

On Some Well Known Fixed Point Theorems in b-2- metric Spaces

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ABSTRACT: Kir and Kiziltunc show that Kannan Type and Chatterjea Type ontractive mappings have unique fixed point in b-metric spaces. The aim of this paper is to show that Kannan Type and Chatterjea Type contractive mappings have a unique fixed point in b-2-metric spaces

KEYWORDS: fixed point, b-metric spaces, contractive mappings, b-2-metric spaces.

I. INTRODUCTION

The contraction principle of Banach [1], proved in 1922, was followed by diverse works about fixed points theory regarding different classes of contractive conditions on some spaces such as: quasi-metric spaces [2,3], cone metric spaces [4,5], partially ordered metric spaces [6,7], Menger spaces [8], and fuzzy metric spaces [9,10].

The concept of 2-metric space was initiated by S. Gahler [11,12,13]. The study was further enhanced by Iseki [14], B.E. Rhoades [15], Miczko, Palezewski [16], and Khan [17]. They introduced several properties of 2-metric spaces and proved some fixed point and common fixed point theorems for contractive and expansion mappings.

The concept of b-metric space was introduced by Bakhtin [18] in 1989, who used it to prove a generalization of the Banach principle in a spaces endowed with such kind of metric. Since then, this notion has been used by many authors to obtain various fixed point theorems. Aydi *et al*, in [19] proved common fixed point results for single-valued and multi-valued mappings satisfying φ -contractions in b-metric spaces. Roshan *et al*, in [20] used the notation of almost generalized contractive mappings in ordered complete b-metric space and established some fixed and common fixed point results. In [21] Pacurar proved the existence and uniqueness of fixed point of φ -contractions on b-metric spaces. Hussain and shah in [22] introduced the notation of cone b-metric space, generalizing both notations of b-metric spaces and cone metric spaces. Fixed point theorems of contractive mappings in cone b-metric spaces without assumption of the normality of a corresponding cone are proved by Huang and Xu in [23]. The setting of partially ordered b-metric spaces was used by Husain et al, in [24] to study tripled coincidence points of mappings which satisfy nonlinear contractive conditions, extending those results of Berinde and Borcut [25] for metric spaces to b-metric spaces. Using the concept of a g-monotone mapping, Shah and Hussain in [26] proved common fixed point theorems involving g-non-decreasing mappings in b-metric spaces, generalizing several results of Agarwal et al [27] and Ciric et al [28]. Some results of Suzuki [29] are extended to the case of metric-type spaces and cone metric-type spaces. In [30] Shatanawi et al consider and establish on setting of b-metric spaces, regarding common fixed points of two mappings, using a contraction condition defined by means of a comparison function.

In [31] Kir and Kiziltunc show that Kannan Type and Chatterjea Type contractive mappings have unique fixed point in b-metric spaces. The aim of this paper is to extend the results in [31] to b-2-metric spaces.

II. PRELIMINARIES

Definition 2.1. Let X be a non empty set. A real valued function d on $X \times X \times X$ is said to be 2-metric if

- (1) To each of distinct points x, y, z in X , $d(x, y, z) > 0$
- (2) $d(x, y, z) = 0$, when at least two of x, y, z are equal

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$$(3) d(x, y, z) = d(z, y, x) = d(x, z, y)$$

$$(4) d(x, y, z) \leq d(x, y, a) + d(x, a, z) + d(a, y, z) \text{ for all } x, y, z, a \in X$$

when d is a 2-metric on X , then the order pair (X, d) is called 2-metric space.

Definition 2.2. A sequence $\{x_n\}$ in 2-metric space (X, d) is said to be convergent to an element $x \in X$ if $\lim_{n \rightarrow \infty} d(x_n, x, a) = 0$ for all $a \in X$.

It follows that if the sequence $\{x_n\}$ converges to x then $\lim_{n \rightarrow \infty} d(x_n, a, b) = d(x, a, b)$

for all $a, b \in X$

Definition 2.3. A sequence $\{x_n\}$ in 2-metric space (X, d) is said to be Cauchy sequence if $\lim_{m, n \rightarrow \infty} d(x_m, x_n, a) = 0$ as $m, n \rightarrow \infty$ for all $a \in X$

Proposition 2.4. If a sequence is convergent in a 2-metric space then it is a Cauchy sequence.

Definition 2.5. A 2-metric space (X, d) is said to be complete if every Cauchy sequence in X is convergent.

Proposition 2.6. If a sequence $\{x_n\}$ in a 2-metric space converges to x then every subsequence of $\{x_n\}$ also converges to the same limit x

Proposition 2.7. Limit of a sequence in 2-metric space, if exists, is unique.

Definition 2.8. Let X be nonempty set and $d : X \times X \rightarrow [0, \infty)$. A function d is called b-metric space with constant (base) $s \geq 1$ if:

$$(1) d(x, y) = 0 \Leftrightarrow x = y$$

$$(2) d(x, y) = d(y, x) \text{ for all } x, y \in X$$

$$(3) d(x, y) \leq s(d(x, z) + d(z, y)) \text{ for all } x, y, z \in X$$

The pair (X, d) is called b-metric space.

It is obvious that a b-metric space with base $s = 1$ is a metric space. spaces (see, e.g., Singh and Prasad [32].

Definition 2.9. Let X be a non empty set. A real valued function d on $X \times X \times X$ is said to be b-2-metric with constant (base) $s \geq 1$ if

$$(1) \text{ To each of distinct points } x, y, z \text{ in } X, d(x, y, z) > 0$$

$$(2) d(x, y, z) = 0, \text{ when at least two of } x, y, z \text{ are equal}$$

$$(3) d(x, y, z) = d(z, y, x) = d(x, z, y)$$

$$(4) d(x, y, z) \leq s(d(x, y, a) + d(x, a, z) + d(a, y, z)) \text{ for all } x, y, z, a \in X$$

when d is a b-2-metric on X , then the order pair (X, d) is called b-2-metric space.

III. MAIN RESULTS

Our first theorem about Banach's contraction principle in b-2-metric space.

Theorem 3.1. Let (X, d) be a complete b-2-metric space with constant (base) $s \geq 1$ and define the sequence

$$\{x_n\}_{n=1}^{\infty} \subset X \quad \text{by} \quad \text{the} \quad \text{recursion}$$

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots \quad (3.1) \quad \text{Let}$$

$T : X \rightarrow X$ be a contraction with restrictions $k \in [0, 1)$ and $ks < 1$. Then, there exists $x^* \in X$ such that $x_n \rightarrow x^*$ and x^* is unique fixed point of T .

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Proof: Let $x_0 \in X$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X defined as following

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots$$

Since T is a contraction with constant $k \in [0, 1)$ then, we obtain

$$d(T^2 x_0, T^2 x_1, a) \leq kd(Tx_0, Tx_1, a) \leq k^2 d(x_0, x_1, a) \quad \text{for all } a \in X$$

continuing in this process, we easily arrive at

$$d(T^n x_0, T^n x_1, a) \leq k^n d(x_0, x_1, a) \tag{3.2}$$

Now, we show that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X .

Let $m, n > 0$ with $m > n$,

$$d(x_n, x_m, a) \leq sd(x_n, x_m, x_{n+1}) + sd(x_n, x_{n+1}, a) + sd(x_{n+1}, x_m, a) \tag{3.3}$$

Using (3.2) in (3.3) and using triangular inequality, we have

$$\begin{aligned} d(x_n, x_m, a) &\leq sd(x_n, x_m, x_{n+1}) + sd(x_n, x_{n+1}, a) + sd(x_{n+1}, x_m, a) \\ &= sd(T^n x_0, x_m, T^n x_1) + sd(T^n x_0, T^n x_1, a) + sd(x_{n+1}, x_m, a) \\ &\leq sk^n d(x_0, x_1, x_m) + sk^n d(x_0, x_1, a) + s^2 d(x_{n+1}, x_m, x_{n+2}) + s^2 d(x_{n+1}, x_{n+2}, a) + s^2 d(x_{n+2}, x_m, a) \\ &= sk^n d(x_0, x_1, x_m) + sk^n d(x_0, x_1, a) + s^2 k^{n-1} d(x_0, x_1, x_m) + s^2 k^{n-1} d(x_0, x_1, a) + s^2 d(x_{n+2}, x_m, a) \\ &\dots\dots\dots(3.4) \end{aligned}$$

Continuing in this process, using (3.2) in (3.4) and using triangular inequality, we have

$$\begin{aligned} d(x_n, x_m, a) &\leq sk^n d(x_0, x_1, x_m) + sk^n d(x_0, x_1, a) + s^2 k^{n-1} d(x_0, x_1, x_m) + s^2 k^{n-1} d(x_0, x_1, a) + \\ &\quad s^3 k^{n-2} d(x_0, x_1, x_m) + s^3 k^{n-2} d(x_0, x_1, a) + \dots + s^{n+1} d(x_0, x_1, x_m) + s^{n+1} d(x_0, x_1, a) \\ &= [d(x_0, x_1, x_m) + d(x_0, x_1, a)](sk^n)[1 + (sk) + (sk)^2 + (sk)^3 + \dots] \quad \dots\dots\dots(3.5) \end{aligned}$$

When we take $m, n \rightarrow \infty$ in (3.5) we arrive at $\lim_{n \rightarrow \infty} d(x_n, x_m, a) = 0$

Hence, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X . In view of completeness of X , we consider that $\{x_n\}_{n=1}^{\infty}$ converge to $x^* \in X$.

$$\text{Since } Tx^* = \lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Tx_{n+1} = x^*$$

Therefore x^* is the fixed point of T

Suppose x' is another the fixed point of T . Then $Tx' = x'$.

For all $a \in X$ we have,

$$d(x^*, x', a) = d(Tx^*, Tx', a) \leq kd(x^*, x', a)$$

This implies $k \geq 1$ then, which a contradiction to $k \in [0, 1)$. Therefore the fixed point is unique.

Our next theorem about Kannan's type in b-2-metric space.

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Theorem3.2. Let (X, d) be a complete b-2-metric space with constant (base) $s \geq 1$ and define the sequence

$$\left\{ \mathcal{X}_n \right\}_{n=1}^{\infty} \subset X \quad \text{by} \quad \text{the} \quad \text{recursion} \quad \text{Let} \quad (3.1)$$

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots$$

$T : X \rightarrow X$ be a mapping for which there exists $\mu \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty, a) \leq \mu[d(x, Tx, a) + d(y, Ty, a)] \quad (3.6)$$

$\forall x, y, a \in X$. Then, there exists $x^* \in X$ such that $x_n \rightarrow x^*$ and x^* is unique fixed point of T .

Proof: Let $x_0 \in X$ and $\left\{ \mathcal{X}_n \right\}_{n=1}^{\infty}$ be a sequence in X defined as following

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots$$

By using (3.6) and (3.1) we obtain that

$$d(x_n, x_{n+1}, a) = d(Tx_{n-1}, Tx_n, a) \leq \mu[d(x_{n-1}, x_n, a) + d(x_n, x_{n+1}, a)]$$

and we obtain

$$d(x_n, x_{n+1}, a) \leq \frac{\mu}{1 - \mu} d(x_{n-1}, x_n, a) \quad (3.7) \quad \text{Similarly,}$$

we have

$$d(x_n, x_{n+1}, a) \leq \left(\frac{\mu}{1 - \mu}\right)^n d(x_0, x_1, a) \quad (3.8) \quad \text{Since}$$

$\mu \in [0, \frac{1}{2})$ then $\frac{\mu}{1 - \mu} \in [0, 1)$. Thus T is contraction mapping. We deduce, in similar manner to that in proof of

Theorem3.1. that $\left\{ \mathcal{X}_n \right\}_{n=1}^{\infty}$ is a Cauchy sequence in X , and hence a convergent sequence. We consider that

$\left\{ \mathcal{X}_n \right\}_{n=1}^{\infty}$ converge to $x^* \in X$, then we have

$$d(x^*, Tx^*, a) \leq s[d(x^*, Tx^*, x_n) + d(x^*, x_n, a) + d(x_n, Tx^*, a)]$$

$$\leq s[d(x^*, Tx^*, x_n) + d(x^*, x_n, a)] + s\mu[d(x_{n-1}, x_n, a) + d(x^*, Tx^*, a)]$$

and we arrive at

$$d(x^*, Tx^*, a) \leq s[d(x^*, Tx^*, x_n) + d(x^*, x_n, a) + d(x_n, Tx^*, a)]$$

$$\leq s[d(x^*, Tx^*, x_n) + d(x^*, x_n, a)] + s\mu[d(x_{n-1}, x_n, a) + d(x^*, Tx^*, a)] \quad (3.9) \quad \text{By using}$$

(3.8) in (3.9) we obtain that

$$d(x^*, Tx^*, a) \leq s[d(x^*, Tx^*, x_n) + d(x^*, x_n, a)] + s\left(\frac{\mu}{1 - \mu}\right)^n d(x_0, x_1, a) + s\mu d(x^*, Tx^*, a) \quad \text{and we}$$

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have

$$d(x^*, Tx^*, a) \leq \frac{s}{1-s\mu} [d(x^*, Tx^*, x_n) + d(x^*, x_n, a)] + \left(\frac{s}{1-s\mu}\right) \left(\frac{\mu}{1-\mu}\right)^n d(x_0, x_1, a) \quad (3.10) \quad \text{Letting}$$

$n \rightarrow \infty$ in (3.10) we obtain

$$\lim_{n \rightarrow \infty} d(x^*, Tx^*, a) = 0$$

Therefore, $x^* = Tx^*$ and implies that $x^* = Tx^*$ is a fixed point of T .

Suppose x' is another the fixed point of T . Then $Tx' = x'$.

For all $a \in X$ we have,

$$d(x^*, x', a) = d(Tx^*, Tx', a) \leq \mu [d(x^*, Tx^*, a) + d(x', Tx', a)]$$

implies that $d(x^*, x', a) = d(Tx^*, Tx', a) \leq 0$. Therefore the fixed point is unique.

Our next theorem about Chatterjea type fixed point theorem in b-2-metric spaces.

Theorem3.3. Let (X, d) be a complete b-2-metric space with constant (base) $s \geq 1$ and define the sequence

$$\{x_n\}_{n=1}^{\infty} \subset X \quad \text{by} \quad \text{the} \quad \text{recursion} \quad (3.1) \quad \text{Let}$$

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots$$

$T : X \rightarrow X$ be a mapping for which there exists $s\lambda \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty, a) \leq \lambda [d(x, Ty, a) + d(y, Tx, a)] \quad (3.11)$$

$\forall x, y, a \in X$. Then, there exists $x^* \in X$ such that $x_n \rightarrow x^*$ and x^* is unique fixed point of T .

Proof: Let $x_0 \in X$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X defined as following

$$x_n = Tx_{n-1} = T^n x_0, \quad n = 1, 2, 3, \dots$$

By using (3.11) and (3.1) we obtain that

$$\begin{aligned} d(x_n, x_{n+1}, a) &= d(Tx_{n-1}, Tx_n, a) \leq \lambda [d(x_{n-1}, Tx_n, a) + d(x_n, Tx_{n-1}, a)] \\ &= \lambda [d(x_{n-1}, x_{n+1}, a) + d(x_n, x_n, a)] \\ &\leq s\lambda [d(x_{n-1}, x_{n+1}, x_n) + d(x_{n-1}, x_n, a) + d(x_n, x_{n+1}, a)] \end{aligned} \quad (3.12)$$

Thus we have

$$d(x_n, x_{n+1}, a) \leq \frac{s\lambda}{1-s\lambda} [d(x_{n-1}, x_{n+1}, x_n) + d(x_{n-1}, x_n, a)] \quad (3.13)$$

Since $s\lambda \in [0, \frac{1}{2})$ then $\frac{s\lambda}{1-s\lambda} \in [0, 1)$. Thus T is contraction mapping. We deduce, in similar manner to that in

proof of Theorem3.1 and Theorem3.2. that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X , and hence a convergent sequence.

We consider that $\{x_n\}_{n=1}^{\infty}$ converge to $x^* \in X$. We have

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$$\begin{aligned} d(x^*, Tx^*, a) &\leq s[d(x^*, Tx^*, x_{n+1}) + d(x^*, x_{n+1}, a) + d(x_{n+1}, Tx^*, a)] \\ &\leq s[d(x^*, Tx^*, x_{n+1}) + d(x^*, x_{n+1}, a)] + s\lambda[d(x_n, Tx^*, a) + d(x^*, x_{n+1}, a)] \end{aligned} \quad (3.14)$$

Letting $n \rightarrow \infty$ in (3.14) we obtain

$$d(x^*, Tx^*, a) \leq s\lambda d(x^*, Tx^*, a) \quad (3.15)$$

The inequality (3.15) is false unless $d(x^*, Tx^*, a) = 0$

Thus $x^* = Tx^*$

Suppose x' is another the fixed point of T . Then $Tx' = x'$.

For all $a \in X$ we have,

$$d(x^*, x', a) = d(Tx^*, Tx', a) \leq \lambda[d(x^*, Tx', a) + d(x', Tx^*, a)] \quad (3.16) \quad \text{From}$$

(3.16) we obtain $d(x^*, x', a) \leq 2\lambda d(x^*, x', a)$. This implies that $x^* = x'$. This complete the proof.

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