

The Study on a Nonlinear Problem arising in Infiltration Phenomenon by Homotopy Perturbation Sumudu Transform Method (HPSTM)

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ABSTRACT:In this article, we have discussed an infiltration phenomenon, which gives a nonlinear partial differential equation in moisture content as a governing equation. We solved the governing equation by a combined technique of Sumudu transform and Homotopy Perturbation method (HPM), called Homotopy Perturbation Sumudu transform method (HPSTM). Also, the analysis between Homotopy Perturbation Sumudu transform method (HPSTM) and Homotopy Analysis method (HAM) has been discussed with their initial approximation $\theta(Z, 0) = (1 - e^{-Z})$ and $\theta(Z, T, \epsilon) = (1 - e^{-Z})T + \epsilon^m$, as well as with their numerical values respectively.

KEYWORDS:Infiltration phenomenon, Sumudu transform, Homotopy Perturbation method.

I. INTRODUCTION

Infiltration is a name given to a phenomenon of liquid intake by porous media. A simple definition of infiltration is a flow of water through the soil surface. It is one of the most important components of hydrological phenomena. It has a great influence on rainfall, runoff, transpiration of plants, evaporation from the soil surface, in groundwater recharge, etc. It is the main component of abstraction losses of rainfall. Infiltration refills soil moisture deficiency and excess water moves downward towards the groundwater storage by a force of gravity.

NOMENCLATURE:

$\theta(z, t)$	Moisture Content
z	Depth in the vertical downward direction of a flow
t	Time
ϵ, Z, T	Parameters
θ^*	Reduced water content
θ_s	Saturation of the moisture
θ_r	Residual of the moisture
α, m, n	Empirical constants

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ψ	Pressure potential (soil matric suction)
$D(\theta)$	Soil water diffusivity coefficient
$a, b, \beta, \gamma, \lambda$	Constants
$K(\theta)$	Coefficient of the volumetric water content
L	Length of a large basin
\bar{V}	Volume flux of the moisture
∇H	Gradient of the whole moisture potential
ρ_s	Bulk density of the soil on dry weight basis
M	Mass of flux of the water at any time $t \geq 0$
ϕ	Porosity
S	Saturation of the soil
ρ	Flux density
G	Gravitational constant

Different relationships between the diffusivity coefficient and volumetric water content problems in unsaturated porous media have been defined by many researchers. The exponential hydraulic conductivity function has been widely used; however, it has a limited range of applications to many real soils. The reduced water content θ^* is given by

$\theta^* = \frac{\theta - \theta_r}{\theta_s - \theta_r}$ (where $\theta_r < \theta < \theta_s, 0 < \theta^* < 1$). A relationship between the reduced water content and the soil suction is

given by the Brooks-Corey model [6] and a relationship between the saturation degree and the soil suction defined by Van Genuchten model [17] is given as

$$\theta^* = \begin{cases} \left(1 + (\alpha\psi)^n\right)^{-m} & \text{if } \psi > 0 \\ 1 & \text{if } \psi \leq 0 \end{cases}$$

where parameters m and n are related by the relation as $m = 1 - \frac{1}{n}$.

Van Genuchten [17] developed other functions for practical applications. Such particular forms of the hydraulic functions enable to linearize the governing flow equations, and thus they can be solved analytically. Such solutions of the linearized unsaturated flow equations are usually restricted to the steady flow in semi-infinite, homogeneous soils [5]; and to transient flow in homogeneous and layered soils [15]. The Broadbridge and White Model [5] has used the soil water diffusivity $D(\theta)$ given by

$$D(\theta) = \frac{a}{(b - \theta)^2} \tag{1.1}$$

Also for the solution of the nonlinear flow problem, Broadbridge and White [5] established an expression for the coefficient of the volumetric water content $K(\theta)$ given by

$$K(\theta) = \beta + \gamma(b - \theta) + \frac{\lambda}{2(b - \theta)^3} \tag{1.2}$$

which is in conjunction with the assumed function of $D(\theta)$. It converts equation (1.8) into a weakly nonlinear Burger's equation.

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Many researchers discussed groundwater recharge problem with different viewpoints. Kajal et al. [8] applied Homotopy Analysis method (HAM) to determine pressure head in the unsaturated soil during infiltration phenomenon. Parikh & et al. [11] discussed a transcendental solution of the Fokker-Planck equation of vertical groundwater recharge in unsaturated homogeneous porous media. Borana et al. [4] have discussed a numerical solution of the Boussinesq equation arising in one-dimensional infiltration phenomenon by using finite difference method. Swartzendruber used Philip's method [13] to get a graphical illustration of the mathematical solution for horizontal water function. Using the singular Perturbation technique, Mehta et al. [9, 10] provided an approximate solution, where the change in average diffusivity coefficient being very small; it has been treated as a constant. H. A. Basha [1] worked on multidimensional linearized nonsteady infiltration with prescribed boundary conditions at the soil surface.

II. STATEMENT OF THE PROBLEM

Consider the unsaturated flow in a vertical downward direction up to the length of a large basin L in a homogeneous porous medium and neglect the flow spreading in the other directions. There is no air resistance to the flow (i.e. The porous medium contains only the flowing liquid water and empty voids of air). The air in the void space is approximately at atmospheric pressure. Consider the moisture content at the soil surface as a time-dependent function. So applying the standard laws as Darcy's law [7] and the continuity equation [2], we get a one-dimensional nonlinear partial differential equation as a governing equation.

The purpose of this article is to discuss the approximate analytical solution of a nonlinear partial differential equation arising in the ground water recharge phenomenon and to examine the moisture content in homogeneous porous media. The change in moisture content and distribution of pore pressure can be calculated using HPSTM.

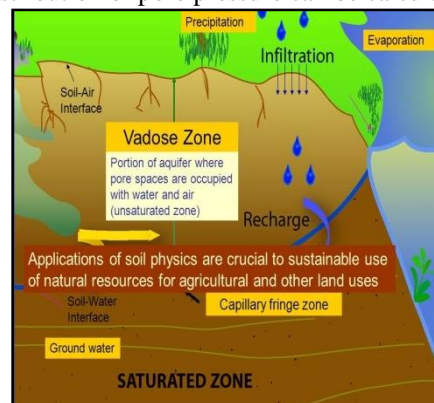


Figure-1:Representation of groundwater phenomenon

III. MATHEMATICAL STRUCTURE OF THE PROBLEM

When water flows through unsaturated porous media in the vertical downward direction, hydraulic conductivity varies nonlinearly with the volumetric water content: $K = K(\theta)$. For the small Reynolds number, the volume of the flow of water described by Darcy's law [7] as

$$\vec{V} = -K(\theta) \nabla H \quad (1.3)$$

Such groundwater flow satisfies the equation of continuity given by

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$$\frac{\partial}{\partial t}(\rho_s \phi S) = -\nabla M \tag{1.4}$$

Consider that the water is incompressible, $M = \rho \vec{V}$ and also consider the fact that the water content of the soil is given by the standard relation with the saturation of the soil S as $\theta = \phi S$ [3].

Equation (1.4) reduces to

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\nabla(\rho \vec{V}) \tag{1.5}$$

By equations (1.3) and (1.5), we get

$$\frac{\partial}{\partial t}(\rho_s \theta) = -\nabla(\rho(-K(\theta)\nabla H)) \tag{1.6}$$

Also consider that the flow takes place only in the vertical downward direction [14]. Then from equation (1.6), we get

$$\rho_s \frac{\partial \theta}{\partial t} = \rho \frac{\partial}{\partial z} \left(K(\theta) \frac{\partial H}{\partial z} \right) \tag{1.7}$$

For unsaturated porous media H , total soil moisture potential is given by $H = \psi - gz$ [14].

By equation (1.7), we get

$$\frac{\partial \theta}{\partial t} = \frac{\rho}{\rho_s} \frac{\partial}{\partial z} \left(K(\theta) \frac{\partial \psi}{\partial z} \right) - \frac{\rho}{\rho_s} g \frac{\partial K(\theta)}{\partial z} \tag{1.8}$$

Equation (1.8) can be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right) - \frac{\rho g}{\rho_s} K'(\theta) \frac{\partial \theta}{\partial z} \tag{1.9}$$

where $D(\theta) = \frac{\rho K(\theta)}{\rho_s} \frac{\partial \psi}{\partial \theta}$ and $K'(\theta) = \frac{dK}{d\theta}$.

The equation (1.9) is a θ dependent equation. Generally, the θ dependent equation is called one-dimensional Fokker-Planck equation. This equation (1.9) is modeled based on Darcy-Buckingham approach in the vertical downward direction flow of water in unsaturated porous media.

Assume that the moisture content in uniform soil takes place in a positive downward direction from $z = 0$ top to the bottom $z = L$, where the water table is saturated as shown in figure-1 and consider the dimensionless variables as

$$Z = \frac{z}{L} \text{ and } T = \frac{\rho g}{\rho_s L} t$$

By equation (1.9), we get

$$\frac{\partial \theta}{\partial T} = \varepsilon \frac{\partial}{\partial Z} \left(D(\theta) \frac{\partial \theta}{\partial Z} \right) - K'(\theta) \frac{\partial \theta}{\partial Z} \tag{1.10}$$

where $\varepsilon = \frac{\rho_s L}{\rho g}$ is an auxiliary parameter.

By Broadridge and White model [5], from the equations (1.1) and (1.2), the soil water diffusivity $D(\theta)$ and hydraulic conductivity $K(\theta)$ can be written as

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$$D(\theta) = \frac{a}{b^2} \left(1 - \frac{\theta}{b}\right)^{-2} = \frac{a}{b^2} \left(1 + \frac{2\theta}{b}\right), \text{ since } \theta \text{ being very small.} \quad (1.11)$$

$$\text{and } K(\theta) = \beta + \gamma b + \frac{\lambda}{2b} + \left(\frac{\lambda}{2b} - \gamma\right)\theta \quad (1.12)$$

By equations (1.10), (1.11) and (1.12), we get

$$\frac{\partial \theta}{\partial T} = \varepsilon \frac{a}{b^3} \frac{\partial}{\partial Z} \left((b + 2\theta) \frac{\partial \theta}{\partial Z} \right) - \left(\frac{\lambda}{2b} - \gamma \right) \frac{\partial \theta}{\partial Z} \quad (1.13)$$

The equation (1.13) is a nonlinear second order partial differential equation, which is a governing equation of the moisture content of the soil for the one-dimensional unsteady flow in the unsaturated porous medium in a downward direction. Assume that the initial amount of the moisture content for any Z and T given by $\theta(Z, T) = \theta_0(Z, T)$.

Equation (1.13) is rewritten as

$$\frac{\partial \theta}{\partial T} = \left[A \left(\frac{\partial \theta}{\partial Z} \right)^2 + B \frac{\partial^2 \theta}{\partial Z^2} + A\theta \frac{\partial^2 \theta}{\partial Z^2} \right] - C \frac{\partial \theta}{\partial Z} \quad (1.14)$$

$$\text{where } A = \frac{2\varepsilon a}{b^3}, B = \frac{\varepsilon a}{b^2} \text{ and } C = \left(\frac{\lambda}{2b} - \gamma \right).$$

Since the moisture content θ is increasing as depth Z increasing for any $T > 0$. It is appropriate to choose initial guess value of moisture content of the solution as

$$\theta(Z, 0) = 1 - e^{-Z} \quad [16] \quad (1.15)$$

IV. THE SOLUTION OF THE PROBLEM BY HOMOTOPY PERTURBATION SUMUDU TRANSFORM METHOD (HPSTM)

Applying Sumudu transform of both sides of the equation (1.14) and using condition, we get

$$S[\theta(Z, T)] = (1 - e^{-Z}) + uS \left[A \left(\frac{\partial \theta}{\partial Z} \right)^2 + B \frac{\partial^2 \theta}{\partial Z^2} + A\theta \frac{\partial^2 \theta}{\partial Z^2} - C \frac{\partial \theta}{\partial Z} \right]$$

Apply inverse Sumudu transform, we get

$$\theta(Z, T) = (1 - e^{-Z}) + S^{-1} \left\{ uS \left[A \left(\frac{\partial \theta}{\partial Z} \right)^2 + B \frac{\partial^2 \theta}{\partial Z^2} + A\theta \frac{\partial^2 \theta}{\partial Z^2} - C \frac{\partial \theta}{\partial Z} \right] \right\}$$

Apply Homotopy Perturbation method (HPM), we get

$$\theta(Z, T) = \sum_{n=0}^{\infty} p^n \theta_n(Z, T) = (1 - e^{-Z}) + pS^{-1} \left\{ uS \left[A \left(\frac{\partial \theta}{\partial Z} \right)^2 + B \frac{\partial^2 \theta}{\partial Z^2} + A\theta \frac{\partial^2 \theta}{\partial Z^2} - C \frac{\partial \theta}{\partial Z} \right] \right\}$$

where p is the perturb parameter.

Equating like power terms of ' p ', we get

$$\begin{aligned} p^0: \theta_0 &= (1 - e^{-Z}) \\ p^1: \theta_1 &= \{2Ae^{-2Z} - (A - B + C)e^{-Z}\}T \\ p^2: \theta_2 &= A(Ce^{-Z} - 4e^{-2Z})T + \{4A(2A + C)e^{-2Z} - (A + C)(A - B + C)e^{-Z}\} \frac{T^2}{2} \\ p^3: \theta_3 &= A(6e^{-2Z} - e^{-Z})T + \frac{A}{2} [(BC + C^2)e^{-Z} - 8(2B - C)e^{-2Z}]T^2 + \frac{4A}{3} (2A + C)e^{-2Z}T^3 \\ p^4: \theta_4 &= -A(8e^{-2Z} + e^{-Z})T + \frac{A}{2} [(-8B + 28C)e^{-2Z} - (B + C)e^{-Z}]T^2 + \frac{8A}{3} (4A + C)e^{-2Z}T^3 \end{aligned}$$

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$$p^5: \theta_5 = A(10e^{-2Z} - e^{-Z})T - \frac{A}{2}[16(2B + C)e^{-2Z} + (B + C)e^{-Z}]T^2 + \frac{A}{3}(-8B + 28C)e^{-2Z}T^3 + \frac{4A}{3}(4A + C)e^{-2Z}T^4, \dots$$

The required solution of equation (1.14) is

$$\begin{aligned} \theta(Z, T) = & (1 - e^{-Z}) + \{2Ae^{-2Z} - (A - B + C)e^{-Z}\}T + A(Ce^{-Z} - 4e^{-2Z})T + \{4A(2A + C)e^{-2Z} - (A + C)(A - \\ & B + C)e^{-Z}\} \frac{T^2}{2} + A(6e^{-2Z} - e^{-Z})T + \frac{A}{2}[(BC + C^2)e^{-Z} - 8(2B - C)e^{-2Z}]T^2 + \frac{4A}{3}(2A + C)e^{-2Z}T^3 - \\ & A(8e^{-2Z} + e^{-Z})T + \frac{A}{2}[(-8B + 28C)e^{-2Z} - (B + C)e^{-Z}]T^2 + \frac{8A}{3}(4A + C)e^{-2Z}T^3 + A(10e^{-2Z} - e^{-Z})T - \\ & \frac{A}{2}[16(2B + C)e^{-2Z} + (B + C)e^{-Z}]T^2 + \frac{A}{3}(-8B + 28C)e^{-2Z}T^3 + \frac{4A}{3}(4A + C)e^{-2Z}T^4 + \dots \end{aligned} \quad (1.16)$$

V. NUMERICAL AND GRAPHICAL SOLUTION

Solution (1.16) represents the moisture content of soil in the one-dimensional downward direction. The numerical, as well as graphical representations of the solution (1.16), have been discussed using MATLAB. Figure-2 represents the graph of moisture content $\theta(Z, T)$ vs time T for fixed depth $Z = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$ and Figure-3 represents the graph of moisture content $\theta(Z, T)$ vs depth Z for fixed time $T = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$. Also, the following Table-1 indicates their numerical values.

**TABLE-1: Moisture Content $\theta(Z, T)$ for Different Depth Z for Fixed Time
 $T = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$.**

T	Moisture content of the soil: $\theta(Z, T)$									
	$Z=0.1$	$Z=0.2$	$Z=0.3$	$Z=0.4$	$Z=0.5$	$Z=0.6$	$Z=0.7$	$Z=0.8$	$Z=0.9$	$Z=1$
0.1	0.5357	0.5270	0.5288	0.5382	0.5532	0.5720	0.5933	0.6161	0.6396	0.1000
0.2	0.9710	0.8715	0.8001	0.7506	0.7182	0.6991	0.6901	0.6888	0.6932	0.2000
0.3	1.4668	1.2685	1.1169	1.0026	0.9179	0.8565	0.8136	0.7850	0.7675	0.3000
0.4	2.0884	1.7714	1.5232	1.3303	1.1817	1.0685	0.9835	0.9208	0.8757	0.4000
0.5	2.9014	2.4339	2.0629	1.7696	1.5392	1.3591	1.2195	1.1123	1.0310	0.5000
0.6	3.9712	3.3097	2.7798	2.3565	2.0196	1.7523	1.5414	1.3758	1.2467	0.6000
0.7	5.3633	4.4523	3.7178	3.1269	2.6524	2.2724	1.9689	1.7274	1.5359	0.7000
0.8	7.1432	5.9153	4.9209	4.1166	3.4670	2.9432	2.5217	2.1831	1.9119	0.8000
0.9	9.3764	7.7525	6.4330	5.3618	4.4929	3.7891	3.2195	2.7593	2.3880	0.9000
1.0	12.1283	10.0173	8.2979	6.8982	5.7595	4.8339	4.0820	3.4719	2.9772	1.0000

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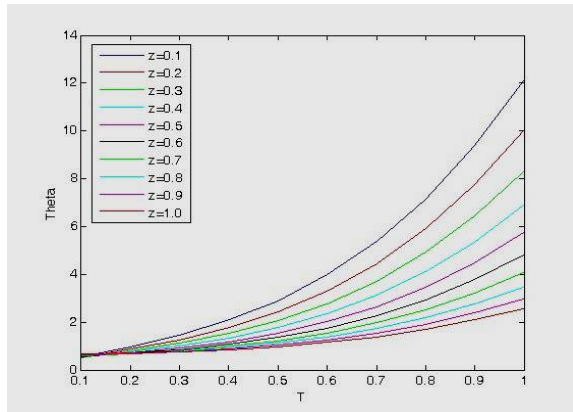


Figure-2: Moisture content $\theta(Z, T)$ vs time T with fixed depth Z

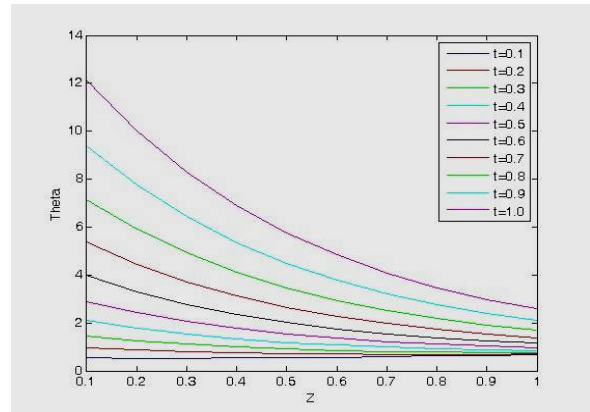


Figure-3: Moisture content $\theta(Z, T)$ vs depth Z with fixed time T

VI. CONCLUSION

The solution (1.16) represents the moisture content of the soil for any depth Z and for any time $T > 0$. The moisture content θ will be zero from guess value of the exact solution for $Z = 0$ and $T = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$. The figure-3 for moisture content θ vs depth Z and fixed time T shows that the moisture content of the soil is increasing as depth Z increasing and $T > 0$, which shows the level of the moisture at different depth when time is fixed. Figure-2 shows the moisture at different time T when depth Z is fixed, and analysis between HPSTM and HAM have been included with their initial approximation $\theta(0) = (1 - e^{-Z})$ and $\theta(Z, T, \epsilon) = (1 - e^{-Z})T + \epsilon^m$ [12] respectively. In which, with initial approximation $(1 - e^{-Z})T + \epsilon^m$, the level of moisture content in the downward direction is leading towards the trivial solution, so initial approximation $\theta(0) = (1 - e^{-Z})$ [16] is appropriate to the physical phenomenon. From the following Table-2, it has been concluded that the level of moisture is too fast in HPSTM compared to the level of moisture content in HAM [12].

TABLE-2: Comparison of Tabular Values of Moisture Content in HPSTM and HAM

T	Moisture content $\theta(Z, T)$																			
	Z=0.1		Z=0.2		Z=0.3		Z=0.4		Z=0.5		Z=0.6		Z=0.7		Z=0.8		Z=0.9		Z=1	
	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM	HPSTM	HAM
0.1	0.5357	0.0125	0.5270	0.0226	0.5288	0.0317	0.5382	0.0399	0.5532	0.0474	0.5720	0.0542	0.5933	0.0603	0.6161	0.0658	0.6396	0.0708	0.1000	0.0753
0.2	0.9710	0.0273	0.8715	0.0472	0.8001	0.0653	0.7506	0.0817	0.7182	0.0964	0.6991	0.1098	0.6901	0.1219	0.6888	0.1328	0.6932	0.1427	0.2000	0.1516
0.3	1.4668	0.0439	1.2685	0.0738	1.1169	0.1007	1.0026	0.1251	0.9179	0.1471	0.8565	0.1669	0.8136	0.1849	0.7850	0.2011	0.7675	0.2158	0.3000	0.2291
0.4	2.0884	0.0624	1.7714	0.1019	1.5232	0.1378	1.3303	0.1701	1.1817	0.1993	1.0685	0.2256	0.9835	0.2493	0.9208	0.2708	0.8757	0.2902	0.4000	0.3077
0.5	2.9014	0.0823	2.4339	0.1318	2.0629	0.1765	1.7696	0.2167	1.5392	0.2529	1.3591	0.2857	1.2195	0.3152	1.1123	0.3418	1.0310	0.3658	0.5000	0.3874
0.6	3.9712	0.1033	3.3097	0.1629	2.7798	0.2166	2.3565	0.2648	2.0196	0.3082	1.7523	0.3473	1.5414	0.3824	1.3758	0.4141	1.2467	0.4427	0.6000	0.4685
0.7	5.3633	0.1254	4.4523	0.1954	3.7178	0.2581	3.1269	0.3144	2.6524	0.3649	2.2724	0.4104	1.9689	0.4512	1.7274	0.4879	1.5359	0.5209	0.7000	0.5508
0.8	7.1432	0.1482	5.9153	0.2289	4.9209	0.3009	4.1166	0.3654	3.4670	0.4232	2.9432	0.4749	2.5217	2.1831	1.9119	0.5632	1.5359	0.6007	0.8000	0.6344
0.9	9.3764	0.1716	7.7525	0.2633	6.4330	0.3449	5.3618	0.4178	4.4929	0.4829	3.7891	0.5411	3.2195	2.7593	2.3880	0.6399	1.9119	0.6819	0.9000	0.7195
1	12.1283	0.1952	10.0173	0.2985	8.2979	0.3902	6.8982	0.4716	5.7595	0.5442	4.8339	0.6089	4.0820	3.4719	2.9772	0.7183	2.9772	0.7646	1.0000	0.8059

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