

Energy of Fuzzy Labeling Graph [EF₁(G)]- Part II

A.Nagarani¹, S.Vimala²

Ph.D Research Scholar, Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamilnadu,
India¹

Assistant Professor, Department of Mathematics, Mother Teresa Women's University, Kodaikanal, Tamilnadu, India²

ABSTRACT: In [3,1] were introduced and calculation of bounds for finite energy graphs and finite fuzzy energy graphs. This motivation provided the some results of energy of fuzzy labeling graphs[12]. In this paper, we extend the concept of fuzzy labeling graph to the energy of fuzzy labeling of Franklin family graph, and generalized Petersen graph (GP) and also discussed the characters.

KEYWORD: Labeling, fuzzy labeling graph, energy graph, energy of fuzzy labeling graph, Franklin graph, Generalized Petersen graph

AMS CODE: 05C22, 05C90, 94D, 05C50

I. INTRODUCTION

Let the graph $G(V;E)$ where V be vertices and E edge. Graph theory involves lot of applications such as geometry, algebra, number theory, topology, operations research, and optimization and computer science. In 1967, most graph labeling methods trace their origin introduced by Rosa [11] and Graham and Sloane [5] introduced harmonious graceful graph by additive bases in 1980. Gallian [6] in a Dynamic Survey of Graph Labeling. Pradhan and Kumar [10] proved that graphs obtained by adding a pendent vertex of hair cycle $C_n \circ K_1$ are graceful if $n \equiv 0 \pmod{4m}$. They further provide a rule for determining the missing numbers in the graceful labeling of $C_n \circ K_1$ and of the graph obtained by adding pendent edges to each pendent vertex of $C_n \circ K_1$. Abhyanker [2] proved that the graph obtained by deleting the branch of length 1 from an olive tree with $2n$ branches and identifying the root of the edge deleted tree with a vertex of a cycle of the form C_{2n+3} is graceful. In 1985 Koh and Yap [8] generalized this by defining a cycle with a P_k -chord to be a cycle with the path P_k joining two nonconsecutive vertices of the cycle.

Fuzzy relation on a set was defined by Zadeh in 1965. Based on fuzzy relation the first definition of a fuzzy graph was introduced by Rosenfeld and Kaufmann in 1973. Nagoor gani [9] – Novel Properties of Fuzzy Labeling Graphs with path, cycle and star).

The concept of energy graph was defined by I. Gutman in 1978. The energy graph simply denoted by $E(G)$, is defined to be the sum of the absolute values of its eigen values. Hence if the adjacency matrix $A(G)$ of G , and $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigen values of $A(G)$, then $E(G) = \sum_{i=1}^n |\lambda_i|$. The set $\{\lambda_1, \dots, \lambda_n\}$ is the spectrum of G and denoted by $\text{Spec } G$ in Balakrishnana [3] introduce the bounds of energy and Energy of Fuzzy Graph introduced by Anjali Narayana [1]

The totally disconnected graph K_n^c has zero energy while complete graph K_n with the maximum possible number of edges has energy $2(n-1)$. It was therefore conjectured in P.Prabhan and A. Kumar [10] that all graphs have energy at most $2(n-1)$.

S.Vimala and A.Nagarani [12] provide the results of energy of fuzzy labeling graphs such as butterfly graph, book graph, wheel graph, caterpillar graph, theta graph, Hamiltonian circuit graph, $K_3 \Join \overline{K_3}$ graph.

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In this paper we developed energy of fuzzy labeling graph of Franklin family graphs and generalized Petersen family graph. And find upper bounds and lower bounds for the same graphs.

II. PRELIMINARIES

A **labeling** of a graph is an assignment of values to the vertices and edges of a graph. Given a graph G , an injective function $f : V(G) \rightarrow \mathbb{N}$ has been called a **vertex labeling** of G .

An **edge labeling** of a graph is a bijection from $E(G)$ to the set $\{1, 2, \dots, |E(G)|\}$.

A **fuzzy graph** $G = (\sigma, \mu)$ is a pair of function $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Fuzzy labeling graph:

A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Franklin graph:

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges.

Generalized Petersen graph:

The generalized Petersen graph is a family of cubical graphs formed by connecting the vertices of a regular polygon to the corresponding vertices of a star polygon.

Energy:

Energy of a simple graph $G = (V, E)$ with adjacency matrix A is defined as the sum of absolute values of eigen values of A denoted by $E(G)$. i.e., $E(G) = \sum_{i=1}^n |\lambda_i|$ where λ_i is an eigen value of A , $i = 1, 2, \dots, n$.

Energy of fuzzy graph:

Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. The eigenvalues of A are called eigenvalues of G . The spectrum of A is called the spectrum of G . It is denoted by $\text{Spec } G$. Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. Energy of G is defined as the sum of absolute values of eigenvalues of $G[1]$.

Energy of fuzzy labeling graph:

The following three conditions are true if the graph is called a energy of fuzzy labeling graph.

- (i) $E(G) = \sum_{i=1}^n |\lambda_i|$
- (ii) $\mu(u, v) > 0$
- (iii) $\mu(u, v) < \sigma(u) \wedge \sigma(v)$

III. MAIN RESULT

THEOREM:1

Let $F_1(G)$ be a fuzzy labeling of with $|V| = n$ vertices and $\mu = \{e_1, e_2, e_3, \dots, e_i\}$ edges. If $m_i = \mu(e_i)$, then

$$\sqrt{2 \sum_{i=1}^m m_i^2 + n(n-1) |A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^m m_i^2)n}$$

Proof:

By the definition of Energy of fuzzy labeling graph we find $EF_1(G)$

$$\text{Lower bound} = \sqrt{2 \sum_{i=1}^m m_i^2 + n(n-1) |A|^{\frac{2}{n}}}$$

$$\text{Upper bound} = \sqrt{2(\sum_{i=1}^m m_i^2)n}$$

$$\sqrt{2 \sum_{i=1}^m m_i^2 + n(n-1) |A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^m m_i^2)n} \text{ for all cases.}$$

The above theorem is satisfied for Franklin family graphs and Generalized Petersen Graph (GP)

Example 1:

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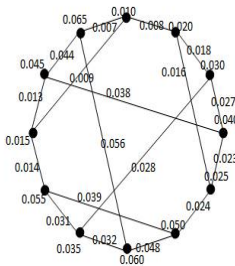


FIG 1: Franklin graph 1

Adjacency matrix of the fuzzy labeling of Franklin graph is

$$A[F_1(G)] = \begin{bmatrix} 0 & 0.008 & 0 & 0 & 0 & 0 & 0 & 0 & 0.009 & 0 & 0.007 & 0 \\ 0.008 & 0 & 0.018 & 0 & 0.016 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.018 & 0 & 0.027 & 0 & 0 & 0 & 0.032 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.027 & 0 & 0.025 & 0 & 0 & 0.038 & 0 & 0 & 0 & 0 \\ 0 & 0.016 & 0 & 0.023 & 0 & 0.024 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.024 & 0 & 0.048 & 0 & 0.039 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.048 & 0 & 0.032 & 0 & 0 & 0 & 0.056 \\ 0 & 0 & 0.028 & 0 & 0 & 0 & 0.032 & 0 & 0.031 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.039 & 0 & 0.031 & 0 & 0.014 & 0 & 0 \\ 0.009 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.014 & 0 & 0.013 & 0 \\ 0 & 0 & 0 & 0.038 & 0 & 0 & 0 & 0 & 0 & 0.013 & 0 & 0.044 \\ 0.007 & 0 & 0 & 0 & 0 & 0 & 0.056 & 0 & 0 & 0 & 0.044 & 0 \end{bmatrix}$$

The eigen values are = 0.0967, -0.0940, -0.0506, 0.0428, -0.0319, -0.0204, 0.0224, -0.0224, 0.02014, 0.0076, -0.0076, -0.0048 .

Example 2:

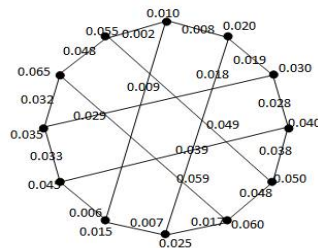


FIG 2: Franklin graph 2

The eigen value are = -0.01169, 0.1169, -0.0759, 0.0759, -0.0356, -0.0237, 0.0356, 0.0237, -0.0108, 1.000, -1.000

Example 3:

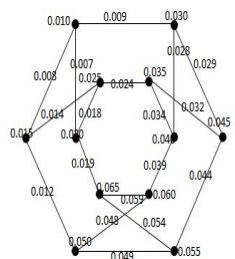


FIG 3: Franklin graph 3

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The eigen value are = -0.1167, 0.1167, -0.0588, 0.0588, -0.0419, -0.0269, -0.0244, -0.0115, 0.0115, 0.0419, 0.0244, 0.0269 .

Example 4:

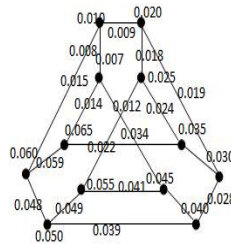


FIG 4: Franklin graph 4

The eigen value are = -0.1028, 0.1028, -0.0644, 0.0644, -0.0470, 0.0470, -0.0245, -0.0110, -0.0160, 0.0245, 0.0110, 0.0160.

Example 5:

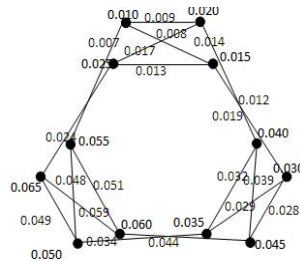


FIG 5: Franklin graph 5

The eigen value are = 0.01170, -0.1121, 0.0586, -0.0603, 0.0386, -0.0386, 0.0115, -0.0115, 0.0128, -0.0128 .

Example 6:

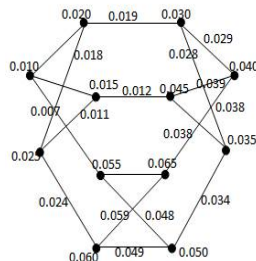


FIG 6: Franklin graph 6

The eigen value are = -0.1138, -0.0614, -0.0357, -0.0302, -0.0152, -0.0107, 0.0107, 0.0152, 0.0302, 0.0357, 0.0614, 0.1138 .

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Example 7:

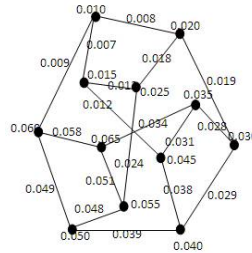


FIG 7: Franklin graph 7

The eigen value are = -0.1160, 0.1160, -0.0641, 0.0641, -0.0289, -0.0358, -0.0149, 0.0149, 0.0114, 0.0289, 0.0358,

Example 8:

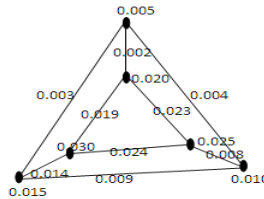


FIG 8: Generalized Petersen Graph (3,1)

The Adjacency matrix of the fuzzy labeling graph is

$$A[F_1(G)] = \begin{bmatrix} 0 & 0.004 & 0.003 & 0.002 & 0 & 0 \\ 0.004 & 0 & 0.009 & 0 & 0.008 & 0 \\ 0.003 & 0.009 & 0 & 0 & 0 & 0.014 \\ 0.002 & 0 & 0 & 0 & 0.023 & 0.019 \\ 0 & 0.008 & 0 & 0.023 & 0 & 0.024 \\ 0 & 0 & 0.014 & 0.019 & 0.024 & 0 \end{bmatrix}$$

The eigen value are = -0.0302, -0.0219, -0.0040, -0.0006, 0.0101, 0.0465.

Example 9:

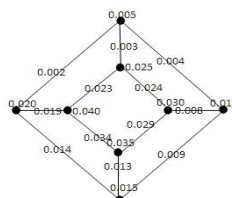


FIG 9: Generalized Petersen Graph (4,1)

The eigen value are = -0.0596, -0.0178, -0.0106, -0.0038, 0.0106, 0.0178, 0.0596.

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Example 10:

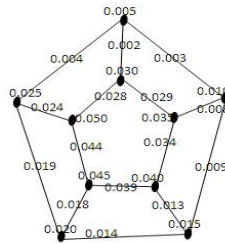


FIG 10: Generalized Petersen Graph (5,1)

The eigen value are = 0.0714, -0.0518, -0.0179, -0.0090,-0.0012, 0.0008, 0.0150, 0.0244, 0.0346, 0.0766 .

Example 11:

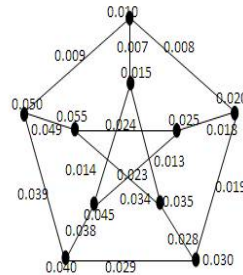


FIG 11: Generalized Petersen Graph (5,2)

The eigen value are = -0.0745, -0.0595, -0.0390, -0.0186, 0.0070, 0.0124, 0.0217,0.0288, 0.0374, 0.0844.

Example 12:

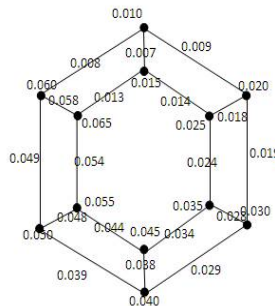


FIG 12: Generalized Petersen Graph (6,1)

The eigen value are = 0.1065, -0.0808, 0.0644, -0.0644, 0.0603, -0.0336 0.0392, 0.0275, -0.0085, -0.0038, 0.0047, 0.0022 .

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Example 13:

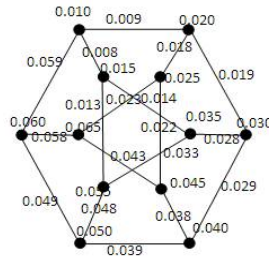


FIG 13: Generalized Petersen Graph (6,2)

The eigen value are = -0.0918, -0.0734, -0.0510, -0.0298, -0.0157, 0.0032, 0.0076, 0.0141, 0.0263, 0.0456, 0.0619, 0.1030 .

Example 14:

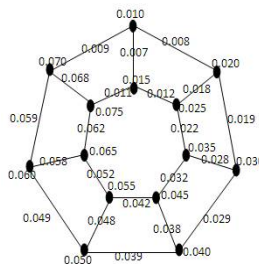


FIG 14: Generalized Petersen Graph (7,1)

The eigen value are = -0.1433, 0.1433, -0.0881, 0.0878, 0.0517, -0.0495, -0.0335, -0.0265, 0.0338, 0.0209, -0.0094, -0.0035, 0.0081, -0.0081 .

Example 15:

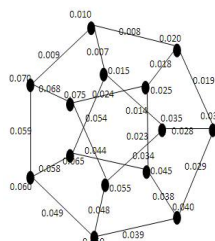


FIG 15: Generalized Petersen Graph (7,2)

The eigen value are = -0.1067, 0.1256, -0.0845, -0.0640, 0.0589, 0.0554, -0.0316, 0.0307, 0.0234, 0.0171, -0.0149, 0.0032, 0.0100, -0.0100 .

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List of Energy ,Upper and Lower Bound

Graph name	Energy	Upper bound	Lower bound
Franklin graph 1	0.414	0.03919	15.3843
Franklin graph 2	0.5258	0.17034	15.5062
Franklin graph 3	0.5604	0.3600	14.02026
Franklin graph 4	0.5488	0.35205	13.8441
Franklin graph 5	0.4738	0.45147	13.1030
Franklin graph 6	0.5340	0.1743	13.9854
GP of (3,1)	0.1133	0.1996	11.36070
GP of (4,1)	0.1798	0.2781	10.548
GP of (5,1)	0.3027	0.6073	15.015
GP of (5,2)	0.1723	0.40097	12.1775
GP of (6,1)	0.4959	0.8964	16.215
GP of (6,2)	0.5234	0.7619	15.3619
GP of (7,1)	0.7075	0.62113	17.5643
GP of (7,2)	0.6360	0.87842	18.6138

IV. CONCLUSION

In this paper, we find some energy of fuzzy labeling graphs for franklin family and generalized Petersen graphs and also to find the energy level is $\sqrt{2 \sum_{i=1}^m m_i^2 + n(n-1) |A|_n^2} > \sqrt{2(\sum_{i=1}^m m_i^2)n}$. Our future work is to apply π electron in fuzzy labeling will be discussed in the forthcoming papers.

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