

Topologized Euler and Supereulerian Graph

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ABSTRACT: In this paper the concept of topology extended to the topologized graph by the S_1 space and the boundary of every vertex and edge of a graph G . The space is called S_1 space if every singleton in the topological space is either open or closed. Boundary of a vertex contains the adjacent vertices and incident edges, and the boundary of an edge consists both of its end vertices. This paper contains some results about the topological approach of the Euler graph and Supereulerian graph and also the results were generalized through this work.

KEYWORDS: Topology, Topologized graph, Euler graph and Supereulerian graph.

I. INTRODUCTION

Topological approach of the graphs deals with a different type of geometry. Topological graph theory deals with embedding the graphs in Surfaces, and the graphs considered as a topological spaces. It studies the embedding of graphs in surfaces, spatial embeddings of graphs, and graphs as topological spaces. It was developed to the other invariants such as skewness, thickness, crossing number, local crossing, coloring and covering[5], [13], [14], [16]. In [1] a graph G a closed walk contains all the edges of the graph, and then the walk is called an Euler line and the graph an Euler graph [20]. Through the comparative study between the Euler and Hamiltonian graphs [15] the idea of Euler graph topologically developed. Through spanning subgraph the new class of graphs were developed and named as supereulerian graph. A graph is supereulerian if it has a spanning Euler subgraph which was developed by Paul A. Catlin[11]. Some reduction methods and algorithms were found to extract the spanning subgraphs and supereulerian graph[12]. In 2005 Antoine Vella [2] tried to express combinatorial concepts in topological language as a part of investigation in the field of classical topology using hypergraph [3], topologized graph, prepath, precycle, edge spaces, separation axioms were defined. It is interesting to note that all these topologies are either defined on the vertex set, or on the union of vertex set and edge set. For topological graph theory the more important applications can be found in printing electronic circuits where the aim is to print (embed) a circuit (the graph) on a circuit board (the surface) without two connections crossing each other and resulting in a short circuit. All graphs are finite, simple, undirected graphs with no loops, multiple edges and planar. Every topological space is finite. And the topology is defined on the set X which is the union of vertices (V) and edges (E) of the graph G .

II. PRELIMINARIES

Definition 1: [7] A topology on a set X is a collection τ of a subset of X with the following properties:

- i. Φ and X are in τ .
- ii. The union of the element of any subcollection of τ is in τ (arbitrary union).
- iii. The intersection of the element of any finite subcollection of τ is in τ .

The set X for which a topology τ has been specified is called a topological space.

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Definition 2: [2] A topologized graph is a topological space X such that

- Every singleton is open or closed.
- For all $x \in X$, $|\partial(x)| \leq 2$, where $\partial(x)$ is the boundary of a point x .

Definition 3: [2] A hyperedge of a topological space is a point which open but not closed. A hyperedge of a topological space is an edge if its boundary consists of at most two points. An edge of a topological space is a loop if it has precisely one boundary point, a proper edge otherwise.

Definition 4: [10] A simple graph in which there exists an edge between every pair of vertices is called a complete graph.

Definition 5: [10] In a graph G a closed walk contains all the edges of the graph, and then the walk is called an Euler line and the graph an Euler graph.

Definition 6: [11] A Graph is supereulerian if it has a spanning Euler subgraph.

Lemma 1: If G is a Complete graph K_n , then it is not a topologized graph, for $n \geq 4$. [18]

Lemma 2: A graph G is topologized graph if and only if it is a path or a circuit. [18]

Lemma 3: Every Hamiltonian circuit in a complete graph is topologized graph. [18]

Lemma 4: If G has a two edge – disjoint spanning subgraphs then G is supereulerian. [11]

III. MAIN RESULTS

Theorem: 1 Every Euler circuit of a topologized graph is topologized.

Proof: Let G be any topologized graph with topology defined on $V \cup E$. Let (X, τ) be a topological space. A graph G is called Euler if it is possible to start at a vertex and move along each path so as to pass along each edge without going over any of them more than once the graph has an Euler path. If the path ends at the same vertex at which started it is called an Euler circuit. So, the whole graph is covered by the Euler circuit. By Lemma 2, A graph is a topologized graph if and only if it is a path or a circuit. Hence, Every Euler circuit in a topologized graph is topologized.

Corollary: 1.1 Every graph with Euler Circuit is not topologized graph.

Theorem: 2 For $n \geq 4$, Every Euler Circuit in a complete graph K_n is not a topologized graph.

Proof: Let G be complete graph K_n . Let (X, τ) be a topological space with topology defined on $V \cup E$. By Lemma 1 “If G is a Complete graph K_n , then it is not a topologized graph, for $n \geq 4$ ” and by the Theorem 1 it is easy to verify that Euler circuit in a complete graph is not topologized for $n \geq 4$.

Example: 2.1 Let G be complete graph K_3 which is topologized graph. The vertex set and the edge sets are defined as $V = \{v_1, v_2, v_3\}$, $E = \{a, b, c\}$, the edges are labeled as $f_G(a) = \{v_1, v_2\}$, $f_G(b) = \{v_2, v_3\}$, $f_G(c) = \{v_1, v_3\}$. Let $X = \{v_1, v_2, v_3, a, b, c\}$ be a topological space and a topology defined by $\tau = \{X, \phi, \{a\}, \{b\}, \{v_1, a\}, \{a, b\}, \{v_1, a, b\}\}$ and every singleton is open or closed. Let $v_1 a v_2 b v_3 c v_1$ be an Euler circuit in K_3 . Clearly it is a topologized graph, since the boundary for each vertex and each edge is at most 2. Let $H = \{v_1, v_2, v_3, a, b, c\}$ be a topological space with the topology

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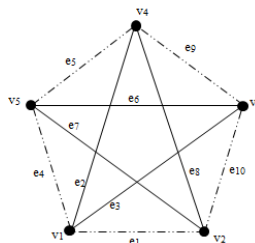
defined by $\tau = \{X, \phi, \{v_2\}, \{v_3\}, \{v_3, v_2\}, \{a, b\}, \{v_2, a, b\}, \{v_3, a, b\}, \{v_3, v_2, a, b\}\}$. Here for every $x \in H, |\partial(x)| \leq 2$. Hence it is a topologized graph. Hence an Euler circuit in a topologized graph is a topologized.

Example:2.2 Let G be a complete graph K_5 with topology defined on $V \cup E$ and it is an Euler graph. The vertex set and the edge sets are defined as $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$, the edges are labeled as $f_G(e_1) = \{v_1, v_2\}, f_G(e_2) = \{v_1, v_4\}, f_G(e_3) = \{v_1, v_3\}, f_G(e_4) = \{v_1, v_5\}, f_G(e_5) = \{v_4, v_5\}, f_G(e_6) = \{v_3, v_5\}, f_G(e_7) = \{v_2, v_5\}, f_G(e_8) = \{v_2, v_4\}, f_G(e_9) = \{v_3, v_4\}$ and $f_G(e_{10}) = \{v_2, v_3\}$. Let $X = \{v_1, v_2, v_3, v_4, v_5, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ be a topological space. The Euler circuit in K_5 is $v_1 e_1 v_2 e_{10} v_3 e_9 v_4 e_5 v_5 e_4 v_1 e_2 v_4 e_8 v_2 e_7 v_5 e_6 v_3 e_3 v_1$. We know that K_5 is not a topologized graph. From this can easily say that the Euler circuit is not a topologized graph.

Theorem: 3 Every Euler graph in a complete graph with n vertices is a $(n-1)/2$ edge-disjoint union of a topologized Hamiltonian circuits.

Proof: Let G be a complete graph with topology defined on $V \cup E$. Let (X, τ) be a topological space. A complete graph G of n vertices has $n(n-1)/2$ edges, and it contains more than one Hamiltonian Circuits. Now we have to determine the number of edge-disjoint Hamiltonian circuits. Let the Hamiltonian circuit in G consists of n edges. Therefore, the number of edge-disjoint Hamiltonian circuits in G cannot exceed $(n-1)/2$. That there are $(n-1)/2$ edge-disjoint Hamiltonian circuits, when n is odd. When n is even, then the degree of each vertex is odd. So, that graph does not contain the Euler circuit. We know that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. By Lemma 3, Every Hamiltonian circuit is a topologized graph. Hence Every Euler graph is edge-disjoint union of topologized Hamiltonian circuits.

Example:3.1 Let G be a complete graph K_5 with topology defined on $V \cup E$. The vertex set and the edge sets are defined as $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$, the edges are labeled as $f_G(e_1) = \{v_1, v_2\}, f_G(e_2) = \{v_1, v_4\}, f_G(e_3) = \{v_1, v_3\}, f_G(e_4) = \{v_1, v_5\}, f_G(e_5) = \{v_4, v_5\}, f_G(e_6) = \{v_3, v_5\}, f_G(e_7) = \{v_2, v_5\}, f_G(e_8) = \{v_2, v_4\}, f_G(e_9) = \{v_3, v_4\}$ and $f_G(e_{10}) = \{v_2, v_3\}$. Let $X = \{v_1, v_2, v_3, v_4, v_5, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ be a connected topological space with topology defined by, $\tau = \{X, \phi, \{e_1, e_2\}, \{v_1, v_2, e_3, e_4, e_5\}, \{v_3, v_4, v_5, e_6, e_7\}, \{v_1, v_2, e_1, e_2, e_3, e_4, e_5\}, \{v_3, v_4, v_5, e_1, e_2, e_6, e_7\}, \{v_1, v_2, v_3, v_4, v_5, e_3, e_4, e_5, e_6, e_7\}, \{v_1, v_2, v_3, v_4, v_5, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}\}$



Edge – Disjoint Hamiltonian Circuits

Among all Hamiltonian circuits in the K_5 complete graph there are only two edge-disjoint Hamiltonian circuits. Those circuits are given below.

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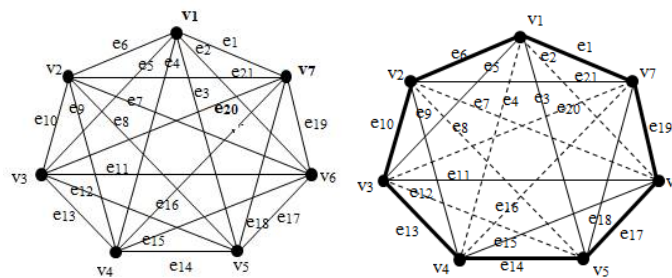
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(i) $v_1 e_1 v_2 e_{10} v_3 e_9 v_4 e_5 v_3 e_4 v_1$ (ii) $v_1 e_2 v_4 e_8 v_2 e_7 v_5 e_6 v_3 e_3 v_1$

These two circuits union gives the Euler graph for the K_5 .

Example: 3.2 Let G be a complete graph K_7 with topology defined on $V \cup E$. The vertex set and the edge sets are defined as $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$, the edges are labeled as $f_G(e_1) = \{v_1, v_7\}, f_G(e_2) = \{v_1, v_6\}, f_G(e_3) = \{v_1, v_5\}, f_G(e_4) = \{v_1, v_4\}, f_G(e_5) = \{v_1, v_3\}, f_G(e_6) = \{v_1, v_2\}, f_G(e_7) = \{v_2, v_6\}, f_G(e_8) = \{v_2, v_5\}, f_G(e_9) = \{v_2, v_4\}, f_G(e_{10}) = \{v_2, v_3\}, f_G(e_{11}) = \{v_6, v_3\}, f_G(e_{12}) = \{v_5, v_3\}, f_G(e_{13}) = \{v_4, v_3\}, f_G(e_{14}) = \{v_4, v_5\}, f_G(e_{15}) = \{v_4, v_6\}, f_G(e_{16}) = \{v_4, v_7\}, f_G(e_{17}) = \{v_5, v_6\}, f_G(e_{18}) = \{v_5, v_7\}, f_G(e_{19}) = \{v_6, v_7\}, f_G(e_{20}) = \{v_7, v_3\}$ and $f_G(e_{21}) = \{v_2, v_7\}$. Let $X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}\}$ be a connected topological space with topology defined by $\tau = \{X, \phi, \{v_1, v_2, e_{10}, e_6\}, \{v_7, v_6, e_{16}, e_{18}, e_{20}\}, \{v_3, v_4, e_5, e_{11}, e_{12}\}, \{v_1, v_1, v_6, v_7, e_6, e_{10}, e_{16}, e_{18}, e_{20}\}, \{v_1, v_2, v_3, v_4, e_5, e_6, e_{10}, e_{11}, e_{12}\}, \{v_3, v_4, v_6, v_7, e_5, e_{11}, e_{12}, e_{16}, e_{18}, e_{20}\}, \{v_1, v_2, v_3, v_4, v_6, v_7, e_5, e_6, e_7, e_{10}, e_{11}, e_{12}, e_{16}, e_{18}, e_{20}\}\}$



Euler graph of K_7

Edge – Disjoint Hamiltonian Circuits

Among all Hamiltonian circuits in the K_7 complete graph there exists $(n-1)/2$ (i.e.,) three edge – disjoint Hamiltonian circuits. These circuits union gives the Euler circuit for the complete graph K_7 . Those circuits are given below.

(i) $v_1 e_6 v_2 e_{10} v_3 e_{13} v_4 e_{14} v_5 e_{17} v_6 e_{19} v_7 e_1 v_1$ (ii) $v_1 e_4 v_4 e_{16} v_7 e_{20} v_3 e_{12} v_5 e_8 v_2 e_7 v_6 e_2 v_1$

(iii) $v_1 e_5 v_3 e_{11} v_6 e_{15} v_4 e_9 v_2 e_{21} v_7 e_{18} v_5 e_3 v_1$

Theorem: 4 Every complete graph K_n is supereulerian.

Proof: Let G be a complete graph K_n with n vertices. An Euler path is a path that crosses each edges of the graph exactly once. If the path is closed, we have an Euler circuit. A graph is supereulerian if it has a spanning Eulersubgraph. Let S be any subgraph in G . Removing a subgraph S from G leaves the graph as a spanning subgraph of G . Here $G - S$ remains as spanning subgraph of G , and which is an Euler graph for $G - S$. Therefore, we get the Euler spanning subgraph $G - S$ for G . Hence, G is a supereulerian graph.

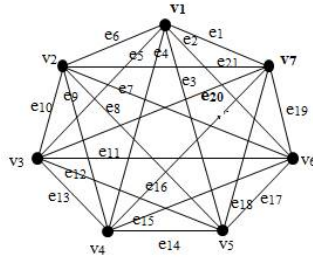
Example:4.1 Let G be a complete graph K_5 . Let $S_1 = \{e_2, e_3, e_4, e_5, e_7, e_8, e_9, e_{11}, e_{12}, e_{15}, e_{16}, e_{18}, e_{20}, e_{21}\}$ be any subgraph of G .

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Supereulerian graph of K_7

Removal of the subgraph S from G results $v_1 e_6 v_2 e_{10} v_3 e_{13} v_4 e_{14} v_5 e_{17} v_6 e_{19} v_7 e_1 v_1$. The above $G - S_1$ is the spanning subgraph and which is also an Eulerian. Let $S_2 = \{e_1, e_3, e_5, e_6, e_9, e_{10}, e_{11}, e_{13}, e_{14}, e_{15}, e_{17}, e_{18}, e_{19}, e_{21}\}$ be any subgraph of G . Removal of this subgraph S from G resulting as $v_1 e_4 v_4 e_{16} v_7 e_{20} v_3 e_{12} v_5 e_8 v_2 e_7 v_6 e_2 v_1$. The above $G - S_2$ is the spanning subgraph and which is also an Eulerian. Hence K_7 is a supereulerian graph.

Theorem: 5 Every topologized Complete graph is a topologized supereulerian graph.

Proof: Let G be a complete graph K_n with n vertices. By Lemma 1, If G is a Complete graph K_n , then it is not a topologized graph, for $n \geq 4$. By Theorem 4, every complete graph K_n is Supereulerian graph. Hence a topologized complete graph is topologized supereulerian.

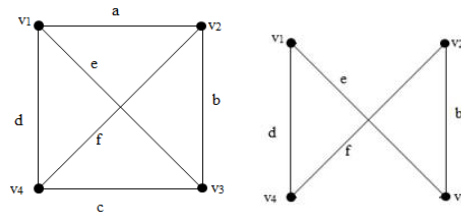
Example: 5.1 Let G be complete graph K_1 . Let $X = \{v_1\}$ be a topological space and a topology defined by $\tau = \{X, \phi\}$ and every singleton is open or closed. Clearly is a topologized supereulerian graph.

Theorem: 6 Every Euler graph is a supereulerian graph.

Proof: Let G be any graph with n vertices. Let G be an Euler graph. So, can easy to say that G is a supereulerian graph.

Remark: 6.1 The converse of the above theorem is not true. That is, every supereulerian graph need not be Euler.

Example: 6.2 Let G be a complete graph K_4 . The vertex set and the edge sets are defined as $V = \{v_1, v_2, v_3, v_4\}$, $E = \{a, b, c, d, e, f\}$, the edges are labeled as $f_G(a) = \{v_1, v_2\}$, $f_G(b) = \{v_2, v_3\}$, $f_G(c) = \{v_3, v_4\}$, $f_G(d) = \{v_1, v_4\}$, $f_G(e) = \{v_1, v_3\}$ and $f_G(f) = \{v_2, v_4\}$. Let $X = \{v_1, v_2, v_3, v_4, a, b, c, d, e, f\}$ be a topological space and a topology defined by $\tau = \{X, \phi, \{v_1\}, \{v_4\}, \{v_1, v_4, c\}, \{v_1, v_4\}\}$ and every singleton is open or closed. But it is not an Euler graph.



Complete Graph K_4 Spanning Eulersubgraph of K_4

Let $S = \{v_1, v_3, v_2, v_4, e, b, d, f\}$ be a spanning subgraph in K_4 , which is also an Euler Graph. Hence K_4 is a supereulerian graph.

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Theorem: 7 If a complete graph K_n is supereulerian if and only if it has

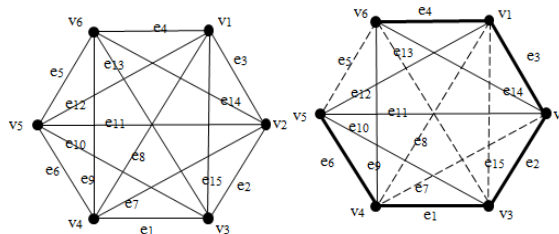
- $(n-1)/2$ edge – disjoint spanning subgraphs, when n is odd.
- $(n/2)$ edge– disjoint spanning subgraphs, when n is even.

Proof: Let G be a complete graph K_n with n vertices.

Let G be a supereulerian graph with n vertices and $n(n-1)/2$ edges, since G is complete graph. A complete graph G with n vertices has $n(n-1)/2$ edges, and it contains more than one spanning subgraph. Now we have to determine the number of edge-disjoint spanning subgraph. If G has spanning subgraph S , then it consist atleast $n-1$ edges. Removal of the subgraph S from G results the subgraph $G - S$ with atmost n edges disjoint from S . $G - S$ may also contain the spanning subgraph S_1 . Removing S_1 from $G - S$ results the subgraph with some edges which are different from the edges in S_1 . Continue the process until we get all the edge-disjoint spanning subgraphs in G . Now, we will get the number of edge – disjoint spanning subgraphs as $(n-1)/2$, when n is odd. When n is even, the number of edge-disjoint spanning subgraphs in G is $(n/2)$.

Conversely, Suppose a complete graph G has $(n-1)/2$ edge-disjoint spanning subgraphs, when n is odd and $(n/2)$ edge – disjoint spanning subgraphs when n is even. To prove G is supereulerian graph. By the Lemma 4, If G has a two edge – disjoint spanning subgraphs then G is supereulerian. It is easy to say that G is supereulerian graph. Hence the proof.

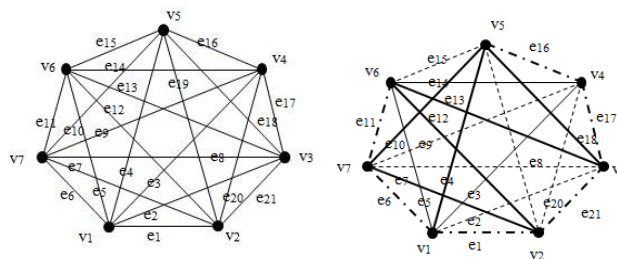
Example: 7.1 Let G be a complete graph K_6 , which is supereulerian but not Eulerian.



Supereulerian graph K_6 Edge – Disjoint spanning subgraphs

Clearly, G has 3 edge – disjoint spanning subgraphs, since n is even.

Example: 7.2 Let G be a complete graph K_7 , which is supereulerian and Eulerian.



Supereulerian graph K_7 Edge – Disjoint spanning subgraphs

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Clearly, G has 3 edge – disjoint spanning subgraphs, since n is odd.

Theorem: 8 If a complete graph K_n is supereulerian if and only if it has

- $(n-1)/2$ edge – disjoint spanning Euler subgraphs when n is odd.
- $\lfloor (n-1)/2 \rfloor$ edge – disjoint spanning Euler subgraphs when n is even.

Proof: Let G be a complete graph K_n with n vertices. Let G be a supereulerian graph with n vertices and $n(n-1)/2$ edges, since G is complete graph. A complete graph G with n vertices has $n(n-1)/2$ edges, and it contains more than one spanning Euler subgraph. Now we have to determine the number of edge-disjoint spanning Euler subgraph.

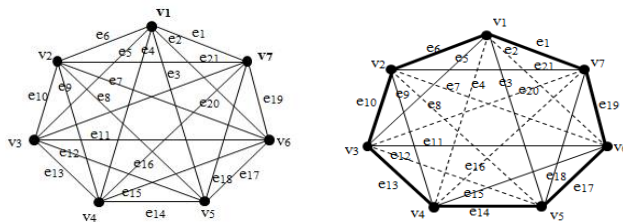
By Theorem 7, and the fact given below, If a complete graph K_n is supereulerian if and only if it has

- $(n-1)/2$ edge – disjoint spanning subgraphs, when n is odd.
- $(n/2)$ edge – disjoint spanning subgraphs, when n is even.

Already know that Hamiltonian circuits are the closed spanning subgraphs in G . The number of edge – disjoint spanning Euler subgraphs as $(n-1)/2$, when n is odd. When n is even, the number of edge – disjoint spanning subgraphs in G is $\lfloor (n-1)/2 \rfloor$.

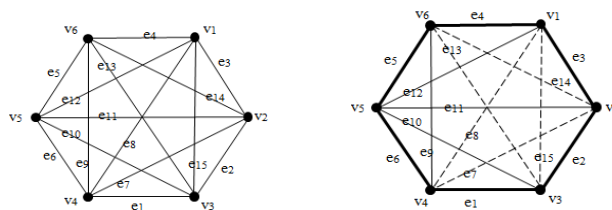
Conversely, Suppose a complete graph G has $(n-1)/2$ edge – disjoint spanning Euler subgraphs, when n is odd and $\lfloor (n-1)/2 \rfloor$ edge – disjoint spanning Euler subgraphs when n is even. To prove G is supereulerian graph. By the Lemma 4, If G has a two edge – disjoint spanning subgraphs then G is supereulerian. So that G is supereulerian graph. Hence the proof.

Example: 8.1 Let G be a complete graph K_7 , which is supereulerian and Euler. Clearly, G has 3 edge – disjoint spanning Euler subgraphs, since n is odd.



Supereulerian graph K_7 Edge – Disjoint spanning Euler subgraph

Example: 8.2 Let G be a complete graph K_6 , which is supereulerian but not Euler.



Supereulerian graph K_6 Edge – Disjoint spanning Euler subgraphs

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Clearly, G has 2 edge – disjoint spanning Euler subgraphs, since n is even.

IV. CONCLUSION

In this work the concept of topologized graph was introduced and investigated through the Euler graph, Supereulerian graph and complete graph. Also, the topologized Euler and topologized Supereulerian graphs were investigated through the Hamiltonian graph of Complete graph.

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