

Four Bar Linkage Synthesis and Design for Generation of Five Coupler Positions

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ABSTRACT: Four-bar planar mechanisms have wide applications in industry and thus receive more attention from machinery design researchers. A mechanical system is made-up of several components, which can be divided into two major groups namely links and joints. This work presents an analytical approach depends on forming mathematical model for mechanism position incorporating five coupler positions. The model is solved using MATLAB software for mechanism unknowns. A case study is presented so as to justify the approach. The deviation of the transmission angle of the mechanism for different coupler positions is calculated.

KEYWORDS: Synthesis of Mechanism, Four-Bar Mechanism, Coupler Positions Generation

I. INTRODUCTION

A four-bar chain is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanization and control of motion due to its simplicity and versatility. A four-bar linkage is a versatile mechanism that is widely used in machines to transmit motion or to provide mechanical advantage.

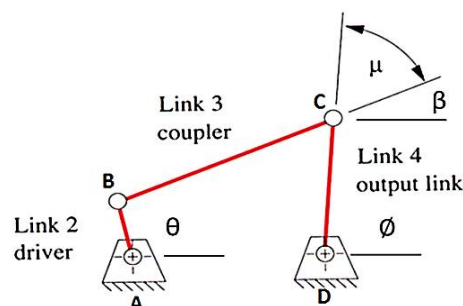


Fig. 1: Four bar mechanism

Their low friction, higher capacity to carry load, ease of manufacturing, and reliability of performance in spite of manufacturing tolerances make them preferable over other mechanisms in certain applications. It is also the most fundamental linkage mechanism, and many more complex mechanisms contain the four-bar linkage as elements. Therefore, a basic understanding of its characteristics is essential.

Basically it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints. Generally, the joints are configured so the links move in parallel planes, and the assembly is called a planar four-bar linkage. When one of the links is fixed, it is known as linkage or mechanism. A link that makes a complete revolution is called the crank. The link which makes a partial rotation or oscillates is known as lever or rocker or follower and the link opposite to the fixed link or the link which connects the crank and lever is called as coupler. Coupler maintains

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constant distance between movable ends of crank and lever. This position of the coupler link can be used as the output of the four bar mechanism [6].

Mechanism is a system of rigid elements (linkages) arranged and connected to transmit motion and/or force in a predetermined fashion. Synthesis of Mechanism means designing a mechanism to obtain a specified motion or force. The transmission angle is defined as angle between output link and the coupler.

The importance of the transmission angle is indicated by the following two statements [7].

1. The force transmission from the connecting rod to the output crank is most effective when the transmission angle is 90°. Of course, it is most desirable to have this angle deviate from 90° as little as possible throughout the range of operation.

2. In some precision machinery, when the transmission angle is too small, the accuracy of the output motion becomes very sensitive to the manufacturing tolerances of the link lengths and clearances between joints. Also a poor transmission angle can cause objectionable noise and jerk at high speed.

The minimum permissible transmission angle of a linkage mechanism depends on many factors, such as the magnitude of the transmitted forces, the manufacturing tolerances, the friction between the joints, speed, etc. A minimum transmission angle of 40° is suggested as a good practical limit by some designers. In some high-speed machinery, a minimum value of 45° is necessary [4].

If links BC and CD become coincident, the transmission angle is zero and mechanism would lock or jam. If μ deviates significantly from 90o, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45o. For a crank-rocker mechanism generally the best results the designers recognize when the transmission angle is close to 90 degree as much as possible during entire rotation of the crank. Alternatively, the transmission angle during entire rotation of crank should lie between the minimum and maximum values.

The following sections are organized as follows. Section II, briefly describes the mathematical model of 4 bar mechanism for five coupler positions. Section III, explains the results and discussion. Section IV gives conclusion.

II. FORMATION OF MATHEMATICAL MODEL

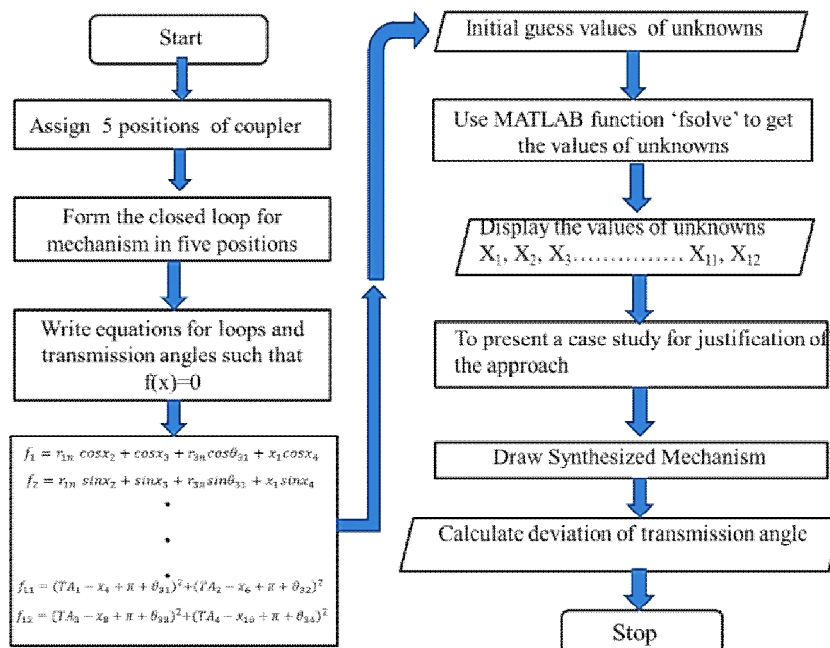


Fig. 2: Flow chart of methodology

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Nomenclature

$f_1, f_2, f_3, \dots, f_{12}$	Nonlinear mechanism functions
r_1, r_2, r_3, r_4	Lengths of links 1,2,3 and 4
r_{1n}, r_{3n}, r_{4n}	Normalized lengths of links 1,3 and 4
x_1, x_2, \dots, x_{12}	Mechanism unknown parameters
$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$	Mechanism transmission angle in the first, second, third, fourth and fifth coupler position respectively
θ_1	Orientation of link 1 (frame)
θ	Angle made by input link AB (crank) with fixed link
β	Angle made by link BC (coupler) with fixed link
ϕ	Angle made by output link CD (rocker) with fixed link
$\theta_{21}, \theta_{31}, \theta_{41}$	Orientation of links 2,3 and 4 in the first mechanism position
$\theta_{22}, \theta_{32}, \theta_{42}$	Orientation of links 2,3 and 4 in the second mechanism position
$\theta_{23}, \theta_{33}, \theta_{43}$	Orientation of links 2,3 and 4 in the third mechanism position
$\theta_{24}, \theta_{34}, \theta_{44}$	Orientation of links 2,3 and 4 in the fourth mechanism position
$\theta_{25}, \theta_{35}, \theta_{45}$	Orientation of links 2,3 and 4 in the fifth mechanism position

Methodology for 5 Coupler Positions

The methodology is applied to standard 4-bar mechanisms having fixed lengths [2, 3]. The steps are as follows:

1. The desired 5 positions of the coupler are assigned in the motion plane.
2. Closed loops are formed for the mechanism in the 5 positions.
3. 2 equations are written for each loop in the x and y directions.
4. 5 equations are written for the 5 transmission angles (one per mechanism position).
5. 12 equations are written in a normalized form by dividing each link dimension by r_2 .
6. The final model consists of 12 nonlinear equations with 12 unknowns which are solved using MATLAB.

Requirements

It is desired to have a coupler of a known length in 5 positions: $A_1B_1, A_2B_2, A_3B_3, A_4B_4$ and A_5B_5 with known orientations $\theta_{31}, \theta_{32}, \theta_{33}, \theta_{34}$ and θ_{35} as in Fig.3

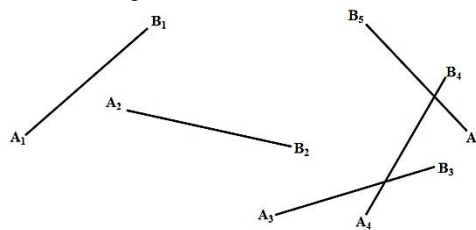


Fig. 3: Mechanism coupler in the 5 positions

Mechanism

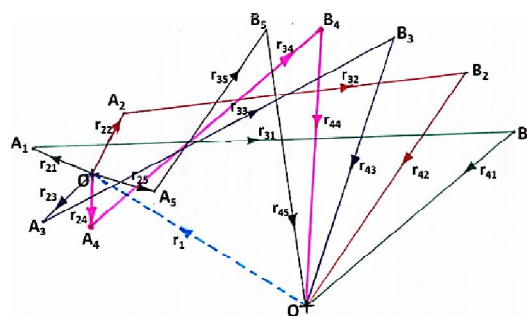


Fig. 4: 4-Bar Mechanism in 5 Positions

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Assumptions made:

1. Length of the fixed link is equal to 240 mm.
2. All the links are rigid and capable of transmitting forces

Analysis: The 5 coupler positions are: $A_1B_1, A_2B_2, A_3B_3, A_4B_4$ and A_5B_5

Polygon 1: OA_1B_1QO . The displacement equation across the polygon is:

$$r_1 + r_{21} + r_{31} + r_{41} = 0$$

The vector components in the x direction; $\sum r_x = 0$ gives:

$$r_1 \cos\theta_1 + r_2 \cos\theta_{21} + r_3 \cos\theta_{31} + r_4 \cos\theta_{41} = 0$$

The vector components in the y direction; $\sum r_y = 0$ gives:

$$r_1 \sin\theta_1 + r_2 \sin\theta_{21} + r_3 \sin\theta_{31} + r_4 \sin\theta_{41} = 0$$

Polygon 2: OA_2B_2QO . The displacement equation across the polygon is:

$$r_1 + r_{22} + r_{32} + r_{42} = 0$$

The vector components in the x direction; $\sum r_x = 0$ gives:

$$r_1 \cos\theta_1 + r_2 \cos\theta_{22} + r_3 \cos\theta_{32} + r_4 \cos\theta_{42} = 0$$

The vector components in the y direction; $\sum r_y = 0$ gives:

$$r_1 \sin\theta_1 + r_2 \sin\theta_{22} + r_3 \sin\theta_{32} + r_4 \sin\theta_{42} = 0$$

Polygon 3: OA_3B_3QO . The displacement equation across the polygon is:

$$r_1 + r_{23} + r_{33} + r_{43} = 0$$

The vector components in the x direction; $\sum r_x = 0$ gives:

$$r_1 \cos\theta_1 + r_2 \cos\theta_{23} + r_3 \cos\theta_{33} + r_4 \cos\theta_{43} = 0$$

The vector components in the y direction; $\sum r_y = 0$ gives:

$$r_1 \sin\theta_1 + r_2 \sin\theta_{23} + r_3 \sin\theta_{33} + r_4 \sin\theta_{43} = 0$$

Polygon 4: OA_4B_4QO . The displacement equation across the polygon is:

$$r_1 + r_{24} + r_{34} + r_{44} = 0$$

The vector components in the x direction; $\sum r_x = 0$ gives:

$$r_1 \cos\theta_1 + r_2 \cos\theta_{24} + r_3 \cos\theta_{34} + r_4 \cos\theta_{44} = 0$$

The vector components in the y direction; $\sum r_y = 0$ gives:

$$r_1 \sin\theta_1 + r_2 \sin\theta_{24} + r_3 \sin\theta_{34} + r_4 \sin\theta_{44} = 0$$

Polygon 5: OA_5B_5QO . The displacement equation across the polygon is:

$$r_1 + r_{25} + r_{35} + r_{45} = 0$$

The vector components in the x direction; $\sum r_x = 0$ gives:

$$r_1 \cos\theta_1 + r_2 \cos\theta_{25} + r_3 \cos\theta_{35} + r_4 \cos\theta_{45} = 0$$

The vector components in the y direction; $\sum r_y = 0$ gives:

$$r_1 \sin\theta_1 + r_2 \sin\theta_{25} + r_3 \sin\theta_{35} + r_4 \sin\theta_{45} = 0$$

- Unknowns in Equations: $r_1, r_2, r_4, \theta_1, \theta_{21}, \theta_{41}, \theta_{22}, \theta_{23}, \theta_{43}, \theta_{24}, \theta_{44}, \theta_{25}$ and θ_{45}
- Number of unknowns: 14
- Number of equations so far: 10

The number of design parameters is reduced through:

1. Assigning the ground length r_1 .
2. Using normalized dimensions by referring all the dimensions to r_2 .

In this case, the unknown design parameters are:

$$x_1 = r_{4n}, x_2 = \theta_1, x_3 = \theta_{21}, x_4 = \theta_{41}, x_5 = \theta_{22}, x_6 = \theta_{42}, x_7 = \theta_{23}, x_8 = \theta_{43}, x_9 = \theta_{24}, x_{10} = \theta_{44}, x_{11} = \theta_{25}, x_{12} = \theta_{45}$$

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Number of unknowns is reduced to 12.

Two more equations may be written for the transmission angle in 5 positions of the mechanism.

The transmission angle is related to links 3 and 4 orientation angles through:

$$\mu_1 = \theta_{41} - \pi - \theta_{31}$$

$$\mu_2 = \theta_{42} - \pi - \theta_{32}$$

$$\mu_3 = \theta_{43} - \pi - \theta_{33}$$

$$\mu_4 = \theta_{44} - \pi - \theta_{34}$$

Now, the kinematical model equations are written in the normalized form as: [1]

$$\begin{aligned} f_1 &= r_{1n} \cos x_2 + \cos x_3 + r_{3n} \cos \theta_{31} + x_1 \cos x_4 \\ f_2 &= r_{1n} \sin x_2 + \sin x_3 + r_{3n} \sin \theta_{31} + x_1 \sin x_4 \\ f_3 &= r_{1n} \cos x_2 + \cos x_5 + r_{3n} \cos \theta_{32} + x_1 \cos x_6 \\ f_4 &= r_{1n} \sin x_2 + \sin x_5 + r_{3n} \sin \theta_{32} + x_1 \sin x_6 \\ f_5 &= r_{1n} \cos x_2 + \cos x_7 + r_{3n} \cos \theta_{33} + x_1 \cos x_8 \\ f_6 &= r_{1n} \sin x_2 + \sin x_7 + r_{3n} \sin \theta_{33} + x_1 \sin x_8 \\ f_7 &= r_{1n} \cos x_2 + \cos x_9 + r_{3n} \cos \theta_{34} + x_1 \cos x_{10} \\ f_8 &= r_{1n} \sin x_2 + \sin x_9 + r_{3n} \sin \theta_{34} + x_1 \sin x_{10} \\ f_9 &= r_{1n} \cos x_2 + \cos x_{11} + r_{3n} \cos \theta_{35} + x_1 \cos x_{12} \\ f_{10} &= r_{1n} \sin x_2 + \sin x_{11} + r_{3n} \sin \theta_{35} + x_1 \sin x_{12} \\ f_{11} &= (TA_1 - x_4 + \pi + \theta_{31})^2 + (TA_2 - x_6 + \pi + \theta_{32})^2 \\ f_{12} &= (TA_3 - x_8 + \pi + \theta_{33})^2 + (TA_4 - x_{10} + \pi + \theta_{34})^2 \end{aligned}$$

Equations (f_{11}) and (f_{12}) are functions of the 4 transmission angles of the 4-bar mechanism in its 5 positions corresponding to each coupler position.

III. RESULTS AND DISCUSSION

A case study is presented in order to justify the approach. It is required to design a 4-bar planar mechanism to move the coupler AB from position A_1B_1 to A_2B_2 to A_3B_3 to A_4B_4 to A_5B_5 shown in Fig.5

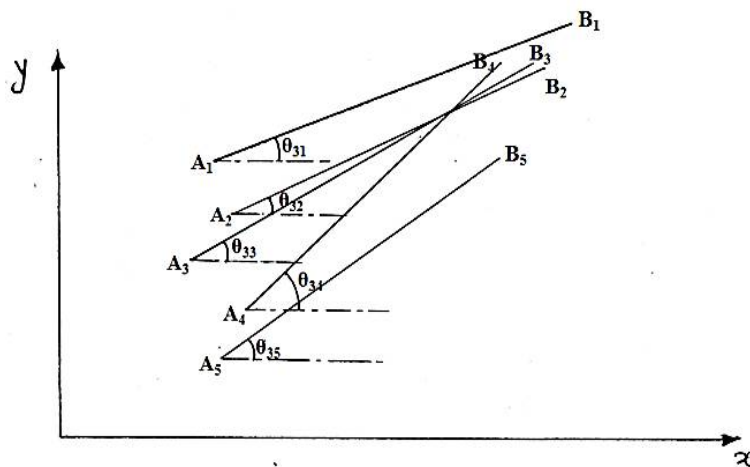


Fig. 5: Desired 5-coupler positions

$$r_3 = AB = 200 \text{ mm}, \theta_{31} = 20^\circ, \theta_{32} = 25^\circ, \theta_{33} = 30^\circ, \theta_{34} = 43^\circ \text{ and } \theta_{35} = 35^\circ, X_{B1} = 500 \text{ mm}, Y_{B1} = 400 \text{ mm}$$

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Mechanism Synthesis

A MATLAB code is written to solve the equations in the normalized form satisfying there right hand side which is zero for those equations. The output is as follows.

Code inputs:

$$r_{3n} = 5, r_{1n} = 6, \theta_{31} = 20^\circ, \theta_{32} = 25^\circ, \theta_{33} = 30^\circ, \theta_{34} = 43^\circ \text{ and } \theta_{35} = 35^\circ$$

$$TA_1 = TA_2 = TA_3 = TA_4 = 90^\circ$$

Code output:

2.3853 (r_{4n})	-3.0134 (θ_1)
-4.7711 (θ_{21})	5.2583 (θ_{41})
-5.2352 (θ_{22})	5.1078 (θ_{42})
0.5603 (θ_{23})	5.0445 (θ_{43})
-0.9318 (θ_{24})	5.4597 (θ_{44})
4.4650 (θ_{25})	5.7906 (θ_{45})

Mechanism Dimensions

Coupler length	: $r_3 = 200$ mm (required)	
Crank length	: $r_2 = r_3/r_{3n} = 200/5 = 40$ mm	
Rocker length	: $r_4 = r_{4n} \times r_2 = 2.3853 \times 40 = 95.412$ mm	
Ground length	: $r_1 = r_{1n} \times r_2 = 240$ mm	
Ground angle	: $\theta_1 = 187.34^\circ$	
Crank orientation :	$\theta_{21} = 86.63^\circ$	Rocker orientation : $\theta_{41} = 301.27^\circ$
	$\theta_{22} = 60.04^\circ$	$\theta_{42} = 292.65^\circ$
	$\theta_{23} = 32.10^\circ$	$\theta_{43} = 289.02^\circ$
	$\theta_{24} = 306.61^\circ$	$\theta_{44} = 312.81^\circ$
	$\theta_{25} = 255.82^\circ$	$\theta_{45} = 331.77^\circ$

The designed mechanism in its 5 positions is shown in Fig.6

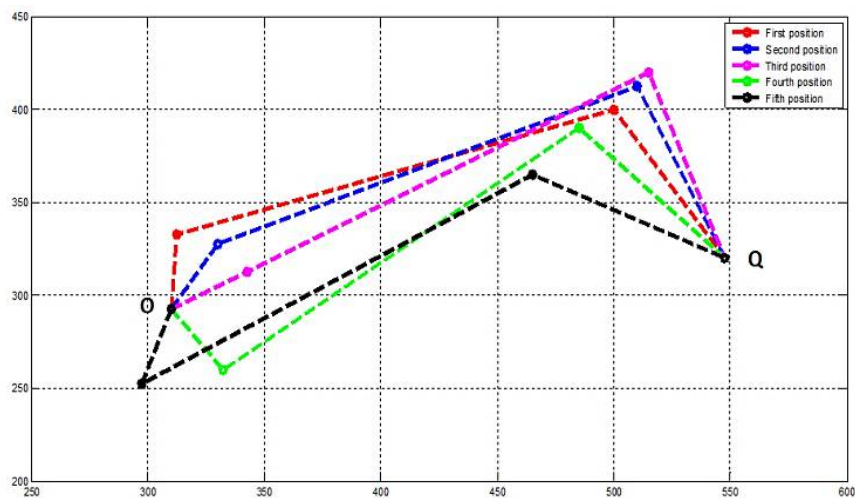


Fig. 6: The synthesized 4-bar mechanism in its 5 positions

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Mechanism Type

The Grashof's criterion is used to identify the type of mechanism.

$$\begin{aligned} L_{\min} &= 40 \text{ mm (crank)} \\ L_{\max} &= 240 \text{ mm} \\ L_a &= 95.412 \text{ mm} \\ L_b &= 200 \text{ mm} \\ L_{\min} + L_{\max} &= 280 \text{ mm} \\ L_a + L_b &= 295.412 \text{ mm} \end{aligned}$$

This means that: $L_{\min} + L_{\max} < L_a + L_b$ and the crank is the minimum.

Therefore, the designed mechanism is a crank-rocker Grashof's mechanism.

Transmission angles for each position

The values of transmission angle (μ), % error in transmission angle, type of mechanism for different mechanism positions are given in the following table.

Table 1: Transmission angles for each position

Mechanism Positions	Transmission Angle (μ)	% Error in (μ)	Mechanism Type
First	$\mu_1 = 101.27^\circ$	-12.52	Crank-rocker Grashof Mechanism
Second	$\mu_2 = 87.65^\circ$	2.61	
Third	$\mu_3 = 79.02^\circ$	12.2	
Fourth	$\mu_4 = 89.81^\circ$	0.211	
Fifth	$\mu_5 = 116.77^\circ$	-29.7	

IV. CONCLUSION

In this paper, synthesis of four bar mechanism is done for five coupler positions. The approach is very accurate in synthesizing 4-bar planar mechanisms for different positions of its coupler. It is found that the coupler traces exactly the desired positions with transmission angles not more than 29.7% of the optimum value of 90° . There is large deviation in the fifth mechanism position for five coupler positions because its transmission angle was not included in the model equations. The transmission angle in all the positions of mechanism is within the recommended range of $45^\circ \leq \mu \leq 135^\circ$

REFERENCES

- [1] Galal Ali Hassaan, Mohammed A. Al-Gamil&Maha M. Lashin, "New Approach for the Synthesis of 4-Bar Mechanism for 2 Coupler-Positions Generation", New York Science Journal, 5(10) pp.86-90, 2012.
- [2] Galal Ali Hassaan, "Synthesis of Planar Mechanisms; Part III: Four –bar Mechanisms for Three Coupler Positions Generation", Global Journal of Advanced Research, Volume 2, Issue IV, pp. 726-734, April 2015
- [3] Galal Ali Hassaan, "Synthesis of Planar Mechanisms; Part IV: Four –bar Mechanisms for Four Coupler Positions Generation". International Journal for Research in Applied Science & Engineering Technology, Volume 3, Issue V, pp.224-229, May 2015
- [4] Hassaan, G. A. "Synthesis of Planar Mechanisms, Part II: Specified stroke, time ratio and transmission angle", International Journal of Applied Sciences and Engineering Research, Vol.2, issue 3, pp.25-32, May-June 2015.
- [5] A. Smaili and F. Zeineddine, "A Matlab/Simulink based Code for the Analysis, Synthesis, Optimization and Simulation of Mechanisms", Proceedings of the 2003 American Society for Engineering Education Annual Conference and Exposition, Session 2666, 2003.
- [6] R. Norton, "Design of machinery", McGraw-Hill, 2011
- [7] Deh Chang Tao, "Applied Linkage Synthesis", Addison Wesley Publication
- [8] P. L. Ballaney, "Theory of Machines and Mechanisms", Khanna Publication, 2009
- [9] S. S. Rattan, "Theory of Machines", Tata McGraw-Hill Publication, 2008