

# Matching and Coloring in Topologized Bipartite Graph

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**ABSTRACT:** Let  $G$  be bipartite graph with  $V$  vertices and  $E$  edges. A topological space  $X$  is a topologized graph if it satisfies the following conditions:

- Every singleton is open or closed.
- $\forall x \in X, |\partial(x)| \leq 2$ , where  $\partial(x)$  is a boundary of  $x$ .

Here the topology is defined on the graph, since the space  $X$  is the union of vertices and edges. This work is extended from topologized graph to Bipartite graph, Matching, maximum matching, Perfect matching, vertex coloring and edge coloring.

**KEYWORDS:** Topology, Topologized graph, labeling, Bipartite graph, Matching graph, Coloring graph

## I. INTRODUCTION

In this paper discussed about topologized graphs referred to Antoine Vella [1]. In 2005 Antoine Vella [1] tried to express combinatorial concepts in topological language. As a part the investigation classical topology, pre path, path [2], pre cycle, compact space, cycle space and bond space [3], locally connectedness and ferns were defined. Given a hypergraph  $H$  [4], the classical topology on  $V_H \cup E_H$  is the collection of all sets  $U$  such that, if  $U$  contains a vertex  $v$ , then it also contains all hyperedges incident with  $v$ . It is interesting to note that all these topologies are either defined on the vertex set, or on the union of vertex set and edge set. For Topological graph theory, more important application can be found in printing electronic circuits where the aim is to print (embed) a circuit on a circuit board without two connections crossing each other and resulting in a short circuit. And the Bipartite graphs are extensively used in modern coding theory, especially to decode code words received from the channel. Factor graphs and Tanner graphs are examples of this. A Tanner graph is a bipartite graph in which the vertices on one side of the bipartition represent digits of a codeword, and the vertices on the other side represent combinations of digits that are expected to sum to zero in a codeword without errors. In this paper discussed about some new methods of topology for bipartite graphs.

All graphs are finite, simple, undirected graphs with no loops and multiple edges and planar. Every topological space considered here are finite. And the topology is defined on the set  $X$  which is the union of vertices ( $V$ ) and edges ( $E$ ) of the graph  $G$ .

## II. PRELIMINARIES

**Definition [5]:** A topology on a set  $X$  is a collection  $\tau$  of a subset of  $X$  with the following properties:

- i.  $\Phi$  and  $X$  are in  $\tau$ .
- ii. The union of the element of any sub collection of  $\tau$  is in  $\tau$  (arbitrary union).

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iii. The intersection of the element of any finite sub collection of  $\tau$  is in  $\tau$ .

The set  $X$  for which a topology  $\tau$  has been specified is called a topological space. The element of  $\tau$  are called open set in  $X$ . A subset of  $X$  is said to be closed if its complement is in  $\tau$ . (i.e., its complement is open) A subset of  $X$  may be open, closed both (clopen set), or neither. The empty set and  $X$  itself are always both closed and open.

**Definition [1]:** A topologized graph is a topological space  $X$  such that

- Every singleton is open or closed.
- $\forall x \in X, |\partial(x)| \leq 2$ . since  $\partial(x)$  is denoted by the boundary of a point  $x$

**Definition [2]:** A hyperedge of a topological space is a points which open but not closed. A hyperedge of a topological space is an edge if its boundary consists of at most two points. An edge of a topological space is a loop if it has precisely one boundary point, proper edge otherwise.

**Definition [6]:** A graph  $G$  is called a bigraph or bipartite graph if  $V$  can be partitioned into two disjoint subsets  $V$  and  $W$  such that every line of  $G$  joins a point of  $V$  to a point  $W$ .  $(V, W)$  is called a bipartition of  $G$ .

**Theorem [7]:** If  $G$  is a bipartite graph if and only if it contains only even cycle.

**Definition [8]:** Let ' $G$ ' =  $(V, E)$  be a graph. A subgraph is called a matching  $M(G)$ , if each vertex of  $G$  is incident with at most one edge in  $M$ , i.e.,  $\deg(V) \leq 1 \forall V \in G$ . which means in the matching graph  $M(G)$ , the vertices should have a degree of 1 or 0, where the edges should be incident from the graph  $G$ . Notation:  $M(G)$

**Definition [9]:** The number of edges in the maximum matching of ' $G$ ' is called its matching number. It is also known as largest maximal matching. Maximum matching is defined as the maximal matching with maximum number of edges.

**Definition [8]:** A matching  $(M)$  of graph  $(G)$  is said to be a perfect match, if every vertex of graph  $g$   $(G)$  is incident to exactly one edge of the matching  $(M)$ , i.e.,  $\deg(V) = 1 \forall V$ . The degree of each and every vertex in the subgraph should have a degree of 1.

**Definition [8]:** An assignment of colors to the vertices of a graph so that no two adjacent vertices get the same color is called a coloring of the graph. For each color, the set of all points which get that color is independent and is called a color class. A coloring of a graph  $G$  using at most  $n$  coloring is called an  $n$  coloring. The chromatic number  $\chi(G)$  of a graph  $G$  is the minimum number of colors needed to color  $G$ . A graph  $G$  is called,  $n$ - colorable if  $\chi(G) \leq n$ .

**Definition [8]:** Each  $n$ - colorable of  $G$  partitions  $V(G)$  into  $n$  independent sets called color classes. Such a partitioning induced by a  $\chi(G)$  coloring of  $G$  is called a chromatic partitioning. In other words a partition of  $V(G)$  into the smallest possible number off independent sets is called a chromatic partitioning of  $G$ .

**Definition [8]:** A graph  $G$  is called critical if  $\chi(H) < \chi(G)$  for every proper subgraph  $H$  of  $G$ . A  $k$ - chromatic graph that is critical is called  $k$ - critical. It is obvious that every  $k$ - chromatic graph has a  $k$ - critical subgraph.

**Definition [9]:** Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called proper coloring.

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**Definition [9]:** If  $\chi(G) = n$  and every  $n$ - coloring of  $G$  induces the same partition on  $V(G)$  then  $G$  is called uniquely  $n$ -colorable or uniquely 2-colourable.

**Definition [8]:** An assignment of colors to the edges of a graph  $G$  so that no two adjacent edges get the same color is called an edge coloring or line coloring of  $G$ . An edge coloring of  $G$  using  $n$  colors is called a  $n$ -edge coloring (or  $n$ -line coloring). The edge chromatic number (also called line chromatic number or chromatic index)  $\chi'(G)$  is the minimum number of colors needed to edge color  $G$ . A graph  $G$  is called  $n$ - edge colorable if  $\chi'(G) \leq n$ .

**Theorem [8]:** Any connected bipartite graph is uniquely 2-colourable.

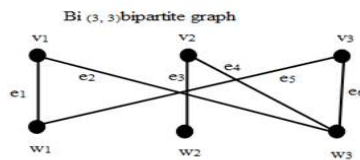
**Theorem [8]:** Every tree with two or more vertices is 2-chromatic.

### III. MAIN RESULTS

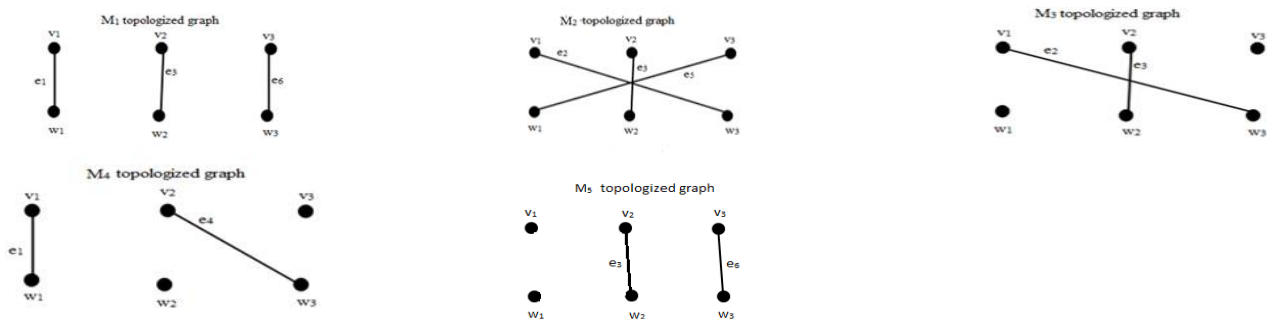
**Theorem 1:** Matching of a bipartite graph is topologized.

**Proof:** Let  $G$  be a bipartite graph. Let  $(X, \tau)$  be a topological space, with topology defined on  $V \cup E$ . A subgraph is called a matching  $M(G)$ , if each vertex of  $G$  is incident with at most one edge in  $M$ . So vertices should have a degree of 1 or 0. Therefore, the boundary of every vertices and edges must be at most two. And every singleton is open or closed. Hence, the matching of a bipartite graph is topologized.

**Example 1.1:** Let  $Bi_{(3,3)}$  be a bipartite graph.



Let  $Bi_{(3,3)}$  be a bipartite graph it is not a topologized but the matching of a  $Bi_{(3,3)}$  bipartite graph is topologized



$M_1, M_2, M_3, M_4, M_5$  From the above graph are the matching of  $G$ . here take Matching of a bipartite graph  $M_1$ .  $M_1$  of the vertices  $V = \{v_1, v_2, v_3\}$ ;  $W = \{w_1, w_2, w_3\}$ ;  $\forall V, W \in V$ . and edges  $E = \{e_1, e_3, e_6\}$ ; Let  $v_1, v_2, v_3, w_1, w_2, w_3$  denote the vertices and  $e_1, e_3, e_6$  the edges labeled so that  $f_G(e_1) = \{v_1, w_1\}$ ;  $f_G(e_3) = \{v_2, w_2\}$ ;  $f_G(e_6) = \{v_3, w_3\}$ ;

Let  $X = \{v_1, v_2, v_3, w_1, w_2, w_3, e_1, e_3, e_6\}$  be a topological space with the topology defined on  $\tau = \{X, \emptyset, \{e_1\},$

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$\{e_3\}, \{e_1, e_3\}, \{v_1, e_1, w_1\}, \{w_2, e_3, v_2, v_3, e_6, w_3\}, \{e_3, v_1, e_1, w_1\}, \{w_2, e_3, v_2, v_3, e_6, w_3, e_1\}$ . Here every  $\{x\} \in X$  is open or closed.  $\forall x \in X, |\partial(x)| \leq 2$

$\partial(v_1) = \{w_1\}$ ; so that we have  $|\partial(v_1)| = 1$ .  $\partial(v_2) = \{w_2\}$ ; so that we have  $|\partial(v_2)| = 1$ .

$\partial(v_3) = \{w_3\}$ ; so that we have  $|\partial(v_3)| = 1$ .  $\partial(w_1) = \{v_1\}$ ; so that, we have  $|\partial(w_1)| = 1$ .

$\partial(w_2) = \{v_2\}$ ; so that, we have  $|\partial(w_2)| = 1$ .  $\partial(w_3) = \{v_3\}$ ; so that, we have  $|\partial(w_3)| = 1$ .

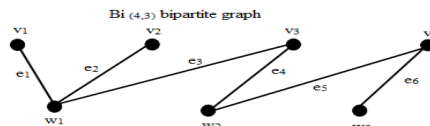
$\partial(e_1) = \{v_1, w_1\}$ ; so that, we have  $|\partial(e_1)| = 2$ .  $\partial(e_3) = \{v_2, w_2\}$ ; so that, we have  $|\partial(e_3)| = 2$ .

$\partial(e_6) = \{v_3, w_3\}$ ; so that, we have  $|\partial(e_6)| = 2$ . The boundary of every vertices and edges is at most two. Therefore  $M_1$  is a topologized. Similarly  $M_2, M_3, M_4, M_5$  is topologized. Hence, The matching of a  $Bi_{(3,3)}$  bipartite graph is topologized.

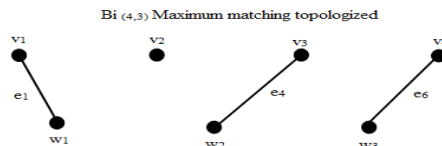
**Theorem 2:** Maximum matching of a bipartite graph is topologized.

**Proof:** Let  $G$  be a bipartite graph. Let  $(X, \tau)$  be a topological space, with topology defined on  $V \cup E$ .  $G$  is a maximum matching if each vertex of  $G$  is incident with at most one edge in  $M$ . So vertices should have a degree of 1 or 0, where the edges should be incident from the graph  $G$ . Therefore, the boundary of every vertices and edges must be at most two. And every singleton is open or closed. Hence, the maximum matching of a bipartite graph is topologized.

**Example 2.1:**  $Bi_{(4,3)}$  be a bipartite graph.



Let  $Bi_{(4,3)}$  be a bipartite graph it is not a topologized but the maximum matching of a  $Bi_{(4,3)}$  bipartite graph is topologized.



$\forall V, W \in V \quad V = \{v_1, v_2, v_3, v_4\}; W = \{w_1, w_2, w_3\}; E = \{e_1, e_4, e_6\}$ ; Let  $v_1, v_2, v_3, v_4, w_1, w_2, w_3$  denote the vertices and  $e_1, e_4, e_6$  the edges labelled so that  $f_G(e_1) = \{v_1, w_1\}; f_G(e_4) = \{v_3, w_2\}; f_G(e_6) = \{v_4, w_3\}$ ; Let  $X = \{v_1, v_2, v_3, v_4, w_1, w_2, w_3, e_1, e_4, e_6\}$  be a topological space with the topology defined on  $\tau = \{X, \emptyset, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_1, e_1, w_1\}, \{w_2, e_4, v_3, v_4, e_6, w_3\}, \{v_2, v_1, e_1, w_1\}, \{w_2, e_4, v_2, v_3, v_4, e_6, w_3\}, \{v_1, w_2, e_4, v_2, v_3, v_4, e_6, w_3\}, \{w_2, e_4, v_3, v_4, e_6, w_3, v_1\}, \{w_2, e_4, v_3, v_4, e_6, w_3, v_1, e_1, w_1\}\}$

Every  $\{x\} \in X$  is open or closed.  $\forall x \in X, |\partial(x)| \leq 2$

$\partial(v_1) = \{w_1\}$ ; so that we have  $|\partial(v_1)| = 1$ .  $\partial(v_2) = \{\emptyset\}$ ; so that we have  $|\partial(v_2)| = 0$ .

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$\partial(v_3) = \{w_2\}$ ; so that we have  $|\partial(v_3)| = 1$ .  $\partial(v_4) = \{w_3\}$ ; so that we have  $|\partial(v_4)| = 1$ .

$\partial(w_1) = \{v_1\}$ ; so that we have  $|\partial(w_1)| = 1$ .  $\partial(w_2) = \{v_3\}$ ; so that we have  $|\partial(w_2)| = 1$ .

$\partial(w_3) = \{v_4\}$ ; so that we have  $|\partial(w_3)| = 1$ .  $\partial(e_1) = \{v_1, w_1\}$ ; so that we have  $|\partial(e_1)| = 2$ .

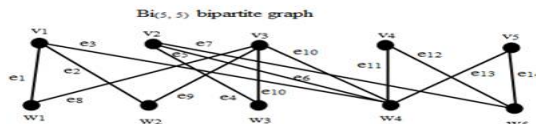
$\partial(e_4) = \{v_3, w_2\}$ ; so that we have  $|\partial(e_4)| = 2$ .  $\partial(e_6) = \{v_4, w_3\}$ ; so that we have  $|\partial(e_6)| = 2$ .

The boundary of every vertices and edges is at most two. Therefore the Maximum Matching of a  $Bi_{(4,3)}$  bipartite graph is topologized.

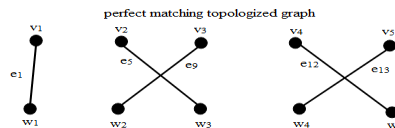
**Theorem 3:** Perfect matching of a bipartite graph is topologized.

**Proof:** Let  $G$  be a bipartite graph. Let  $(X, \tau)$  be a topological space, with topology defined by  $V \cup E$ . A matching  $(M)$  of graph  $(G)$  is a perfect matching if every vertex of graph  $g(G)$  is incident to exactly one edge of the matching  $(M)$ , so the degree of each and every vertex in the subgraph should have a degree of 1. Therefore, the boundary of every vertex is one and edge is two. And every singleton is open or closed. Hence, the perfect matching of a bipartite graph is topologized.

**Example 3.1:**  $Bi_{(5,5)}$  be a bipartite graph



Given  $Bi_{(5,5)}$  be a bipartite graph the graph is not topologized but its perfect matching is topologized as shown in the figure



$\forall V, W \in V$ .  $V = \{v_1, v_2, v_3, v_4, v_5\}$ ;  $W = \{w_1, w_2, w_3, w_4, w_5\}$ ;  $E = \{e_1, e_5, e_9, e_{12}, e_{13}\}$ ; Let  $v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3, w_4, w_5$  denote the vertices and  $e_1, e_5, e_9, e_{12}, e_{13}$  the edges labelled so that  $f_G(e_1) = \{v_1, w_1\}$ ;  $f_G(e_5) = \{v_2, w_3\}$ ;  $f_G(e_9) = \{v_3, w_2\}$ ;  $f_G(e_{12}) = \{v_4, w_5\}$ ;  $f_G(e_{13}) = \{v_5, w_4\}$ ; Let  $X = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3, w_4, w_5, e_1, e_5, e_9, e_{12}, e_{13}\}$  be a topological space with the topology defined on  $\tau = \{X, \emptyset, \{e_1\}, \{e_5\}, \{e_9\}, \{e_1, e_5\}, \{v_1, e_1, w_1, v_2, v_3, w_2, w_3, e_5, e_9\}, \{e_1, e_9\}, \{e_9, e_5\}, \{e_9, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_5, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_1, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_1, e_5, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_1, e_9, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_9, e_5, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_1, e_5, e_9, v_4, v_5, e_{12}, e_{13}, w_4, w_5\}, \{e_1, e_5, e_9\}\}$  Every  $\{x\} \in X$  is open or closed.  $\forall x \in X, |\partial(x)| \leq 2$

$\partial(v_1) = \{w_1\}$ ; so that we have  $|\partial(v_1)| = 1$ .  $\partial(v_2) = \{w_3\}$ ; so that we have  $|\partial(w_3)| = 1$ .

$\partial(v_3) = \{w_2\}$ ; so that we have  $|\partial(v_3)| = 1$ .  $\partial(v_4) = \{w_5\}$ ; so that we have  $|\partial(v_4)| = 1$ .

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$\partial(v_5) = \{w_4\}$ ; so that we have  $|\partial(v_5)| = 1$ .  $\partial(w_1) = \{v_1\}$ ; so that, we have  $|\partial(w_1)| = 1$ .

$\partial(w_2) = \{v_3\}$ ; so that, we have  $|\partial(w_2)| = 1$ .  $\partial(w_3) = \{v_2\}$ ; so that, we have  $|\partial(w_3)| = 1$ .

$\partial(w_4) = \{v_5\}$ ; so that, we have  $|\partial(w_4)| = 1$ .  $\partial(w_5) = \{v_4\}$ ; so that, we have  $|\partial(w_5)| = 1$ .

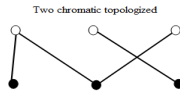
$\partial(e_1) = \{v_1, w_1\}$ ; so that, we have  $|\partial(e_1)| = 2$ .  $\partial(e_5) = \{v_2, w_3\}$ ; so that, we have  $|\partial(e_5)| = 2$ .

$\partial(e_9) = \{v_3, w_2\}$ ; so that, we have  $|\partial(e_9)| = 2$ .  $\partial(e_{12}) = \{v_4, w_5\}$ ; so that, we have  $|\partial(e_{12})| = 2$ .

$\partial(e_{13}) = \{v_5, w_4\}$ ; so that, we have  $|\partial(e_{13})| = 2$ . The boundary of every vertices and edges is at most two. Therefore the perfect matching of a  $Bi_{(5,5)}$  bipartite graph is topologized.

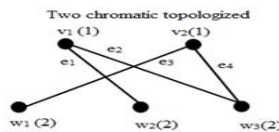
**Theorem 4:** Every two chromatic graph is topologized for all  $\deg(v) \leq 2$ .

**Proof:** Let  $G$  be a bipartite graph. Let  $(X, \tau)$  be a topological space, with topology defined on  $V \cup E$ . We know that every bipartite graph is two chromatic. But two chromatic of all bipartite graphs is not topologized. Here we take each vertex of degree at most two. Consider the  $V$  paint every vertex in  $V$  is white color. Paint all vertices adjacent to  $V$  with  $(W)$  black color. For  $\forall V, W \in V$ . Thus  $G$  has been properly colored with two colors.



Therefore the boundary of every point is at most two. And every singleton is open or closed. Hence, two chromatic of a bipartite graph is topologized.

**Example 4.1:** Let  $G$  be a bipartite graph. Consider the  $V$  paint every vertex in  $V$  is color 1. Paint all vertices adjacent to  $V$  with  $(W)$  color 2. For  $\forall V, W \in V$ . Thus  $G$  has been properly colored with two color.



Here  $V = \{v_1, v_2\}$ ;  $W = \{w_1, w_2, w_3\}$ ;  $E = \{e_1, e_2, e_3, e_4\}$ ; Let  $v_1, v_2, w_1, w_2, w_3$  denote the vertices and  $e_1, e_2, e_3, e_4$  the edges labelled so that  $f_G(e_1) = \{v_1, w_2\}$ ;  $f_G(e_2) = \{v_1, w_3\}$ ;  $f_G(e_3) = \{v_2, w_1\}$ ;  $f_G(e_4) = \{v_2, w_3\}$ ; Let  $X = \{v_1, v_2, w_1, w_2, w_3, e_1, e_2, e_3, e_4\}$  be a topological space with the topology defined on  $\tau = \{X, \emptyset, \{v_1\}, \{w_2\}, \{v_1, e_2, e_1, w_1, w_2, e_3\}, \{v_2, e_4, w_3\}, \{v_1, w_2\}, \{v_1, w_2, e_4, e_3, w_3\}, \{v_1, v_2, e_4, w_3\}, \{w_2, v_2, e_4, w_3\}\}$  Every  $\{x\} \in X$  is open or closed.  $\forall x \in X, |\partial(x)| \leq 2$

$\partial(v_1) = \{w_2, w_3\}$ ; so that, we have  $|\partial(v_1)| = 2$ .  $\partial(v_2) = \{w_1, w_3\}$ ; so that, we have  $|\partial(v_2)| = 2$ .

$\partial(w_1) = \{v_2\}$ ; so that, we have  $|\partial(w_1)| = 1$ .  $\partial(w_2) = \{v_1\}$ ; so that, we have  $|\partial(w_2)| = 1$ .

$\partial(w_3) = \{v_1, v_2\}$ ; so that, we have  $|\partial(w_3)| = 2$ .  $\partial(e_1) = \{v_1, w_2\}$ ; so that, we have  $|\partial(e_1)| = 2$ .

$\partial(e_2) = \{v_1, w_2\}$ ; so that, we have  $|\partial(e_2)| = 2$ .  $\partial(e_3) = \{v_2, w_2\}$ ; so that we have  $|\partial(e_3)| = 2$ .

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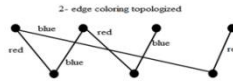
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$\partial(e_4) = \{v_2, w_3\}$ ; so that we have  $|\partial(e_4)| = 2$ . The boundary of every points is at most two. Therefore two chromatic of every bipartite graph is topologized if all vertices of degree at most two.

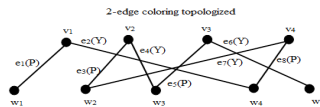
**Theorem 5:** Every 2- edge coloring of a bipartite graph is topologized.

**Proof:** Let  $G$  be a bipartite graph. Let  $(X, \tau)$  be a topological space, with topology defined on  $V \cup E$ . An assignment of colors to the edges of a graph  $G$  so that no two adjacent edges get the same color is called edge coloring. Here we consider the graph with only 2- edge coloring. Paint (E) edge with red color and another non adjacent edge with color blue. Thus  $G$  has been properly colored with two colors.



Therefore the boundary of every point is at most two. And every singleton is open or closed. Hence, two-edge chromatic of all bipartite graphs is topologized.

**Example 5.1:** Let  $G$  be a bipartite graph. Consider the edges paint with one edge in  $E$  is color Pink (P). Paint all non-adjacent edge with color Yellow(Y). Thus  $G$  has been properly colored with two color.



$\forall V, W \in V. V = \{v_1, v_2, v_3, v_4\}; W = \{w_1, w_2, w_3, w_4, w_5\}; E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ ; Let  $v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4, w_5$  denote the vertices and  $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8$  the edges labeled so that  $f_G(e_1) = \{v_1, w_1\}$ ;  $f_G(e_2) = \{v_1, w_4\}$ ;  $f_G(e_3) = \{v_2, w_2\}$ ;  $f_G(e_4) = \{v_2, w_3\}$ ;  $f_G(e_5) = \{v_3, w_3\}$ ;  $f_G(e_6) = \{v_3, w_5\}$ ;  $f_G(e_7) = \{v_4, w_2\}$ ;  $f_G(e_8) = \{v_4, w_4\}$  Let  $X = \{v_1,$

$v_2, v_3, v_4, w_1, w_2, w_3, w_4, w_5, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$  be a topological space with the topology defined on  $\tau = \{X, \phi, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_1, e_1, w_1, w_4, e_2, v_2, e_3, w_2, e_4, w_3, e_5, v_3\}, \{w_5, e_7, e_8, v_4, e_6\}, \{v_1, w_5, e_7, e_8, v_4, e_6\}, \{v_1, v_2, w_5, e_7, e_8, v_4, e_6\}, \{v_2, w_5, e_7, e_8, v_4, e_6\}\}$  Every  $\{x\} \in X$  is open or closed.  $\forall x \in X, |\partial(x)| \leq 2$

$\partial(v_1) = \{w_1, w_4\}$ ; so that we have  $|\partial(v_1)| = 2. \partial(v_2) = \{w_3, w_2\}$ ; so that we have  $|\partial(v_2)| = 2.$

$\partial(v_3) = \{w_3, w_5\}$ ; so that we have  $|\partial(v_3)| = 2. \partial(v_4) = \{w_2, w_4\}$ ; so that we have  $|\partial(v_4)| = 2.$

$\partial(w_1) = \{v_1\}$ ; so that, we have  $|\partial(w_1)| = 1. \partial(w_2) = \{v_2, v_4\}$ ; so that, we have  $|\partial(w_2)| = 2.$

$\partial(w_3) = \{v_2, v_3\}$ ; so that, we have  $|\partial(w_3)| = 2. \partial(w_4) = \{v_1, v_4\}$ ; so that, we have  $|\partial(w_4)| = 2.$

$\partial(w_5) = \{v_3\}$ ; so that, we have  $|\partial(w_5)| = 1. \partial(e_1) = \{v_1, w_1\}$ ; so that, we have  $|\partial(e_1)| = 2.$

Similarly, the boundary edges of  $\{e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$  is two. The boundary of every vertices and edges is at most two. Therefore 2-edge coloring of a  $Bi_{(4,5)}$  bipartite graph is topologized.



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## IV. CONCLUSION

The above results we discussed about some new results on bipartite graph. We have applied and verified a new methods topologized graph for, bipartite graphs, bipartite graph contains all the matching, maximum matching, perfect matching, 2- edge coloring and two colorable.

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