

Gravity Dual D_3 -braneworld and Closed/Open String Duality

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ABSTRACT: A covariantly constant dynamical two-form is exploited on a D_3 -brane to obtain its gravity dual action, governing an S^3 deformed AdS_5 black hole, in a type IIB string theory on $S^1 \times K3$. We invoke the Kaluza-Klein compactification and identify the T -duality with the open/closed stringduality. Interestingly, the Reissner-Nordstrom black hole is obtained on the “non-Riemannian”braneworld.

I. INTRODUCTION

In General Theory of Relativity (GTR), the space-time curvatures are based on the intrinsic notion of geometry, by incorporating a nontrivial metric tensor on a (Riemannian) manifold. The fact that the space-time curvatures are governed by the nonlinear matter field, enforces hidden gravity/gaugeduality in Einstein’s formulation. The duality becomes evident in presence of a fifth dimension, i.e. in Kaluza-Klein gravity theory. Importantly, the duality takes a precise form in AdS gravity and is established as a familiar bulk/boundary or AdS/CFT correspondence [1] in string theory.

In the context, non-perturbative Dirichlet (D-) branes [2] are believed to enlighten the aspects of geometry/matter duality coded in Einstein’s formulation. In fact, the nonlinear U (1) gauge dynamics established on a D-brane [3] is indeed remarkable. Along with the higher derivative corrections, the nonlinear gauge sector is believed to govern the space-time curvature. Most importantly, the underlying duality has been considerably explored through the success of 5D Anti de Sitter (AdS_5) gravity and D_3 -brane gauge theory correspondence established in string theory. Interestingly, the non-commutative open string (NCOS) leading to a deformed D-braneworld, its gravity correspondence [4] and various (non-linear) charged D_3 -braneworld black holes [5]-[9] have been obtained in the past.

In the paper, we attempt to formalize the nonlinear U (1) gauge dynamics of a Kalb-Ramond (KR) field on a D_3 -brane in terms of its geometric dual in a string theory. We explore a D_3 -braneworld in a plausible fundamental open string formulation and obtain a closed string dual of the D_3 -braneworld, i.e. a curved D_3 -braneworld in 5D. We consider a covariantly constant NS form field to incorporate new connections, i.e. a torsion, which in turn leads to a unique modified covariant derivative on a vacuum pair of curved braneworld [10]. Our starting point is a flat D_3 -brane, which is known to govern the gravity decoupling limit $E \rightarrow E_c = g/(2\pi\alpha')$ in an open string theory.

We revisit the decoupling limit as a cancellation of torsion arising from the zeroth order term in B-field to that of all higher orders, keeping the geometrical notion of Einstein’s formulation. In fact, an asymptotic D_3 -brane may be thought of as a transition world-volume connecting the underlying NCOS and a fundamental open string theory. As a result, the D_3 -braneworld in the classical regime, i.e. $g \rightarrow 0$ and $\alpha' \rightarrow 0$, may be seen to be described by the nontrivial generic curvatures underlying a (modified) open string theory.

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II. KALB-RAMOND FIELD AS A SOURCE OF GRAVITY

NEW CONNECTIONS:

A dynamical KR field $B_{\mu\nu}$ incorporates non-trivial connections $\Gamma_{\mu\nu\lambda}$ on a vacuum pair of gravitational brane and anti-brane [10, 11, 12, 13, 14, 15]. Since Einstein's gravity decouples, the generic curvature on a curved D_3 -braneworld may be seen to be governed by the "non-Riemannian" geometries. The vanishing field strength for the dynamical matter introduces the notion of new geometries on the braneworld, underlying a modified open string theory. Formally, the field strength may be expressed as

$$\mathcal{H}_{\mu\nu\lambda} = (\mathcal{D}_\mu B_{\nu\lambda} + F_{\mu\nu} A_\lambda) + \text{cyclic (in } \mu, \nu, \lambda), \quad (1)$$

where the U(1) field strength $F_{\mu\nu} = (2\pi\alpha')^{-1} F_{\mu\nu}$. The covariant derivative \mathcal{D}_μ takes into account a non-dynamical metric $g_{\mu\nu}$ and a dynamical two-form $B_{\mu\nu}$ and $\mathcal{D}_\lambda g_{\mu\nu} = 0$. We rewrite the new connection as

$$\Gamma_{\mu\nu\lambda} = \Gamma_{\mu\nu\lambda}^{(0)} - \Gamma_{\mu\nu\lambda}^{(h)} + (F_{\mu\nu} A_\lambda + \text{cyclic}), \quad (2)$$

where $\Gamma_{\mu\nu\lambda}^{(0)} = -\frac{1}{2}(\partial_\mu B_{\nu\lambda} + \text{cyclic})$ and $\Gamma_{\mu\nu\lambda}^{(h)} = -(T_{\mu\nu}^{\rho\lambda} B_{\rho\lambda} + \text{cyclic})$, respectively, denote the zeroth and higher orders, in two-form, defined on a D_3 -braneworld. The gravity decoupling limit, in the formalism, is defined by $\Gamma_{\mu\nu\lambda}^{(0)} \rightarrow \Gamma_{\mu\nu\lambda}^{(h)}$. In the low energy regime, the non-Riemannian geometry on a 4D braneworld may be viewed in terms of two equivalent open string descriptions. They are: (i) a matter dominated braneworld for $|\Gamma_{\mu\nu\lambda}^{(0)}| \gg |\Gamma_{\mu\nu\lambda}^{(h)}|$ and (ii) a gravity or geometric braneworld for $|\Gamma_{\mu\nu\lambda}^{(h)}| \gg |\Gamma_{\mu\nu\lambda}^{(0)}|$. The matter dominated braneworld precisely corresponds to the non-commutative open string (NCOS) description. On the other hand, the geometric braneworld may be seen to govern by a (modified) fundamental open string. The underlying open string, in an Einstein gravity decoupled braneworld, is a new phenomenon in the formalism.

GRAVITY AND GAUGE CURVATURES ON THE BRANE-WORLD:

The generic gravity and gauge curvatures on the D_3 -braneworld may be computed from the commutator of appropriate gauge covariant derivatives on it. In presence of a gauge field, the appropriate covariant derivative becomes $D_\mu = (\mathcal{D}_\mu - A_\mu)$. A priori, the nontrivial curvatures on a D_3 -braneworld may be given by

$$[D_\mu, D_\nu] A_\lambda = \mathcal{F}_{\nu\mu} A_\lambda + K_{\mu\nu\lambda}{}^\rho A_\rho + T_{\mu\nu}^\rho \mathcal{D}_\rho A_\lambda + \frac{1}{2} (T_{\mu\sigma}^\rho T_{\nu\lambda}^\sigma + T_{\nu\lambda}^\sigma T_{\mu\sigma}^\rho - T_{\mu\lambda}^\sigma T_{\nu\sigma}^\rho - T_{\nu\sigma}^\rho T_{\mu\lambda}^\sigma) A_\rho. \quad (3)$$

The non-linear U(1) field strength $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + T_{\mu\nu}{}^\rho A_\rho$, where $T_{\mu\nu}{}^\rho = -2\Gamma_{\mu\nu}{}^\sigma$ is a torsion on the braneworld. The non-Riemannian curvature tensor is precisely due to the two-form and takes an identical form to the Riemannian-Christoffel curvature tensor $R_{\mu\nu\lambda\rho}$. It is given by

$$K_{\mu\nu\lambda}{}^\rho = \partial_\nu T_{\mu\lambda}^\rho - \partial_\mu T_{\nu\lambda}^\rho + T_{\mu\lambda}^\sigma T_{\nu\sigma}^\rho - T_{\nu\lambda}^\sigma T_{\mu\sigma}^\rho. \quad (4)$$

Importantly for a constant torsion, $K_{\mu\nu\lambda\rho} \rightarrow R_{\mu\nu\lambda\rho}$. Incorporating the irreducible generic non-Riemannian curvature tensor, we rewrite the D_3 -braneworld action as

$$S = \frac{1}{\alpha'} \int d^4x \sqrt{-g} \left(K - \frac{\alpha'}{4} F^2 \right). \quad (5)$$

A priori, simple looking braneworld may be seen to be governed by the intrinsically coupled gravity and gauge curvatures from the $B_{\mu\nu}$ and A_μ equations of motion. They are, respectively, given by

$$\partial_\lambda \Gamma^{\lambda\mu\nu} - \frac{1}{2} (g^{\alpha\beta} \partial_\lambda g_{\alpha\beta} \Gamma^{\lambda\mu\nu}) = 0$$

and

$$a(\nabla_\lambda F^{\lambda\mu}) - 6F_{\lambda\nu} \Gamma^{\lambda\nu\mu} + 12\nabla_\lambda (F^{\lambda\mu\nu} A_\nu) = 0. \quad (6)$$

REISSNER-NORDSTROM ON A PAIR OF BRANE/ANTI-BRANE:

In this section, we address a non-Riemannian formulation leading to Reissner-Nordstrom black hole geometry on the D_3 -braneworld. The ansatz for a two-form and a gauge field leading to a charged black hole may be given by $B_{0\phi} = -M \cos \theta$ and $A_0 = Q_e/r$, where M and Q_e are constants and shall be identified with the mass and the electric charge. The gauge choice leads to a nontrivial connection $\Gamma_{00\phi} = M/2 \sin \theta$ and an extremal charged black hole geometry may be seen to emerge through an effective metric $G_{\mu\nu} = g_{\mu\nu} - B_{\mu\lambda} g^{\lambda\rho} g_{\rho\nu}$ description in an open string theory. However, we consider a (generalized) geodesic expansion for the effective metric components $G_{00} = G^{\text{II}}$, around an S^2 -symmetric vacua, in a non-relativistic limit. The G_{00} component of the effective metric may be obtained from a geodesic expansion represented by the derivative corrections of gauge potentials of increasing rank in each term. It is given by $G_{00} = (g_{00} + h^{(0)}_{00} + h^{(1)}_{00} + h^{(2)}_{00} + \dots)$. The nontrivial scalar quantities in 2nd, 3rd and 4th terms are essentially the distinct contributions, respectively, out of the scalar, vector and two-form fields. To avoid an ambiguous form of $h^{(2)}_{00}$, we rather consider a

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Poincaré dual of B-field, i.e. an axion field $\chi(x)$ as a source of Newtonian potential on the braneworld. The axion and gauge gravitational potentials are, respectively, worked out to yield $h^{(0)}_{00} = -2M/r$ and $h^{(1)}_{00} = Q_e^2/r^2$. The axion source on the D_3 -braneworld leads to an electric charged black hole which in turn may be viewed in terms of both the charges ($Q^2 = Q_e^2 + Q_m^2$) in a boosted coordinate system. In the case, the “non-Riemannian” D_3 -braneworld may be seen to describe a typical Reissner-Nordström black hole geometry known in Riemannian geometry. The nontrivial vacua common to two distinct tensor formulations of gravity in string theory is remarkable. It may provide clues to enhance our understanding of de Sitter vacua.

III. GRAVITY DUAL D_3 -BRANEWORLD

The gravity and gauge curvatures on the curved D_3 -braneworld in 4D may be seen to have its origin in a five dimensional gravity theory governed by a dynamical B-field in a truncated closed string theory. A priori, one may begin with the type IIB string theory on $S^1 \times K3$. We consider a non-dynamical metric and for simplicity, focus on constant scalars and gauge fields. The generic notion of closed string is maintained in 5D by the (covariantly constant) dynamical B-field in absence of the dynamical metric on the curved braneworld. The relevant closed string action may be given by

$$S = \int d^5y \sqrt{-\tilde{g}} \tilde{K} \quad (7)$$

where $\tilde{g}_{\mu\nu}$ is a constant metric in the Cartesian coordinates. An ansatz for the B-field, around an S^3 -symmetric vacua, is worked out to yield

$$B_{0\psi} = B_{r\psi} = r_0, B_{\theta\psi} = q(\cot \theta \sin^2 \psi) \text{ and } B_{\varphi\psi} = q(\cos \theta \sin^2 \psi) \quad (8)$$

where $0 < \theta \leq \pi$, $0 < \psi \leq \pi$, $0 \leq \varphi \leq 2\pi$ and (r_0 and q) are arbitrary constants. The nontrivial connections are given by

$$\Gamma_{\theta\psi\varphi} = (q/2)(\sin \theta \sin^2 \psi) \quad (9)$$

Importantly, the D-braneworld tension turns out to be a significant vacuum energy density in the effective metric description. As a result, the radial coordinate picks up an asymptotic cut-off b , the AdS radius, in the curved D-braneworld formalism. The 5D black hole metric may be constructed using the B-field ansatz around an AdS vacua. The AdS₅ geometry may be seen to be described by an AdS₂, and an S^2 , deformed S^3 -symmetry in the closed string theory. It is given by

$$ds^2 = -\left(1 + \frac{r^2}{b^2} - \frac{r_0^2}{r^2}\right) dt^2 + \left(1 + \frac{r^2}{b^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 \left(1 + \frac{2f^2}{r^4}\right) d\Omega_3^2 - \frac{B_{\theta\psi}^2}{r^2} d\Omega_2^2 \quad (10)$$

where $f = q(\cot \theta \sin \psi)$. A priori, the fundamental closed string action (7), may alternately be viewed in an effective metric description to yield the usual Kaluza-Klein gravity in presence of a cosmological constant. In that case, the geometry may be seen to be governed by the Schwarzschild AdS₅ black hole with S^3 -symmetry [16, 17]. In other words, AdS₅ black hole in the B-field theory of gravity governs a deformed S^3 , where as the 3D-spherical symmetry is preserved in the metric source theory of gravity. As a result, the D_3 -braneworld action (7) possibly governs a generalized gravity dynamics, which is beyond a typical 5D Kaluza-Klein gravity theory. For a constant $\Gamma_{\theta\psi\varphi} \neq 0$, the conformal factor and the S^3 deformation geometry become trivial in the AdS₅ black hole (10). In the case, the Riemannian notion of manifold becomes manifest and the geometry is in agreement with the Schwarzschild AdS₂ \times S^3 black hole obtained in the Kaluza-Klein theory. The Kaluza-Klein compactification of the fifth dimension in the closed string theory (7), in a static gauge, precisely reduces to the gravity dual D_3 -brane action (5). For instance, if ψ denotes a compactified coordinate, the gauge potential may be worked out in 4D, from the B-field ansatz, to yield

$$A_\mu = (r_0, r_0, q \cot \theta, -q \cos \theta) \quad (11)$$

The linear electromagnetic field, in the gravity dual D_3 -braneworld, is a priori governed by the radial component of the magnetic field $B_r = q/r^2$, along with a nontrivial B-field in a Kaluza-Klein compactified closed string theory⁵. It is thought provoking to understand the Kaluza-Klein compactification in a B-field theory of gravity as a T-duality between a, fundamental, open/closed string. It may also be understood as a (bulk) gravity/(boundary) gauge theory correspondence. In fact, the D_3 -braneworld action possesses a dual notion in a type IIB string theory. They are: (i) a fundamental closed string description in 5D and (ii) a fundamental open string in 4D. The geometric braneworld curvature in 4D essentially confirms the fundamental nature of open string there.

Interestingly, all the higher derivative corrections vanish for a self-dual electro-magnetic field [7]. As a result, the deformed AdS 5 (10) in a closed string theory may be seen to reduce to an AdS₄ black hole whose curvature is due to the nonlinear electric/magnetic charge (q) in an open string theory obtained by one of the author in ref.[9].

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IV. CONCLUDING REMARKS AND RESULTS

The gravity dual of a D_3 -brane action is obtained in a type IIB string theory on $S^1 \times K3$. The geometrical description on the braneworld confirms a dual nature of two-form, i.e. a closed string description in 5D and a NCOS description in 4D. The fact is reassured by its underlying deformed AdS black holes geometries both in closed and open string theories. As a bonus, we obtain a fundamental open string theory T -dual to a closed string theory in 5D. Our analysis leads to the birth of a generic curvature tensor $K_{\mu\nu\lambda\rho}$ which retains the properties of Riemann-Christoffel tensor except for the symmetric property under the exchange of its pair of indices. However for a constant torsion, the gravity dual braneworld may be seen to be described by the Riemannian geometry.

A constant two-form ansatz may be obtained for $q = 0$ in the gravity dual D_3 -braneworld at the expense of an effective dynamical theory of gravity. It restores the S^3 -symmetry and describes an AdS_5 Schwarzschild black hole in a typical Kaluza-Klein gravity in presence of a cosmological constant. On the other hand, the global mode of a B-field in an open string theory is known to yield the deformation geometry [3]. In fact, it may be interesting to investigate the thermal aspects associated with the non-commutative geometry in the curved D-braneworld along the spirit of entropic force of gravity discussed recently [18]. It confirms that the deformation geometry, in string theory, is intrinsic to the B-field source of gravity, which is unlike to a metric source of gravity. In other words, the open/closed string duality in presence of B-field retains the notion of the deformation geometry. Interestingly, the torsion does not break the S^3 -symmetry in the AdS_5 black hole (10), rather it is broken solely by the B-field.

⁵The Kaluza-Klein compactification may also be seen to yield an irreducible curvature precisely due to a generic nonlinear $U(1)$ gauge field in the theory.

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