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## On Edge Irregularity Intuitionistic Fuzzy Graphs

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**ABSTRACT:** In this paper, edge irregular intuitionistic fuzzy graphs, highly edge irregular intuitionistic fuzzy graphs, neighbourly edge irregular intuitionistic fuzzy graphs, strongly edge irregular intuitionistic fuzzy graphs are introduced and some of their properties are discussed.

**KEYWORDS:** edge degree in a fuzzy graph, edge irregular fuzzy graph, highly edge irregular fuzzy graph, neighbourly edge irregular fuzzy graphs, strongly edge irregular fuzzy graph, intuitionistic fuzzy graph.

**AMS Mathematics Subject Classification.** 05C12, 03E72, 05C72.

### I. INTRODUCTION

In 1965, Lofti A. Zadeh [16] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Atanassov added a new component ( which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set ( and the non-membership degree equals one minus the degree of membership ), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [1, 2].Azriel Rosenfeld introduced the concept of fuzzy graphs in 1975 [10]. It has been growing fast and has numerous application in various fields. Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Morderson and Peng [9]. Krassimir T Atanassov [2] introduced the intuitionistic fuzzy graph theory. R.Parvathi and M.G.Karunambigai [8] introduced intuitionistic fuzzy graphs as a special case of Atanassov's IFG and discussed some properties of regular intuitionistic fuzzy graphs [6]. M. Akram, W. Dudek [3] introduced the regular intuitionistic fuzzy graphs. M.Akram and Bijan Davvaz [4] introduced the notion of strong intuitionistic fuzzy graphs and discussed some of their properties. M.G. Karunambigai and R.Parvathi and R.Buvaneswari introduced constant intuitionistic fuzzy graphs [7]. Also, M.G.Karunambigai, S. Sivasankar and K.Palnivel studied some properties of regular intuitionistic fuzzy graphs and edge regular intuitionistic fuzzy graphs[19]. This is the background to introduce the strongly edge irregular intuitionistic fuzzy graphs and strongly edge totally irregular intuitionistic fuzzy graphs, neighbourly edge irregular intuitionistic fuzzy graphs and neighbourly edge totally irregular intuitionistic fuzzy graphs, an edge irregular intuitionistic fuzzy graphs and edge totally irregular intuitionistic fuzzy graphs are introduced and discussed some of their properties.

### II. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

**Definition 2.1.** A Fuzzy graph denoted by  $G : (\sigma, \mu)$  on the graph  $G^* : (V, E)$ : is a pair of functions  $(\sigma, \mu)$  where  $\sigma : V \rightarrow [0; 1]$  is a fuzzy subset of a set  $V$  and  $\mu : V \times V \rightarrow [0; 1]$  is a symmetric fuzzy relation on  $\sigma$  such that for all  $u, v$  in  $V$  the relation  $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$  is satisfied[7].

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**Definition 2.2.** Let  $G : (\sigma, \mu)$  be a connected fuzzy graph  $G^* : (V, E)$ . The degree of an edge  $uv$  is defined as  $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$ . The minimum degree of an edge is  $\delta_E(G) = \wedge \{d_G(uv) : uv \in E\}$ . The maximum degree of an edge is  $\Delta_E(G) = \vee \{d_G(uv) : uv \in E\}$  [10]

**Definition 2.3.** Let  $G : (\sigma, \mu)$  be a connected fuzzy graph  $G^* : (V, E)$ . The total degree of an edge  $uv$  is defined as  $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$ . The minimum total degree of an edge is  $\delta_{tE}(G) = \wedge \{td_G(uv) : uv \in E\}$ . The maximum total degree of an edge is  $\Delta_{tE}(G) = \vee \{td_G(uv) : uv \in E\}$  [10]

**Definition 2.4.** The degree of an edge  $uv$  in the underlying graph is defined as  $d_G(uv) = d_G(u) + d_G(v) - 2[1]$ .

**Definition 2.5.** A fuzzy graph  $G$  is said to be a strongly edge irregular fuzzy graph if every pair of edges of  $G$  have distinct degrees.

**Definition 2.6.** A fuzzy graph  $G$  is said to be a strongly edge totally irregular fuzzy graph if every pair of edges of  $G$  have distinct total degrees.

**Definition 2.7.** A fuzzy graph  $G$  is said to be a neighbourly edge irregular fuzzy graph if every pair of adjacent edges of  $G$  have distinct degrees.

**Definition 2.8.** A fuzzy graph  $G$  is said to be a neighbourly totally edge irregular fuzzy graph if every pair of adjacent edges of  $G$  have distinct total degrees.

**Definition 2.9.** A fuzzy graph  $G$  is said to be a edge irregular fuzzy graph if there exists at least one edge of  $G$  which is adjacent to the edges having distinct degrees.

**Definition 2.10** An intuitionistic fuzzy graph with underlying set  $V$  is defined to be a pair  $G = (V, E)$  where (i)  $V = \{v_1, v_2, v_3, \dots, v_n\}$  such that  $\mu_1 : V \rightarrow [0, 1]$  and  $\gamma_1 : V \rightarrow [0, 1]$  denote the degree of membership and non membership of the element  $v_i \in V$ , ( $i=1,2,3,\dots,n$ ), such that  $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$

(ii)  $E \subseteq V \times V$ , where  $\mu_2 : V \times V \rightarrow [0, 1]$  and  $\gamma_2 : V \times V \rightarrow [0, 1]$  are such that  $\mu_2(v_i, v_j) \leq \min \{\mu_1(v_i), \mu_1(v_j)\}$  and  $\gamma_2(v_i, v_j) \geq \max \{\gamma_1(v_i), \gamma_1(v_j)\}$  and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ , for every  $(v_i, v_j) \in E$ , ( $i, j = 1, 2, \dots, n$ ) [7].

**Definition 2.11** Let  $G = (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then the degree of a vertex  $v_i \in G$  is defined by  $d(v_i) = (d_{\mu_1}(v_i), d_{\gamma_1}(v_i))$ , where  $d_{\mu_1}(v_i) = \sum \mu_2(v_i, v_j)$  and  $d_{\gamma_1}(v_i) = \sum \gamma_2(v_i, v_j)$ , for  $v_i, v_j \in E$  and  $\mu_2(v_i, v_j) = 0$  and  $\gamma_2(v_i, v_j) = 0$  for  $v_i, v_j \notin E$  [7].

**Definition 2.12** Let  $G = (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then the total degree of a vertex  $v_i \in G$  is defined by  $td(v_i) = (td_{\mu_1}(v_i), td_{\gamma_1}(v_i))$ , where  $td_{\mu_1}(v_i) = d_{\mu_1}(v_i) + \mu_1(v_i)$  and  $td_{\gamma_1}(v_i) = d_{\gamma_1}(v_i) + \gamma_1(v_i)$  [7].

**Definition 2.13** Let  $G = (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . The degree of an edge  $uv$  is defined as  $d_G(uv) = (d_{\mu_2}(uv), d_{\gamma_2}(uv))$ , where  $d_{\mu_2}(uv) = d_{\mu_1}(u) + d_{\mu_1}(v) - 2\mu_2(uv)$  and  $d_{\gamma_2}(uv) = d_{\gamma_1}(u) + d_{\gamma_1}(v) - 2\gamma_2(uv)$ .

**Definition 2.13** Let  $G = (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . The total degree of an edge  $uv$  is defined as  $td_G(uv) = (td_{\mu_2}(uv), td_{\gamma_2}(uv))$ , where  $td_{\mu_2}(uv) = d_{\mu_1}(u) + d_{\mu_1}(v) - \mu_2(uv)$  and  $td_{\gamma_2}(uv) = d_{\gamma_1}(u) + d_{\gamma_1}(v) - \gamma_2(uv)$ .

## III STRONGLY EDGE IRREGULAR INTUITIONISTIC FUZZY GRAPHS

**Definition 3.1:** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^*(V, E)$ . Then,  $G$  is said to be a strongly edge irregular intuitionistic fuzzy graph if every pair of edges have distinct degrees (or) no pair of edges have the same degree.

**Definition 3.2 :** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^*(V, E)$ . Then,  $G$  is said to be a strongly edge totally irregular intuitionistic fuzzy graph if every pair of edges have distinct total degrees (or) no pair of edges have the same total degree.

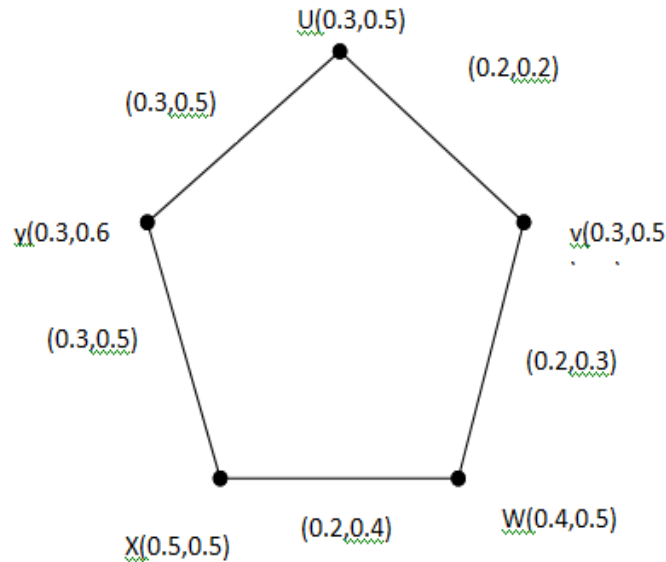
**Example 3.3:** Consider a intuitionistic fuzzy graph on  $G^*(V, E)$ , a cycle of length 5.

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Here,  $d_G(u) = (0.5, 0.7)$ ,  $d_G(v) = (0.4, 0.5)$ ,  $d_G(w) = (0.4, 0.7)$ ,  $d_G(x) = (0.3, 0.9)$ ,  $d_G(y) = (0.6, 1)$ . Now,  $d_G(uv) = (0.5, 0.8)$ ,  $d_G(vw) = (0.4, 0.6)$ ,  $d_G(wx) = (0.3, 0.8)$ ,  $d_G(xy) = (0.3, 0.9)$ ,  $d_G(yu) = (0.5, 0.7)$ . It is noted that all the edges in  $G$  have distinct degrees. Hence  $G$  is strongly edge irregular intuitionistic fuzzy graph.

Also,  $td_G(uv) = (0.7, 1)$ ,  $td_G(vw) = (0.6, 0.9)$ ,  $td_G(wx) = (0.5, 1.2)$ ,  $td_G(xy) = (0.6, 1.4)$ ,  $td_G(yu) = (0.8, 1.2)$ . It is noted that all the edges in  $G$  have distinct total degrees. Hence  $G$  is strongly edge totally irregular intuitionistic fuzzy graph.

**Remark 3.4.** A strongly edge irregular intuitionistic fuzzy graph need not be a strongly edge totally irregular intuitionistic fuzzy graph.

**Remark 3.5.** A strongly edge totally irregular intuitionistic fuzzy graph need not be a strongly edge irregular intuitionistic fuzzy graph.

**Theorem 3.6 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$  and  $B$  is a constant function. If  $G$  is strongly edge irregular intuitionistic fuzzy graph, then  $G$  is strongly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Assume that  $B$  is a constant function, let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $uv$  and  $xy$  be any pair of edges in  $E$ . Suppose that  $G$  is strongly edge irregular intuitionistic fuzzy graph. Then  $d_G(uv) \neq d_G(xy)$ , where

- $\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy))$
- $\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) + (c_1, c_2) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy)) + (c_1, c_2)$
- $\Rightarrow d_G(uv) + B(uv) \neq d_G(xy) + B(xy)$
- $\Rightarrow td_G(uv) \neq td_G(xy)$ , where  $uv$  and  $xy$  are any pair of edges in  $E$ .

Hence  $G$  is strongly edge totally irregular intuitionistic fuzzy graph.

**Theorem 3.7 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$  and  $B$  is a constant function. If  $G$  is strongly edge totally irregular intuitionistic fuzzy graph, then  $G$  is strongly edge irregular intuitionistic fuzzy graph.

**Remark 3.8 :** In Intuitionistic fuzzy graph given in example 2.3, concluded that if  $G$  is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph. Then  $B$  need not be a constant function.

**Theorem 3.9 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$

and  $B(e) = (d_{\mu_2}(e), d_{\gamma_2}(e))$  is a constant function. If  $G$  is a strongly edge irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

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**Proof :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume that  $B(e) = (d_{\mu_2}(e), d_{\gamma_2}(e))$  is a constant function, let  $B(uv) = (c_1, c_2)$ , for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let us suppose that  $G$  is strongly edge irregular intuitionistic fuzzy graph. Then every pair of edges have distinct degrees. Let  $uv$  and  $vw$  are adjacent edges in  $G$  have distinct degrees. Then  $(d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(vw), d_{\gamma_2}(vw))$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) - 2\mu_2(uv), d_{\gamma_1}(u)+d_{\gamma_1}(v)-2\gamma_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2\mu_2(vw), d_{\gamma_1}(v) +d_{\gamma_1}(w)-2\gamma_2(vw))$$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) - 2c_1, d_{\gamma_1}(u) +d_{\gamma_1}(v)-2c_2) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2c_1, d_{\gamma_1}(v) +d_{\gamma_1}(w)-2c_2)$$

$$\Rightarrow (d_{\mu_1}(u), d_{\gamma_1}(u)) \neq (d_{\mu_1}(w), d_{\gamma_1}(w))$$

$\Rightarrow d_G(u) \neq d_G(w)$ , so there exist a vertex  $v$  which is adjacent to a vertices  $u$  and  $w$  have distinct degrees. Hence  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 3.10 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a strongly edge totally irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 3.11 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a strongly edge irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume that  $B$  is a constant function and let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $v$  be any vertex adjacent with  $u, w$  and  $x$ . Then  $uv, vw$  and  $vx$  are adjacent edges in  $G$ . Let us suppose that  $G$  is a strongly edge irregular intuitionistic fuzzy graph so every pair of edges in  $G$  have distinct degrees.

$$\Rightarrow d_G(uv) \neq d_G(vw) \neq d_G(vx)$$

$$\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(vw), d_{\gamma_2}(vw)), \neq (d_{\mu_2}(vx), d_{\gamma_2}(vx))$$

Consider  $(d_{\mu_1}(u)+d_{\mu_1}(v) - 2\mu_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2\mu_2(vw))$  or  $d_{\gamma_1}(u) +d_{\gamma_1}(v)-2\gamma_2(uv) \neq d_{\gamma_1}(v) +d_{\gamma_1}(w)-2\gamma_2(vw)$

$$\Rightarrow d_{\mu_1}(u)+d_{\mu_1}(v) - 2c_1 \neq d_{\mu_1}(v)+d_{\mu_1}(w) - 2c_1 \text{ or } d_{\gamma_1}(u) +d_{\gamma_1}(v)-2c_2 \neq d_{\gamma_1}(v) +d_{\gamma_1}(w)-2c_2$$

$$\Rightarrow d_{\mu_1}(u) +d_{\mu_1}(v) \neq d_{\mu_1}(v)+d_{\mu_1}(w) \text{ (or) } d_{\gamma_1}(u) +d_{\gamma_1}(v) \neq d_{\gamma_1}(v) +d_{\gamma_1}(w)$$

$$\Rightarrow d_{\mu_1}(u) \neq d_{\mu_1}(w) \text{ (or) } d_{\gamma_1}(u) \neq d_{\gamma_1}(w)$$

$$\Rightarrow (d_{\mu_1}(u), d_{\gamma_1}(u)) \neq (d_{\mu_1}(w), d_{\gamma_1}(w))$$

$$\Rightarrow d_G(u) \neq d_G(w)$$

similarly taking  $(d_{\mu_2}(vw), d_{\gamma_2}(vw)), \neq (d_{\mu_2}(vx), d_{\gamma_2}(vx))$

$$\Rightarrow d_G(w) \neq d_G(x)$$

$\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$ , so any vertex  $v$  is adjacent to the vertices with distinct degrees.

**Theorem 3.12 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a strongly edge totally irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

**Remark 3.13 :** converse of the above theorems 3.9,3.10,3.11 and 3.12 need not be true

**Definition 3.14 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . If no two edges have the same edge degree and each edge  $e_i$  have edge degree  $(c_i, k_i)$  with  $c_i = |k_i|$ , then  $G$  is called an equally strongly

edge irregular intuitionistic fuzzy graph. otherwise it is unequally strongly edge irregular intuitionistic fuzzy graph.

**Result 3.15 :** An equally strongly edge irregular intuitionistic fuzzy graph is strongly edge irregular intuitionistic fuzzy graph.

**Result 3.16 :** A strongly edge irregular intuitionistic fuzzy graph need not be equally strongly edge irregular intuitionistic fuzzy graph.

**Theorem 3.17 :** Let  $G : (V,E)$  be a intuitionistic fuzzy graph on  $G^* : (V,E)$ , a path on  $2m(m>1)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ , then  $G$  is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.

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**Proof :** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ , a path on  $2m(m > 1)$  vertices. Let  $e_1, e_2, e_3, \dots, e_{2m-1}$  be the edges of a path  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-1$ ,  $d_G(v_1) = (c_1, k_1)$ ,  $d_G(v_{2m}) = (c_{2m-1}, k_{2m-1})$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $d_G(e_1) = (c_2, k_2) + (c_1, k_1)$ ,  $d_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2}) + (c_{2m-1}, k_{2m-1})$ . Hence  $G$  is strongly edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $td_G(e_1) = (c_2, k_2) + (c_1, k_1)$ ,  $td_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2}) + (c_{2m-1}, k_{2m-1})$ . Hence  $G$  is strongly edge totally irregular intuitionistic fuzzy graph.

**Theorem 3.18 :** Let  $G : (V, E)$  be an intuitionistic fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ , then  $G$  is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V, E)$  be a fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. Let  $e_1, e_2, e_3, \dots, e_n$  be the edges of a cycle  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n$ .  $d_G(v_1) = (c_1, k_1) + (c_n, k_n)$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $d_G(e_1) = (c_2, k_2) + (c_n, k_n)$ ,  $d_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1})$ . Hence  $G$  is strongly edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $td_G(e_1) = (c_2, k_2) + (c_n, k_n) + (c_1, k_1)$ ,  $td_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1}) + (c_n, k_n)$ . Hence  $G$  is strongly edge totally irregular intuitionistic fuzzy graph.

## IV. NEIGHBOURLY EDGE IRREGULAR INTUITIONISTIC FUZZY GRAPHS

**Definition 4.1:** Let  $G : (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then,  $G$  is said to be a neighbourly edge irregular intuitionistic fuzzy graph if every pair of adjacent edges have distinct degrees (or) no pair of adjacent edges have the same degree.

**Definition 4.2 :** Let  $G : (V, E)$  be an intuitionistic fuzzy graph on  $G^*(V, E)$ . Then,  $G$  is said to be a neighbourly edge totally irregular intuitionistic fuzzy graph if every pair of adjacent edges have distinct total degrees (or) no pair of adjacent edges have the same total degree.

**Example 4.3:** Consider an intuitionistic fuzzy graph on  $G^*(V, E)$ , a cycle of length 5 given in example 3.3.

**Remark 4.4.** A neighbourly edge irregular intuitionistic fuzzy graph need not be a neighbourly edge totally irregular intuitionistic fuzzy graph.

**Remark 4.5.** A neighbourly edge totally irregular intuitionistic fuzzy graph need not be a neighbourly edge irregular intuitionistic fuzzy graph.

**Theorem 4.6 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$  and  $B$  is a constant function. If  $G$  is neighbourly edge irregular intuitionistic fuzzy graph, then  $G$  is neighbourly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Assume that  $B$  is a constant function, let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $uv$  and  $xy$  be any pair of edges in  $E$ . Suppose that  $G$  is neighbourly edge irregular intuitionistic fuzzy graph. Then  $d_G(uv) \neq d_G(xy)$ , where  $uv$  and  $xy$  are adjacent edges in  $G$ .

- $\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy))$
- $\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) + (c_1, c_2) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy)) + (c_1, c_2)$
- $\Rightarrow d_G(uv) + B(uv) \neq d_G(xy) + B(xy)$
- $\Rightarrow td_G(uv) \neq td_G(xy)$ , where  $uv$  and  $xy$  are any pair of edges in  $E$ .

Hence  $G$  is neighbourly edge totally irregular intuitionistic fuzzy graph.

**Theorem 4.7 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$  and  $B$  is a constant function. If  $G$  is neighbourly edge totally irregular intuitionistic fuzzy graph, then  $G$  is neighbourly edge irregular intuitionistic fuzzy graph.

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**Remark 4.8 :** In Intuitionistic fuzzy graph given in example 2.3, concluded that if  $G$  is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totallyirregular intuitionistic fuzzy graph. Then  $B$  need not be a constant function.

**Theorem 4.9 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B(e) = (d_{\mu_2}(e), d\gamma_2(e))$  is a constant function. If  $G$  is a neighbourly edge irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

Proof : Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume

that  $B(e) = (d_{\mu_2}(e), d\gamma_2(e))$  is a constant function, let  $B(uv) = (c_1, c_2)$ , for all  $uv \in E$ ,

where  $c_1$  and  $c_2$  are constant. Let us suppose that  $G$  is neighbourly edge irregular intuitionistic fuzzy graph. Then every pair of edges have distinct degrees. Let  $uv$  and  $vw$  are adjacent edges in  $G$  have distinct degrees. Then  $(d_{\mu_2}(uv), d\gamma_2(uv)) \neq (d_{\mu_2}(vw), d\gamma_2(vw))$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) -2\mu_2(uv), d\gamma_1(u)+d\gamma_1(v)-2\gamma_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) -2\mu_2(vw), d\gamma_1(v)+d\gamma_1(w)-2\gamma_2(vw))$$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) -2c_1, d\gamma_1(u)+d\gamma_1(v)-2c_2) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) -2c_1, d\gamma_1(v)+d\gamma_1(w)-2c_2)$$

$$\Rightarrow (d_{\mu_1}(u), d\gamma_1(u)) \neq (d_{\mu_1}(w), d\gamma_1(w))$$

$\Rightarrow d_G(u) \neq d_G(w)$ , so there exist a vertex  $v$  which is adjacent to a vertices  $u$  and  $w$  have distinct degrees. Hence  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 4.10 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a neighbourly edge totally irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 4.11 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a neighbourly edge irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

Proof : Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume that  $B$  is a constant function and let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $v$  be any vertex adjacent with  $u, w$  and  $x$ . Then  $uv, vw$  and  $vx$  are adjacent edges in  $G$ . Let us suppose that  $G$  is a neighbourly edge irregular intuitionistic fuzzy graph so every pair of edges in  $G$  have distinct degrees.

$$\Rightarrow d_G(uv) \neq d_G(vw) \neq d_G(vx)$$

$$\Rightarrow (d_{\mu_2}(uv), d\gamma_2(uv)) \neq (d_{\mu_2}(vw), d\gamma_2(vw)), \neq (d_{\mu_2}(vx), d\gamma_2(vx))$$

Consider  $(d_{\mu_1}(u)+d_{\mu_1}(v) -2\mu_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) -2\mu_2(vw) \text{ or } d\gamma_1(u)+d\gamma_1(v)-2\gamma_2(uv)) \neq d\gamma_1(v)+d\gamma_1(w)-2\gamma_2(vw)$

$$\Rightarrow d_{\mu_1}(u)+d_{\mu_1}(v) -2c_1 \neq d_{\mu_1}(v)+d_{\mu_1}(w) -2c_1 \text{ or } d\gamma_1(u)+d\gamma_1(v)-2c_2 \neq d\gamma_1(v)+d\gamma_1(w)-2c_2$$

$$\Rightarrow d_{\mu_1}(u)+d_{\mu_1}(v) \neq d_{\mu_1}(v)+d_{\mu_1}(w) \text{ (or) } d\gamma_1(u)+d\gamma_1(v) \neq d\gamma_1(v)+d\gamma_1(w)$$

$$\Rightarrow d_{\mu_1}(u) \neq d_{\mu_1}(w) \text{ (or) } d\gamma_1(u) \neq d\gamma_1(w)$$

$$\Rightarrow (d_{\mu_1}(u), d\gamma_1(u)) \neq (d_{\mu_1}(w), d\gamma_1(w))$$

$$\Rightarrow d_G(u) \neq d_G(w)$$

similarly taking  $(d_{\mu_2}(vw), d\gamma_2(vw)), \neq (d_{\mu_2}(vx), d\gamma_2(vx))$

$$\Rightarrow d_G(w) \neq d_G(x)$$

$\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$ , so any vertex  $v$  is adjacent to the vertices with distinct degrees.

**Theorem 4.12 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a neighbourly edge totally irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

**Remark 4.13 :** converse of the above theorems 4.9, 4.10, 4.11 and 4.12 need not be true

**Definition 4.14 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . If no two edges have the same edge degree and each edge  $e_i$  have edge degree  $(c_i, k_i)$  with  $c_i = |k_i|$ , then  $G$  is called an equally neighbourly edge irregular intuitionistic fuzzy graph. otherwise it is unequally neighbourly edge irregular intuitionistic fuzzy graph.

**Result 4.15 :** An equally neighbourly edge irregular intuitionistic fuzzy graph is neighbourly edge irregular intuitionistic fuzzy graph.

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**Result 4.16 :** A neighbourly edge irregular intuitionistic fuzzy graph need not be equally neighbourly edge irregular intuitionistic fuzzy graph.

**Theorem 4.17 :** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^* : (V, E)$ , a path on  $2m(m > 1)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ , then  $G$  is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ , a path on  $2m(m > 1)$  vertices. Let  $e_1, e_2, e_3, \dots, e_{2m-1}$  be the edges of a path  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-1$ ,  $d_G(v_1) = (c_1, k_1)$ ,  $d_G(v_{2m}) = (c_{2m-1}, k_{2m-1})$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $d_G(e_1) = (c_2, k_2)$ ,  $d_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2})$ . Hence  $G$  is neighbourly edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $td_G(e_1) = (c_2, k_2) + (c_1, k_1)$ ,  $td_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2}) + (c_{2m-1}, k_{2m-1})$ . Hence  $G$  is neighbourly edge totally irregular intuitionistic fuzzy graph.

**Theorem 4.18 :** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ , then  $G$  is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V, E)$  be a fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. Let  $e_1, e_2, e_3, \dots, e_n$  be the edges of a cycle  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n$ .  $d_G(v_1) = (c_1, k_1) + (c_n, k_n)$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $d_G(e_1) = (c_2, k_2) + (c_n, k_n)$ ,  $d_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1})$ . Hence  $G$  is neighbourly edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $td_G(e_1) = (c_2, k_2) + (c_n, k_n) + (c_1, k_1)$ ,  $td_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1}) + (c_n, k_n)$ . Hence  $G$  is neighbourly edge totally irregular intuitionistic fuzzy graph.

## V. EDGE IRREGULAR INTUITIONISTIC FUZZY GRAPHS

**Definition 5.1.** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^*(V, E)$ . Then  $G$  is said to be a edge intuitionistic irregular fuzzy graph if there exists at least one edge which is adjacent to the edges with distinct edge degrees.

**Definition 5.2.** If  $G$  is said to be a totally irregular intuitionistic fuzzy graph then there exists at least one edge which is adjacent to the edges with distinct total edge degrees.

**Remark 5.3.** A edge irregular intuitionistic fuzzy graph need not be a edge totally irregular intuitionistic fuzzy graph.

**Remark 5.4.** A edge totally irregular intuitionistic fuzzy graph need not be a edge irregular intuitionistic fuzzy graph.

**Theorem 5.5 :** Let  $G : (V, E)$  be a connected intuitionistic fuzzy graph on  $G^*(V, E)$  and  $B$  is a constant function. If  $G$  is an edge irregular intuitionistic fuzzy graph, then  $G$  is an edge totally irregular intuitionistic fuzzy graph.

**Proof :** Assume that  $B$  is a constant function, let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $uv$  and  $xy$  be any pair of adjacent edges of  $v_x$  in  $E$ . Suppose that  $G$  is an edge irregular intuitionistic fuzzy graph. Then  $d_G(uv) \neq d_G(xy)$ , where  $uv$  and  $xy$  are adjacent edges of  $v_x$  in  $E$ .

$$\begin{aligned} \Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) &\neq (d_{\mu_2}(xy), d_{\gamma_2}(xy)) \\ \Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) + (c_1, c_2) &\neq (d_{\mu_2}(xy), d_{\gamma_2}(xy)) + (c_1, c_2) \\ \Rightarrow d_G(uv) + B(uv) &\neq d_G(xy) + B(xy) \end{aligned}$$

$$\Rightarrow td_G(uv) \neq td_G(xy), \text{ where } uv \text{ and } xy \text{ are any pair of edges of } v_x \text{ in } E.$$

Hence  $G$  is an edge totally irregular intuitionistic fuzzy graph.

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**Theorem 5.6 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is an edge totally irregular intuitionistic fuzzy graph, then  $G$  is an edge irregular intuitionistic fuzzy graph.

**Proof:**Proof is similar to above theorem.

**Remark 5.7:** In Intuitionistic fuzzy graph given in example 3.3, concluded that if  $G$  is both edge irregular intuitionistic fuzzy graph and edge totally irregular intuitionistic fuzzy graph. Then  $B$  need not be a constant function.

**Theorem 5.8 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B(e) = (d_{\mu_2}(e), d_{\gamma_2}(e))$  is a constant function. If  $G$  is a an edge irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  . Assume that  $B(e) = (d_{\mu_2}(e), d_{\gamma_2}(e))$  is a constant function, let  $B(uv) = (c_1, c_2)$ , for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let us suppose that  $G$  is an edge irregular intuitionistic fuzzy graph. Then atleast one edge which is adjacent to the edges ohave distinct degrees. Let  $uv$  and  $vw$  are adjacent edges of  $v_x$  in  $G$  have distinct degrees. Then  $(d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(vw), d_{\gamma_2}(vw))$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) -2\mu_2(uv), d_{\gamma_1}(u)+d_{\gamma_1}(v)-2\gamma_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) -2\mu_2(vw), d_{\gamma_1}(v) +d_{\gamma_1}(w)-2\gamma_2(vw))$$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) -2c_1, d_{\gamma_1}(u)+d_{\gamma_1}(v)-2c_2) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) -2c_1, d_{\gamma_1}(v) +d_{\gamma_1}(w)-2c_2)$$

$$\Rightarrow (d_{\mu_1}(u), d_{\gamma_1}(u)) \neq (d_{\mu_1}(w), d_{\gamma_1}(w))$$

$\Rightarrow d_G(u) \neq d_G(w)$ , so there exist a vertex  $v$  which is adjacent to a vertices  $u$  and  $w$  have distinct degrees. Hence  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 5.9 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is If  $G$  is a edge totally irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Proof:**Proof is similar to above theorem.

## VI. HIGHLY EDGE IRREGULAR INTUITIONISTIC FUZZY GRAPHS

**Definition 6.1.** Let  $G : (V,E)$  be a intuitionistic fuzzy graph on  $G^*(V,E)$ . Then  $G$  is said to be a highly edge irregular intuitionistic fuzzy graph if every edge of  $G$  which is adjacent to the edges with distinct edge degrees.

**Definition 6.2.** If  $G$  is said to be a highly edge totally irregular intuitionistic fuzzy graph then every edge of  $G$  which is adjacent to the edges with distinct total edge degrees.

**Remark 6.3.** A highly edge irregular intuitionistic fuzzy graph need not be a highly edge totally irregular intuitionistic fuzzy graph.

**Remark 6.4.** A highly edge totally irregular intuitionistic fuzzy graph need not be a highly edge irregular intuitionistic fuzzy graph.

**Theorem 6.5 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is highly edge irregular intuitionistic fuzzy graph, then  $G$  is highly edge totally irregular intuitionistic fuzzy graph.

**Proof :** Assume that  $B$  is a constant function, let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $uv$  and  $xy$  be any pair of adjacent edges of  $v_x$  in  $E$ . Suppose that  $G$  is highly edge irregular intuitionistic fuzzy graph. Then  $d_G(uv) \neq d_G(xy)$ , where  $uv$  and  $xy$  are any two adjacent edges of  $v_x$  in  $G$ .

$$\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy))$$

$$\Rightarrow (d_{\mu_2}(uv), d_{\gamma_2}(uv)) + (c_1, c_2) \neq (d_{\mu_2}(xy), d_{\gamma_2}(xy)) + (c_1, c_2)$$

$$\Rightarrow d_G(uv) + B(uv) \neq d_G(xy) + B(xy)$$

$$\Rightarrow t d_G(uv) \neq t d_G(xy), \text{ where } uv \text{ and } xy \text{ are any pair of edges of } v_x \text{ in } E.$$

Hence  $G$  is highly edge totally irregular intuitionistic fuzzy graph.

**Theorem 6.6 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is highly edge totally irregular intuitionistic fuzzy graph, then  $G$  is highly edge irregular intuitionistic fuzzy graph.

**Proof:**Proof is similar to the above theorem.



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**Remark 6.7 :** In Intuitionistic fuzzy graph given in example 3.3, concluded that if  $G$  is both highly edge irregular intuitionistic fuzzy graph and highly edge totally irregular intuitionistic fuzzy graph. Then  $B$  need not be a constant function.

**Theorem 6.8 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B(e) = (d_{\mu_2}(e), d\gamma_2(e))$  is a constant function. If  $G$  is a highly edge irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume that  $B(e) = (d_{\mu_2}(e), d\gamma_2(e))$  is a constant function, let  $B(uv) = (c_1, c_2)$ , for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let us suppose that  $G$  is highly edge irregular intuitionistic fuzzy graph. Then every edge is adjacent to the edges have distinct degrees. Let  $uv$  and  $vw$  are adjacent edges in  $G$  have distinct degrees. Then  $(d_{\mu_2}(uv), d\gamma_2(uv)) \neq (d_{\mu_2}(vw), d\gamma_2(vw))$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) - 2\mu_2(uv), d\gamma_1(u)+d\gamma_1(v)-2\gamma_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2\mu_2(vw), d\gamma_1(v)+d\gamma_1(w)-2\gamma_2(vw))$$

$$\Rightarrow (d_{\mu_1}(u)+d_{\mu_1}(v) - 2c_1, d\gamma_1(u)+d\gamma_1(v)-2c_2) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2c_1, d\gamma_1(v)+d\gamma_1(w)-2c_2)$$

$$\Rightarrow (d_{\mu_1}(u), d\gamma_1(u)) \neq (d_{\mu_1}(w), d\gamma_1(w))$$

$\Rightarrow d_G(u) \neq d_G(w)$ , so there exist a vertex  $v$  which is adjacent to a vertices  $u$  and  $w$  have distinct degrees. Hence  $G$  is an irregular intuitionistic fuzzy graph.

**Theorem 6.9:** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a highly edge totally irregular intuitionistic fuzzy graph, then  $G$  is an irregular intuitionistic fuzzy graph.

**Proof:** Proof is similar to the above theorem.

**Theorem 6.10 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a highly edge irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . Assume that  $B$  is a constant function and let  $B(uv) = (c_1, c_2)$  for all  $uv \in E$ , where  $c_1$  and  $c_2$  are constant. Let  $v$  be any vertex adjacent with  $u, w$  and  $x$ . Then  $uv, vw$  and  $vx$  are adjacent edges in  $G$ . Let us suppose that  $G$  is a highly edge irregular intuitionistic fuzzy graph so every edge adjacent to the edges in  $G$  have distinct degrees.

$$\Rightarrow d_G(uv) \neq d_G(vw) \neq d_G(vx)$$

$$\Rightarrow (d_{\mu_2}(uv), d\gamma_2(uv)) \neq (d_{\mu_2}(vw), d\gamma_2(vw)), \neq (d_{\mu_2}(vx), d\gamma_2(vx))$$

$$\text{Consider } (d_{\mu_1}(u)+d_{\mu_1}(v) - 2\mu_2(uv)) \neq (d_{\mu_1}(v)+d_{\mu_1}(w) - 2\mu_2(vw) \text{ or } d\gamma_1(u) + d\gamma_1(v) - 2\gamma_2(uv)) \neq d\gamma_1(v) + d\gamma_1(w) - 2\gamma_2(vw)$$

$$\Rightarrow d_{\mu_1}(u)+d_{\mu_1}(v) - 2c_1 \neq d_{\mu_1}(v)+d_{\mu_1}(w) - 2c_1 \text{ or } d\gamma_1(u) + d\gamma_1(v) - 2c_2 \neq d\gamma_1(v) + d\gamma_1(w) - 2c_2$$

$$\Rightarrow d_{\mu_1}(u) + d_{\mu_1}(v) \neq d_{\mu_1}(v) + d_{\mu_1}(w) \text{ (or) } d\gamma_1(u) + d\gamma_1(v) \neq d\gamma_1(v) + d\gamma_1(w)$$

$$\Rightarrow d_{\mu_1}(u) \neq d_{\mu_1}(w) \text{ (or) } d\gamma_1(u) \neq d\gamma_1(w)$$

$$\Rightarrow (d_{\mu_1}(u), d\gamma_1(u)) \neq (d_{\mu_1}(w), d\gamma_1(w))$$

$$\Rightarrow d_G(u) \neq d_G(w)$$

similarly taking  $(d_{\mu_2}(vw), d\gamma_2(vw)), \neq (d_{\mu_2}(vx), d\gamma_2(vx))$

$$\Rightarrow d_G(w) \neq d_G(x)$$

$\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$ , so any vertex  $v$  is adjacent to the vertices with distinct degrees.

**Theorem 6.11 :** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$  and  $B$  is a constant function. If  $G$  is a highly edge totally irregular intuitionistic fuzzy graph, then  $G$  is highly irregular intuitionistic fuzzy graph.

**Proof:** Proof is similar to the above theorem.

**Remark 6.12 :** converse of the above theorems 6.8, 6.9, 6.10 and 6.11 need not be true

**Definition 6.13:** Let  $G : (V,E)$  be a connected intuitionistic fuzzy graph on  $G^*(V,E)$ . If no two edges have the same edge degree and each edge  $e_i$  have edge degree  $(c_i, k_i)$  with  $c_i = |k_i|$ , then  $G$  is called an equally an edge irregular intuitionistic fuzzy graph. otherwise it is unequally an edge irregular intuitionistic fuzzy graph.

**Result 6.14 :** An equally an edge irregular intuitionistic fuzzy graph is an edge irregular intuitionistic fuzzy graph.

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**Result 6.15 :** A an edge irregular intuitionistic fuzzy graph need not be equally an edge irregular intuitionistic fuzzy graph.

**Theorem 6.16 :** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^* : (V, E)$ , a path on  $2m(m > 1)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ , then  $G$  is both an edge irregular intuitionistic fuzzy graph and an edge totally irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ , a path on  $2m(m > 1)$  vertices. Let  $e_1, e_2, e_3, \dots, e_{2m-1}$  be the edges of a path  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_{2m-1}$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_{2m-1}, k_{2m-1})$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_{2m-1}, k_{2m-1})$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-1$ ,  $d_G(v_1) = (c_1, k_1)$ ,  $d_G(v_{2m}) = (c_{2m-1}, k_{2m-1})$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $d_G(e_1) = (c_2, k_2)$ ,  $d_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2})$ . Hence  $G$  is an edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, 2m-2$ ,  $td_G(e_1) = (c_2, k_2) + (c_1, k_1)$ ,  $td_G(e_{2m-1}) = (c_{2m-2}, k_{2m-2}) + (c_{2m-1}, k_{2m-1})$ . Hence  $G$  is an edge totally irregular intuitionistic fuzzy graph.

**Theorem 6.17:** Let  $G : (V, E)$  be a intuitionistic fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. If the membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ , then  $G$  is both an edge irregular intuitionistic fuzzy graph and an edge totally irregular intuitionistic fuzzy graph.

**Proof :** Let  $G : (V, E)$  be a fuzzy graph on  $G^* : (V, E)$ , a cycle on  $n(n \geq 4)$  vertices. Let  $e_1, e_2, e_3, \dots, e_n$  be the edges of a cycle  $G^*$  in that order. Let membership value of the edges  $e_1, e_2, e_3, \dots, e_n$  are respectively  $(c_1, k_1), (c_2, k_2), (c_3, k_3), \dots, (c_n, k_n)$  such that  $(c_1, k_1) < (c_2, k_2) < (c_3, k_3), \dots < (c_n, k_n)$ .  $d_G(v_i) = (c_{i-1}, k_{i-1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n$ ,  $d_G(v_1) = (c_1, k_1) + (c_n, k_n)$ ,  $d_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1})$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $d_G(e_1) = (c_2, k_2) + (c_n, k_n)$ ,  $d_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1})$ . Hence  $G$  is an edge irregular intuitionistic fuzzy graph.  $td_G(e_i) = (c_{i-1}, k_{i-1}) + (c_{i+1}, k_{i+1}) + (c_i, k_i)$ , for  $i = 2, 3, 4, 5, \dots, n-1$ ,  $td_G(e_1) = (c_2, k_2) + (c_n, k_n) + (c_1, k_1)$ ,  $td_G(e_n) = (c_1, k_1) + (c_{n-1}, k_{n-1}) + (c_n, k_n)$ . Hence  $G$  is an edge totally irregular intuitionistic fuzzy graph.

**Theorem 6.18.** Every highly edge irregular intuitionistic fuzzy graph is edge irregular intuitionistic fuzzy graph.

**Proof.** Let  $G$  be a highly edge irregular intuitionistic fuzzy graph. Then every edge of  $G$  is adjacent to the edges with distinct edge degrees  $\implies$  there is a edge which is adjacent to the edges with distinct edge degrees. Hence the graph  $G$  is edge irregular intuitionistic fuzzy graph.

**Remark: 6.19.** The converse of the above theorem need not be true.

## IX. CONCLUSION

We have introduced edge irregular intuitionistic fuzzy graphs, highly edge irregular intuitionistic fuzzy graphs, neighbourly edge irregular intuitionistic fuzzy graphs, Strongly edge regular intuitionistic fuzzy graphs in this paper. We can discuss these regularity and irregularity in interval valued fuzzy graphs.

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