m \text{ where } f(v) = 2, \text{ and } f(u_i) = i + 2$$

$$\therefore \text{ the } \gcd\{f(x); x \in N(u)\} = 1$$

Now we consider  $N(v) \supset \{u\}$  but  $f(u) = 1$ , then  $\gcd\{f(x); x \in N(v)\} = 1$  follows.

Hence bistar  $B_{m,n}$  is a neighbourhood prime graph.

**Example 2.3:** Consider the bistar  $B_{4,3}$  with  $f(u) = 1, f(v) = 2, f(u_1) = 3, f(u_2) = 4, f(u_3) = 5, f(u_4) = 6, f(v_1) = 7, f(v_2) = 8, f(v_3) = 9$

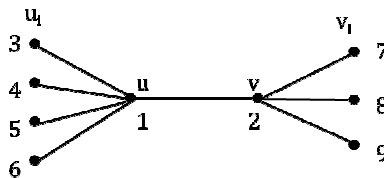


Figure 2.1 Bistar  $B_{4,3}$  with neighbourhood prime labelling

Hence the bistar  $B_{4,3}$  has neighbourhood prime labeling.

**Definition 2.4 :** A subdivision of a graph  $G$  is a graph that can be obtained from  $G$  by a sequence of edge subdivisions.

**Theorem 2.5:** The graph obtained by the subdivision of the central edge of the bistar  $B_{m,n}$  is a neighbourhood prime graph.

**Proof:**

On subdivision of the central edge of the bistar  $B_{m,n}$  we get the graph

$G$ , which has  $m + n + 3$  vertices and  $m + n + 2$  edges. Here the subdivision is at the

bridge  $uv$  forming the new vertex  $w$ . Let the vertex set of  $G$  be  $V(G) = \{u_1, u_2, \dots, u_m, u, w, v, v_1, v_2, \dots, v_n\}$  and the edge set be

$$E(G) = \{uw, vw, uu_i, vv_j; i = 1 \text{ to } m, j = 1 \text{ to } n\}$$

$$f(u) = 2; f(v) = 3; f(w) = 1$$

$$f(u_i) = i + 3, \quad i = 1 \text{ to } m$$

$$f(v_j) = j + 3 + m, \quad j = 1 \text{ to } n$$

The vertices  $u_i$ 's and  $v_j$ 's have degree 1. So we consider the following cases.

**Case(1):** The neighbourhood prime graph  $N(u)$  and  $N(v)$  contains the vertex  $w$  where  $f(w) = 1$ .

$\therefore \gcd$  of the labels of the vertices in  $N(u)$  and  $N(v)$  is 1.

**Case(2):**

$$N(w) = \{uw, vw\} = \{2, 3\}$$

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$\therefore$  gcd of the labels of vertices in  $N(w)$  is 1.  
Hence it has neighbourhood prime graph.

**Example 2.6 :** Consider the bistar  $B_{4,3}$  with subdivision at the center, we find  $f(u) = 2, f(w) = 1, f(v) = 3, f(u_1) = 4, f(u_2) = 5, f(u_3) = 6, f(u_4) = 7, f(v_1) = 8, f(v_2) = 9, f(v_3) = 10$

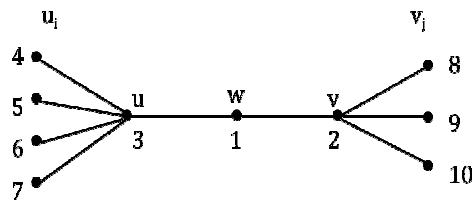


Figure 2.2 Bistar  $B_{4,3}$  with subdivision at the center with neighbourhood prime labelling

Hence the subdivision of bistar  $B_{4,3}$  at the centre has neighbourhood prime labeling.

**Theorem 2.7:** The graph obtained by the subdivision of the edges of stars of the bistar  $B_{m,n}$  is a neighbourhood prime graph.

**Proof:**

The given graph with the subdivision of the edges of stars of the bistar  $B_{m,n}$  in  $G$  which has  $2m + 2n + 2$  vertices and  $2m + 2n + 1$  edges.

Here the subdivision is in the edges except at the bridge.

Let the vertex set be  $\{u_1, u_2, \dots, u_m, u_1', u_2', \dots, u_m', u, v, v_1, v_2, \dots, v_n, v_1', v_2', \dots, v_n'\}$

where  $u_i$ 's and  $v_i$ 's are the vertices due to subdivision between  $u$  and  $u_i$ 's,  $v$  and  $v_i$ 's respectively and the edge set be  $E(G) = \{uv, uu_i, u_i u_i', vv_j, v_j v_j'; i = 1 \text{ to } m, j = 1 \text{ to } n\}$

$$f(u) = 1$$

$$f(v) = 2$$

$$f(u_i) = 2i + 1 \quad i = 1 \text{ to } m$$

$$f(u_i) = 2i + 2, \quad i = 1 \text{ to } m$$

$$f(v_j) = 2j + 1 + 2m, \quad j = 1 \text{ to } n$$

$$f(v_j) = 2j + 2 + 2m, \quad j = 1 \text{ to } n$$

In order to show that  $f$  has a neighbourhood prime labeling, we have to establish that

if  $x$  is an arbitrary vertex of  $G$ , then  $\gcd\{f(p) : p \in N(x)\} = 1$

**Case(1):** If  $x = u$  or  $v$ , then  $\{f(p) : p \in N(x)\} = \{3, 5, \dots, 2m + 1\}$  and so

$\gcd\{f(p) : p \in N(x)\} = 1$  follows.

**Case(2):** If  $x = u_i$  for  $1 \leq i \leq m$ , then  $\gcd\{f(p) : p \in N(x)\} = 1$  follows since  $u \in N(x)$  and  $f(u) = 1$ .

**Case(3):** If  $x = v_j$ , for  $1 \leq j \leq n$ , then  $N(x) \supset \{v, v_j\}$  but

$f(v) = 2, f(v_j) = 2j + 1 + 2m; 1 \leq j \leq n$  and so  $\gcd\{f(p) : p \in N(x)\} = 1$  follows.

**Example 2.8 :** Consider the bistar  $B_{4,3}$  with subdivision at the pendant vertices

we find  $f(u) = 1, f(v) = 2, f(u_1) = 4, f(u_2) = 6, f(u_3) = 8, f(u_4) = 10, f(u_1') = 3, f(u_2') = 5, f(u_3') = 7, f(u_4') = 9, f(v_1) = 11, f(v_2) = 13, f(v_3) = 15, f(v_1') = 12, f(v_2') = 14, f(v_3') = 16$ .

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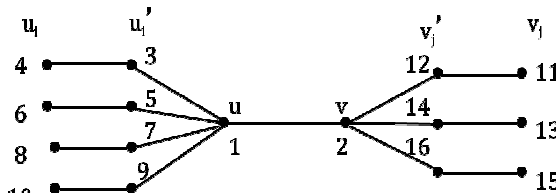


Figure 2.3 Bistar  $B_{4,3}$  with subdivision at the pendant vertices with neighbourhood prime labelling

Hence the subdivision at the pendant vertices of bistar  $B_{4,3}$  at the centre has neighbourhood prime labeling.

**Theorem 2.9:** The graph  $G = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} \rangle$  is a neighbourhood prime graph.

**Proof:**

Let the pendant vertices be  $v_1^{(i)}, v_2^{(i)}, \dots, v_n^{(i)}$  of  $K_{1,n}^{(i)}$  and  $u_i$ 's be the apex vertex of  $K_{1,n}^{(i)}$  for  $i = 1, 2, 3$ .

Now  $u_1$  and  $u_2$  are adjacent to  $w_1$ ,  $u_2$  and  $u_3$  are adjacent to  $w_2$ . This graph  $G$  has  $3n + 5$  vertices and  $3n + 4$  edges.

Let us define the bijective function  $f: V(G) \rightarrow \{1, 2, \dots, 3n + 5\}$  as follows

$$f(u_i) = i, \quad 1 \leq i \leq 3$$

$$f(v_j^{(i)}) = n(i - 1) + j + 3; \quad 1 \leq i \leq 3 \text{ and } 1 \leq j \leq n$$

$$f(w_i) = 3(n + 1) + i; \quad i = 1, 2$$

Consider the apex vertex  $u_i$  of  $K_{1,n}^{(i)}$  then  $\gcd\{f(x) : x \in N(u_i)\} = 1$ , since  $\{f(x) : x \in N(u_i)\} = \{n(i - 1) + 4, n(i - 1) + 5, \dots, n(i - 1) + (n + 3), 3(n + 1) + i\}$

Now consider the vertex  $w_i$  which is adjacent to  $u_i$  and  $u_{i+1}$  for which

$$\gcd\{f(x) : x \in N(w_i)\} = 1 \text{ since } \{f(x) : x \in N(w_i)\} = \{u_i, u_{i+1}\}.$$

Hence the  $G = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} \rangle$  is neighbourhood prime graph.

**Example 2.10:** Consider the graph  $G = \langle K_{1,6}^{(1)}, K_{1,6}^{(2)}, K_{1,6}^{(3)} \rangle$  with  $f(u_1) = 1, f(u_2) = 2, f(u_3) = 3, f(v_1^{(1)}) = 4, f(v_2^{(1)}) = 5, f(v_3^{(1)}) = 6, f(v_4^{(1)}) = 7, f(v_5^{(1)}) = 8, f(v_6^{(1)}) = 9, f(v_1^{(2)}) = 11, f(v_2^{(2)}) = 12, f(v_3^{(2)}) = 13, f(v_4^{(2)}) = 14, f(v_5^{(2)}) = 15, f(v_6^{(2)}) = 16, f(v_1^{(3)}) = 17, f(v_2^{(3)}) = 18, f(v_3^{(3)}) = 19, f(v_4^{(3)}) = 20, f(v_5^{(3)}) = 21, f(v_6^{(3)}) = 22, f(w_1) = 23, f(w_2) = 24$

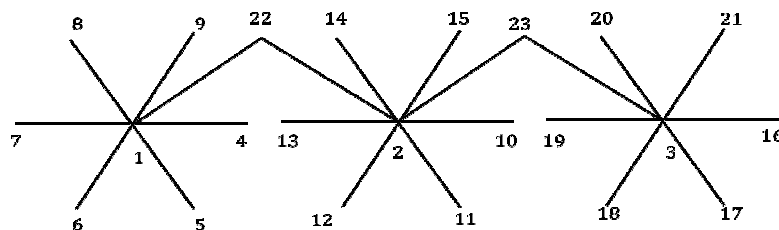


Figure 2.4  $G = \langle K_{1,6}^{(1)}, K_{1,6}^{(2)}, K_{1,6}^{(3)} \rangle$  with neighbourhood prime labelling

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### III. NEIGHBOURHOOD PRIME LABELLING ON SOME REGULAR GRAPHS

**Theorem 3.1:** A Petersen graph  $P(n, m)$  has a neighbourhood prime labeling.

**Proof:**

A Petersen graph  $P(n, m)$  has  $2n$  vertices and  $3n$  edges. Denote the vertices in the outer cycle as  $v_i$ 's and vertices in inner cycle as  $u_i$ 's. Let us define bijective function

$f: V(G) \rightarrow \{1, 2, \dots, 2n\}$  as given below,

$$f(u_i) = i ; 1 \leq i \leq n$$

$$f(v_i) = i + n ; 1 \leq i \leq n$$

$P(n, m)$  is a 3 regular graph which has each of its vertex with degree 3.

Let  $w$  be any arbitrary vertex, consider following cases to establish neighbourhood prime labeling in  $G$ .

**Case (1):** If  $w$  is odd,  $N(w)$  contains 2 even and 1 odd labels. Then  $\gcd\{f(x) : x \in N(w)\} = 1$

**Case(2):** If  $w$  is even,  $N(w)$  contains 2 odd labels and 1 even label. Then  $\gcd\{f(x) : x \in N(w)\} = 1$  holds.

**Example 3.2:** Consider the (5,2) Petersen graph with the function defined as  $f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 4, f(5) = 5, f(6) = 6, f(7) = 7, f(8) = 8, f(9) = 9, f(10) = 10$ .

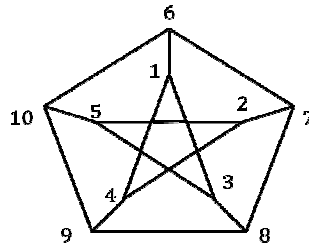


Figure 3.1. (5,2) Petersen graph with neighbourhood prime labeling

We can find that (5,2) Petersen graph has neighbourhood prime labeling.

**Definition 3.3:** A H graph  $H(r)$  is a 3-regular graph with vertex set  $\{(i, j) : 1 \leq i \leq 3; 1 \leq j \leq 2r\}$  and edge  $\{(i, j), (i, j + 1), i = 1, 3, 1 \leq j \leq 2r - 1\} \cup \{(2, j), (2, j + 1), j \text{ is odd}, 1 \leq j \leq 2r - 1\} \cup \{(1, 1), (1, n), (3, 1), (3, n)\} \cup \{(i, j), (i + 1, j), i = 1, 2; 1 \leq j \leq 2r\}$ . A H graph  $H(r)$  has  $6r$  vertices and  $9r$  edges.

**Theorem 3.4:** H- graph has neighbourhood prime labeling.

**Proof:**

Let us redefine the H- graph as below

A H-graph  $H(r)$  is a 3 regular graph with vertex set  $\{v_{ij}, 1 \leq i \leq 3, 1 \leq j \leq 2r\}$  and edge set  $\{(v_{ij}, v_{i(j+1)}) ; i \text{ is odd}, 1 \leq j \leq 2r - 1\} \cup \{(v_{2j}, v_{2(j+1)}) ; j \text{ is odd}, 1 \leq j \leq 2r - 1\} \cup \{(v_{11}, v_{1n}), (v_{31}, v_{3n})\} \cup \{(v_{ij}, v_{(i+1)j}) ; i = 1, 2 ; 1 \leq j \leq 2r\}$

Define the bijective function  $f: V(G) \rightarrow \{1, 2, \dots, 6r\}$  as follows :

If  $r$  is odd,

$$f(v_{ij}) = \begin{cases} 6r + 2i - 7 & \text{for } i = 1, 2, 3 ; j \text{ is odd} \\ 6r + 2i - 6 & \text{for } i = 1, 2, 3 ; j \text{ is even} \end{cases}$$

If  $r$  is even,

$$f(v_{ij}) = \begin{cases} 6r + 2i - 6 & \text{for } i = 1, 2, 3 ; j \text{ is odd} \\ 6r + 2i - 7 & \text{for } i = 1, 2, 3 ; j \text{ is even} \end{cases}$$

To show that  $f$  is a neighbourhood prime label, we should establish that if  $w$  is an arbitrary vertex of the H-graph which is a 3-regular graph all its vertices have degree 3 and hence  $N(w)$  also has 3 vertices.

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**Case(1) :** If  $w$  is odd, by definition of the function  $N(w)$  contains 2 odd label and 1 even label, then the  $gcd\{f(x) : x \in N(w)\} = 1$

**Case(2) :** If  $w$  is even,  $N(w)$  contains 2 even labels and 1 odd label, then the  $gcd\{f(x) : x \in N(w)\} = 1$  follows.  
Hence H-graph is neighbourhood prime graph.

**Example 3.5:** Consider the  $H(2)$  graph. Here we find the function as

$$f(v_{11}) = 1, \quad f(v_{12}) = 2, f(v_{13}) = 8, f(v_{14}) = 7, f(v_{21}) = 3, f(v_{22}) = 4, f(v_{23}) = 10, f(v_{24}) = 9, \\ f(v_{31}) = 5, f(v_{32}) = 6, f(v_{33}) = 12, f(v_{34}) = 11$$

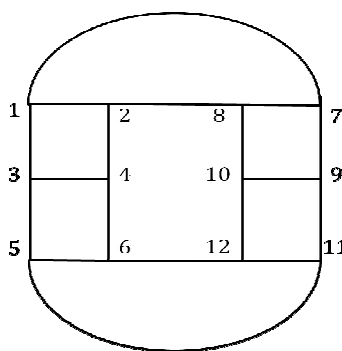


Figure 3.2.  $H(2)$  graph with neighbourhood prime labeling

Hence the  $H(2)$  graph has neighbourhood prime labeling.

## IV. CONCLUSION

The neighbourhood prime labeling of some star related graphs, Petersen graph  $P(n, m)$ , H-graphs  $(m, n)$  – kite graphs, Jelly fish  $J(m, n)$  have been discussed in this paper, which can also be extended for various other graphs .

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