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An Inventory Model for Deteriorating Items with Quadratic Demand, Time Dependent Holding Cost and Salvage Value

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ABSTRACT: This paper deals an inventory model for deteriorating products with quadratic function of time with time dependent holding cost. This model also covers constant salvage value for considering current supermarket scenario and deterioration rate follows an exponential distribution. The aim of this research is to minimize the total cost in which the shortages are not allowed.

KEYWORDS: Deteriorating items, Quadratic Demand, Salvage value, Time-Varying Holding Cost

I. INTRODUCTION

The Studies of deteriorating inventory play an important role in day today life of a common man. The inventory models for deteriorating items have been thoroughly investigated by S.K Goyal and B.C Giri (2001) and Kunianli, HongjieLan and John R. Mawhinnay (2010). While formulating inventory models, the factors such as demand and deterioration rate cannot be ignored. Other factors such as price discount, allow shortages or not, lead time, backlogging trade credit, inflation and the time-value of money also play an important role in deteriorating inventory models. However, products which come under obsolescence and deterioration have short life period. Various inventory models have been suggested in the past by assuming the constant demand rate which can be applicable for manufacturing and sales environment.

AzizulBatenMd and AbdulbasahKamil (2009) expressed inventory model by assuming demand rate as constant. Ghost,S.K, and Chaudhuri,K.S (2004) ,Nita H. Shah and NidhiRaykudaliya (2010) ,Begum R et al (2010,2012) developed inventory models for time-quadratic demand.

Poonam Mishra and Nita H.Shah (2008) examined an inventory model for considering time dependent deterioration rate and salvage value is incorporated with deteriorated units.

Udayakumar R and Sekar T (2014) proposed an EPQ model for imperfect manufacturing system and rework has been discussed mainly in reverse logistic and green supply chain to reduce production cost and environmental problem.Kumar Mishra V.(2012) ,discussed a production inventory model for the business enterprises under reverse logistics in which demand and deterioration considered as a function of time and salvage value is incorporated to the deteriorated items.PanditJagatanandha Mishra et al (2016) focused an optimal policy for an inventory model with Quadratic demand rate, three parameter Weibull distribution deterioration rate,partial backlogging and salvage value. Mishra V.K. (2013) obtained a minimized total cost by reducing the deterioration rate of the inventory items using the preservation technology with salvage value.The factors such as demand rate and deterioration rate and assumed to be constant and time dependent respectively. NitaH.Shah, AjayS.Gor and Wee H.M (2008) suggested a strategy for both the buyer and vendor when the expired stocks have salvage value with respect to constant rate of deterioration.Venkateswaralu R and Mohan R (2014) formulated a model with Quadratic demand rate , constant deterioration rate and salvage value .Srichandran Mishra et al (2014),analysed an instantaneous replenishment model for the items under salvage value is incorporated to the deterioration units.TripathyC.K andPradhan L.M (2012)

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presented an EOQ model for three parameter Weibull distribution with permissible delay in payments associated with salvage value of the deteriorated items.

This paper is an extension of AmuthaR.andChandrasekaran E. (2013) by considering salvage value incorporated to the deteriorated items. In which Holding cost is a linear function of time along with Quadratic demand rate and deterioration rate follows an exponential distribution. Shortages are not allowed .The aim of the model is to obtain the minimum total cost.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been considered in this inventory system.

- The inventory system deals with single item
- The demand rate is assumed to be $D(t) = a + bt + ct^2$; the annual demand as a function of time where $a > 0$, $b > 0$ and $c > 0$.
- The lead time is zero.
- A finite planning horizon is assumed.
- Shortages are not allowed.
- A is set up cost per order.
- $I(t)$ is the inventory level of the product.
- $\theta = \theta(t)$ is the deterioration rate follows an exponential distribution $f(t) = \theta e^{-\theta t}$, $t > 0$ and $\theta > 0$.
- Holding cost is linear function of time $h(t) = \alpha + \beta t$, $\alpha > 0$, $\beta > 0$.
- The salvage value γ ($\gamma \leq 0 < 1$) is associated to deteriorated units during the cycle time.
- C_1 is the deterioration cost per unit time.
- SV is the salvage value per unit time.
- Q is the initial inventory level.
- T^* , Q^* and $K(T^*)$ are the optimal values of the length of time, order quantity and total average cost respectively.

III. MATHEMATICAL MODEL

A mathematical inventory model is developed for determining Economic order Quantity when shortages not occurring. Let $I(t)$ be the inventory level at any instant of time t ($0 \leq t \leq T$). Either by demand or by deterioration the inventory becomes zero at $t = T$. The rate of change of inventory level is governed by the following differential equations.

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt + ct^2), 0 \leq t \leq T \quad (1)$$

with boundary conditions $I(T) = 0$ and $I(0) = Q$

The Solution of equation (2.3.1) with boundary condition $I(T) = 0$ is

$$I(t) = a(T-t) + \frac{(b+a\theta)(T^2-t^2)}{2} + \frac{(c+b\theta)(T^3-t^3)}{3} + \frac{c\theta(T^4-t^4)}{4} + \frac{c\theta^2(T^5-t^5)}{5} \quad (2)$$

Therefore the total cost consists of the following three components SC, HC, DC and SV

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Using the condition $I(0) = Q$ and by (2) we obtain $Q = aT + \frac{(b+a\theta)T}{2} + \frac{(c+b\theta)T^3}{3} + \frac{c\theta T^4}{4} + \frac{c\theta^2 T^5}{5}$ (3)

Set up cost per unit time

$$SC = \frac{A}{T} \quad (4)$$

Inventory holding cost per unit time

$$HC = \frac{1}{T} \int_0^T (\alpha + \beta t) \left[a(T-t) + \frac{(b+a\theta)(T^2-t^2)}{2} + \frac{(c+b\theta)(T^3-t^3)}{3} + \frac{c\theta(T^3-t^3)}{4} + \frac{c\theta^2(T^3-t^3)}{5} \right] dt$$

$$HC = \frac{a\alpha T}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta T^4}{5} + \frac{\alpha c\theta^2 T^5}{6} + \frac{\beta a T^2}{6} + \frac{3\beta(c+b\theta)T^4}{20} + \frac{\beta(b+a\theta)T^3}{8} + \frac{C\theta\beta T^5}{20} \quad (5)$$

Deterioration cost per unit time

$$DC = \frac{C_1}{T} \left[Q - \int_0^T D(t) dt \right]$$

$$DC = \frac{aC_1\theta T}{2} + \frac{bC_1\theta T^2}{3} + \frac{cC_1\theta T^3}{4} + \frac{cC_1\theta^2 T^4}{5} \quad (6)$$

Salvage value per unit time

$$SV = \frac{\gamma C_1}{T} \left[Q - \int_0^T D(T) dt \right]$$

$$SV = \frac{a\gamma C_1\theta T}{2} + \frac{b\gamma C_1\theta T^2}{3} + \frac{c\gamma C_1\theta T^3}{4} + \frac{c\gamma C_1\theta^2 T^4}{5} \quad (7)$$

Therefore by (3), (4), (5), (6) and (7) the average total cost per unit time $K(T)$ as

$$K(T) = \frac{1}{T} [\text{Set up cost} + \text{Deterioration cost} + \text{Holding cost} - \text{Salvage value}]$$

$$= \frac{A}{T} + \frac{aC_1\theta T}{2} + \frac{bC_1\theta T^2}{3} + \frac{cC_1\theta T^3}{4} + \frac{cC_1\theta^2 T^4}{5} + \frac{a\alpha T}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta T^4}{5} + \frac{\alpha c\theta^2 T^5}{6} + \frac{\beta a T^2}{6}$$

$$- \frac{a\gamma C_1\theta T}{2} - \frac{b\gamma C_1\theta T^2}{3} - \frac{c\gamma C_1\theta T^3}{4} - \frac{c\gamma C_1\theta^2 T^4}{5}$$

Our objective is to minimize the total cost per unit time $K(T)$. The necessary and sufficient conditions for total cost $K(T)$ to be minimize are

$$\frac{\partial K(T)}{\partial T} = 0 \text{ and } \frac{\partial^2 K(T)}{\partial T^2} > 0 \text{ for all } T > 0.$$

Therefore, the first order differential equations of $K(T)$ with respect to T is as follows

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$$\frac{\partial K(T)}{\partial T} = \frac{-A}{T^2} + \frac{aC_1\theta}{2} + \frac{3cC_1\theta T^2}{4} + \frac{4cC_1\theta^2 T^3}{5} + \frac{a\alpha}{2} + \frac{2\alpha(b+a\theta)T}{3} + \frac{4\alpha c\theta T^3}{5} + \frac{5\alpha c\theta^2 T^4}{6} + \frac{2\beta a T}{6} + \frac{3\beta(b+a\theta)T^2}{8} + \frac{5c\theta\beta T^4}{12} + \frac{6\beta(c+b\theta)T^3}{10} - \frac{a\theta\gamma C_1}{2} - \frac{2b\theta\gamma C_1}{3} - \frac{3c\theta\gamma^2 C_1}{4} - \frac{4c\theta^2\gamma^3 C_1}{5} \quad (8)$$

The second order differential equation of $K(T)$ with respect to T is as follows

$$\frac{\partial^2 K(T)}{\partial T^2} = \frac{2A}{T^3} + \frac{2b\theta C_1}{3} + \frac{12cC_1\theta^2 T^2}{5} + \frac{2\alpha(b+a\theta)}{3} + \frac{12\alpha c\theta T^2}{5} + \frac{3\alpha(c+b\theta)T}{2} + \frac{10\alpha c\theta^2 T^4}{3} + \frac{\beta a}{3} + \frac{3\beta(b+a\theta)T}{4} + \frac{5c\theta\beta T^3}{3} + \frac{\alpha(T_1-\gamma)^{\beta+1}}{\beta+1} - \frac{3C_1 T c \theta \gamma}{2} - \frac{2C_1 b \theta \gamma}{3} - \frac{12C_1 T^2 c \theta^2 \gamma}{5} > 0 \quad (9)$$

The solution of the equation (2.3.8) gives the optimal value of T (say T^*) and satisfies the condition $\frac{\partial^2 K(T^*)}{\partial T^{*2}} > 0$.

Also, the optimal Economic order quantity Q^* is then obtained from equation (2.3.7) by substituting the value of T = T^* .

$$Q^* = aT^* + \frac{(b+a\theta)T^*}{2} + \frac{(c+a\theta)T^{*2}}{3} + \frac{C\theta T^{*3}}{4} + \frac{C\theta^2 T^{*5}}{5}$$

IV. NUMMERICAL ILLUSTRATION

Considering an inventory system with following parametric values in proper units

$[A, a, b, \alpha, \beta, \theta, c, C_1, \gamma] = [1000, 25, 40, 0.5, 0.011, 0.05, 20, 1.5, 0.2]$ in their respective units. Then we

get $T^* = 2.7416$ $K(T^*) = 513.8090$.

From the above said parameter values, we obtain the numerical values of $K(T^*)$ for various values of T^* . It can be noted that there exists an optimal values of T^* to minimize $K(T^*)$.

TABLE 4.1 OPTIMAL RESULTS FOR DIFFERENT VALUES OF T

T*	K(T*)
2.5	519.5329
2.6	515.8248
2.7416	514.0235
2.8	514.0363
2.9	515.7879

The observation from figure 4.2 is that the total cost function is strictly convex and the total cost per unit time is minimum

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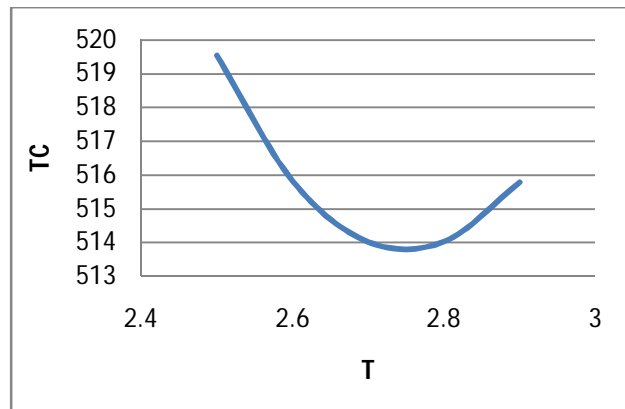


Fig. 4.2 Graphical representation of Total Cost (TC) vs (T)

V. CONCLUSION

In this paper, a quadratic demand based inventory model has been developed for deteriorating products with time dependent holding cost and salvage value. In this model shortages are not allowed. The model is solved analytically to obtain the minimum cost. The result is illustrated with example and the graphical representation of the total cost and time has presented.

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