

Generalization of Summation Formulas Due to Ramanujan with the Help of Generalized Dixon's Theorem

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ABSTRACT: The aim of this research paper is to present generalizations of summation formulas due to Ramanujan. All these results are derived with the help of generalized Dixon theorem for ${}_5F_4$. The derived results in this paper are straightforward, motivating, easily established and may be potentially valuable.

KEYWORDS: Generalized Hypergeometric Series ${}_pF_q$, Dixon's Theorem, Generalized Dixon's Theorem, Ramanujan's Summations.

1. INTRODUCTION

The generalized hypergeometric function is defined by the series [4]

$${}_pF_q \left(\begin{matrix} a_1, a_2, a_3, \dots, a_p \\ b_1, b_2, b_3, \dots, b_q \end{matrix}; z \right) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n \dots (a_p)_n}{(b_1)_n (b_2)_n (b_3)_n \dots (b_q)_n} \frac{z^n}{n!} \quad (1.1)$$

Where $a_1, a_2, a_3, \dots, a_p$ and $b_1, b_2, b_3, \dots, b_q$ are called numerator and denominator parameters respectively. $(a)_n$ is called pochhammer's symbol which is defined by

$$(a)_n = \begin{cases} 1 & n = 0 \\ a(a+1)(a+2)\dots(a+n-1) & n = 1, 2, 3, \dots \end{cases} \quad (1.2)$$

Special cases:

$$\begin{aligned} \Gamma(a+n) &= (a+n-1)\Gamma(a+n-1) \\ &= (a+n-1)(a+n-2)\Gamma(a+n-2) \\ &= (a+n-1)(a+n-2)\dots(a+1)a\Gamma(a) \end{aligned}$$

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$$\Gamma(a+n) = (a)_n \Gamma(a)$$

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$$

Where a is neither zero nor negative integer.

In hypergeometric series the various classical summation theorems for the ${}_2F_1, {}_3F_2, {}_4F_3$ type function plays significant role and have a wide range of applications in the regarded field. The four summation formulas due to Ramanujan [3]

Case 1. $\text{Re}(x) > \frac{1}{4}$

$$1 + \left(\frac{x-1}{x+1}\right)^2 + \left(\frac{(x-1)(x-2)}{(x+1)(x+2)}\right)^2 + \dots = \frac{2x[\Gamma(x+1)]^4 \Gamma(4x+1)}{(4x-1)[\Gamma(2x+1)]^4} \quad (1.3)$$

Case 2. $\text{Re}(x) < \frac{2}{3}$

$$1 + \left(\frac{x}{1!}\right)^3 + \left(\frac{x(x+1)}{2!}\right)^3 + \dots = \frac{6 \sin(\frac{1}{2}\pi x) \sin(\pi x) [\Gamma(\frac{1}{2}x+1)]^3}{\pi^2 x^2 (1+2 \cos \pi x) [\Gamma(\frac{3}{2}x+1)]} \quad (1.4)$$

Case 3. $\text{Re}(x) > 0$

$$1 - \frac{1}{3} \left(\frac{x-1}{x+1}\right) + \frac{1}{5} \left(\frac{(x-1)(x-2)}{(x+1)(x+2)}\right) - \dots = \frac{2^{4x} [\Gamma(x+1)]^4}{4x [\Gamma(2x+1)]^2} \quad (1.5)$$

Case 4. $\text{Re}(x) > 1$

$$1 - 3 \left(\frac{x-1}{x+1}\right) + 5 \left(\frac{(x-1)(x-2)}{(x+1)(x+2)}\right) - \dots = 0 \quad (1.6)$$

All above mentioned summations pointed by Berndt (1985) due to Ramanujan can be achieved by using following Dixon theorem [2]

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$${}_3F_2 \left(\begin{matrix} a, b, c \\ 1+a-b, 1+a-c \end{matrix}; 1 \right) = \frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+\frac{1}{2}a-b-c)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)\Gamma(1+\frac{1}{2}a-c)\Gamma(1+a-b-c)}$$

Provided by $\text{Re}(a-2b-2c) > -2$ (1.7)

Generalized Dixon's Summation Theorem

$${}_5F_4 \left(\begin{matrix} a, \frac{1}{2}a+1, b, c, d \\ \frac{1}{2}a, 1+a-b, 1+a-c, 1+a-d \end{matrix}; 1 \right) = \frac{\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-d)\Gamma(1+a-b-c-d)}{\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

Provided by $\text{Re}(a-b-c-d) > -1$ (1.8)

II. MAIN RESULTS

We established generalized form of equation (1.3) for our results consider $a=1, b=1-x, c=1-x$. Putting these values in equation (1.8) and expand the hypergeometric function as a series

$$\begin{aligned} {}_5F_4 \left(\begin{matrix} 1, \frac{3}{2}, 1-x, 1-x, d \\ \frac{1}{2}, 1+x, 1+x, 2-d \end{matrix}; 1 \right) &= \sum_{n=0}^{\infty} \frac{(1)_n (\frac{3}{2})_n (1-x)_n (1-x)_n (d)_n}{(\frac{1}{2})_n (1+x)_n (1+x)_n (2-d)_n n!} \\ &= \frac{(1)_0 (\frac{3}{2})_0 (1-x)_0 (1-x)_0 (d)_0}{(\frac{1}{2})_0 (1+x)_0 (1+x)_0 (2-d)_0} + \frac{(1)_1 (\frac{3}{2})_1 (1-x)_1 (1-x)_1 (d)_1}{(\frac{1}{2})_1 (1+x)_1 (1+x)_1 (2-d)_1 1!} \\ &\quad + \frac{(1)_2 (\frac{3}{2})_2 (1-x)_2 (1-x)_2 (d)_2}{(\frac{1}{2})_2 (1+x)_2 (1+x)_2 (2-d)_2 2!} + \dots \end{aligned} \tag{2.1}$$

From equation (1.2) the first term of equation (2.1) becomes 1. i.e.

$$\frac{(1)_0 (\frac{3}{2})_0 (1-x)_0 (1-x)_0 (d)_0}{(\frac{1}{2})_0 (1+x)_0 (1+x)_0 (2-d)_0} = 1 \tag{2.2}$$

Now taking second term of (2.1)

$$\frac{(1)_1 (\frac{3}{2})_1 (1-x)_1 (1-x)_1 (d)_1}{(\frac{1}{2})_1 (1+x)_1 (1+x)_1 (2-d)_1 1!}$$

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On solving with the help of equation (1.2) it becomes

$$\frac{(1)_1 \left(\frac{3}{2}\right)_1 (1-x)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 (1+x)_1 (1+x)_1 (2-d)_1 1!} = \frac{3(x-1)^2 d}{(x+1)^2 (2-d)} \quad (2.3)$$

Similarly from the special case of (1.2) the third term of the equation (2.1) becomes

$$\frac{(1)_2 \left(\frac{3}{2}\right)_2 (1-x)_2 (1-x)_2 (d)_2}{\left(\frac{1}{2}\right)_2 (1+x)_2 (1+x)_2 (2-d)_2 2!} = \frac{5(x-1)^2 (x-2)^2 d (d+1)}{(x+1)^2 (x+2)^2 (2-d)(3-d)} \quad (2.4)$$

Now equation (2.1) becomes due to (2.2), (2.3), (2.4)

$$1 + \frac{3(x-1)^2 d}{(x+1)^2 (2-d)} + \frac{5(x-1)^2 (x-2)^2 d (d+1)}{(x+1)^2 (x+2)^2 (2-d)(3-d)} + \dots \quad (2.5)$$

Now putting the values of a, b, c in the right hand side of the equation (1.8) we obtain

$$\begin{aligned} &= \frac{\Gamma(1+x)\Gamma(1+x)\Gamma(2-d)\Gamma(2x-d)}{\Gamma(2)\Gamma(2x)\Gamma(1+x-d)\Gamma(1+x-d)} \\ &= \frac{[\Gamma(1+x)]^2 \Gamma(2-d)\Gamma(2x-d)}{\Gamma(2x)[\Gamma(1+x-d)]^2} \end{aligned} \quad (2.6)$$

By virtue of equation (1.8), equation (2.5) and (2.6) gives

$$\begin{aligned} &1 + \frac{3(x-1)^2 d}{(x+1)^2 (2-d)} + \frac{5(x-1)^2 (x-2)^2 d (d+1)}{(x+1)^2 (x+2)^2 (2-d)(3-d)} + \dots \\ &= \frac{[\Gamma(1+x)]^2 \Gamma(2-d)\Gamma(2x-d)}{\Gamma(2x)[\Gamma(1+x-d)]^2} \end{aligned} \quad (2.7)$$

Which complete the generalization of equation (1.3).

Now we established generalization form of equation (1.4). We consider $a = b = c = x$ putting the values of a, b, c in equation (1.8) and expand the hypergeometric function as a series

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$$\begin{aligned}
 {}_5F_4 \left(x, \frac{x}{2} + 1, x, x, d; 1 \right) &= \sum_{n=0}^{\infty} \frac{(x)_n (\frac{x}{2} + 1)_n (x)_n (x)_n (d)_n}{(\frac{x}{2})_n (1)_n (1)_n (1+x-d)_n n!} \\
 &= \frac{(x)_0 (\frac{x}{2} + 1)_0 (x)_0 (x)_0 (d)_0}{(\frac{x}{2})_0 (1)_0 (1)_0 (1+x-d)_0} + \frac{(x)_1 (\frac{x}{2} + 1)_1 (x)_1 (x)_1 (d)_1}{(\frac{x}{2})_1 (1)_1 (1)_1 (1+x-d)_1 1!} \\
 &\quad + \frac{(x)_2 (\frac{x}{2} + 1)_2 (x)_2 (x)_2 (d)_2}{(\frac{x}{2})_2 (1)_2 (1)_2 (1+x-d)_2 2!} + \dots
 \end{aligned} \tag{2.8}$$

From equation (1.2) the first term of equation (2.8) becomes 1. i.e.

$$\frac{(x)_0 (\frac{x}{2} + 1)_0 (x)_0 (x)_0 (d)_0}{(\frac{x}{2})_0 (1)_0 (1)_0 (1+x-d)_0} = 1 \tag{2.9}$$

Now taking the second term of (2.8)

$$\frac{(x)_1 (\frac{x}{2} + 1)_1 (x)_1 (x)_1 (d)_1}{(\frac{x}{2})_1 (1)_1 (1)_1 (1+x-d)_1 1!}$$

On solving with the help of the equation (1.2) it becomes

$$\frac{(x)_1 (\frac{x}{2} + 1)_1 (x)_1 (x)_1 (d)_1}{(\frac{x}{2})_1 (1)_1 (1)_1 (1+x-d)_1 1!} = \frac{x^2 (x+2)d}{(1+x-d)} \tag{2.10}$$

Similarly from the special case of (1.2) the third term of the equation (2.8) becomes

$$\frac{(x)_2 (\frac{x}{2} + 1)_2 (x)_2 (x)_2 (d)_2}{(\frac{x}{2})_2 (1)_2 (1)_2 (1+x-d)_2 2!} = \frac{x^2 (x+1)^3 (x+4)d(d+1)}{8(2+x-d)(1+x-d)} \tag{2.11}$$

Now equation (2.8) becomes due to (2.9), (2.10), (2.11)

$$1 + \frac{x^2 (x+2)d}{(1+x-d)} + \frac{x^2 (x+1)^3 (x+4)d(d+1)}{8(2+x-d)(1+x-d)} + \dots \tag{2.12}$$

Putting the values of a, b, c in the right hand side of the equation (1.8) we obtain

$$= \frac{\Gamma(1)\Gamma(1)\Gamma(1+x-d)\Gamma(1+x-x-x-d)}{\Gamma(1+x)\Gamma(1-x)\Gamma(1-d)\Gamma(1-d)}$$

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$$= \frac{\Gamma(1+x-d)\Gamma(1-x-d)}{\Gamma(1+x)\Gamma(1-x)[\Gamma(1-d)]^2} \quad (2.13)$$

By virtue of equation (1.8), an equation (2.12) and (2.13) gives

$$1 + \frac{x^2(x+2)d}{(1+x-d)} + \frac{x^2(x+1)^3(x+4)d(d+1)}{8(2+x-d)(1+x-d)} + \dots$$

$$= \frac{\Gamma(1+x-d)\Gamma(1-x-d)}{\Gamma(1+x)\Gamma(1-x)[\Gamma(1-d)]^2} \quad (2.14)$$

Which complete the generalized form of (1.4).

Further, we established generalization form of equation (1.5). For this, we take $a = 1$, $b = \frac{1}{2}$, $c = 1 - x$. Putting the values of a, b, c in equation (1.8) and expand the hypergeometric function as a series

$${}_5F_4 \left(\begin{matrix} 1, \frac{3}{2}, \frac{1}{2}, 1-x, d \\ \frac{1}{2}, \frac{3}{2}, 1+x, 2-d \end{matrix}; 1 \right) = \sum_{n=0}^{\infty} \frac{(1)_n \left(\frac{3}{2}\right)_n \left(\frac{1}{2}\right)_n (1-x)_n (d)_n}{\left(\frac{1}{2}\right)_n \left(\frac{3}{2}\right)_n (1+x)_n (2-d)_n n!}$$

$$= \frac{(1)_0 \left(\frac{3}{2}\right)_0 \left(\frac{1}{2}\right)_0 (1-x)_0 (d)_0}{\left(\frac{1}{2}\right)_0 \left(\frac{3}{2}\right)_0 (1+x)_0 (2-d)_0} + \frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1+x)_1 (2-d)_1 1!}$$

$$+ \frac{(1)_2 \left(\frac{3}{2}\right)_2 \left(\frac{1}{2}\right)_2 (1-x)_2 (d)_2}{\left(\frac{1}{2}\right)_2 \left(\frac{3}{2}\right)_2 (1+x)_2 (2-d)_2 2!} + \dots \quad (2.15)$$

From equation (1.2) the first term of the equation (2.15) becomes 1. i.e.

$$\frac{(1)_0 \left(\frac{3}{2}\right)_0 \left(\frac{1}{2}\right)_0 (1-x)_0 (d)_0}{\left(\frac{1}{2}\right)_0 \left(\frac{3}{2}\right)_0 (1+x)_0 (2-d)_0} = 1 \quad (2.16)$$

Now taking the second term of (2.15)

$$\frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1+x)_1 (2-d)_1 1!}$$

On solving with the help of equation (1.2) it becomes

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$$\frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1+x)_1 (2-d)_1 1!} = -\frac{(x-1)d}{(x+1)(2-d)} \quad (2.17)$$

Similarly from the special case of equation (1.2) the third term of the equation (2.15) becomes

$$\frac{(1)_2 \left(\frac{3}{2}\right)_2 \left(\frac{1}{2}\right)_2 (1-x)_2 (d)_2}{\left(\frac{1}{2}\right)_2 \left(\frac{3}{2}\right)_2 (1+x)_2 (2-d)_2 2!} = \frac{(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} \quad (2.18)$$

Now equation (2.15) becomes due to (2.16), (2.17), (2.18)

$$1 - \frac{(x-1)d}{(x+1)(2-d)} + \frac{(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} - \dots \quad (2.19)$$

Now putting the values of a, b, c in the right hand side of the equation (1.8) we obtain

$$\begin{aligned} &= \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(1+x)\Gamma(2-d)\Gamma\left(\frac{1}{2}+x-d\right)}{\Gamma(2)\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{3}{2}-d\right)\Gamma(1+x-d)} \\ &= \frac{\sqrt{\pi}}{2} \frac{\Gamma(1+x)\Gamma(2-d)\Gamma\left(\frac{1}{2}+x-d\right)}{\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{3}{2}-d\right)\Gamma(1+x-d)} \end{aligned} \quad (2.20)$$

By virtue of equation (1.8), an equation (2.19) and (2.20) gives

$$\begin{aligned} &1 - \frac{(x-1)d}{(x+1)(2-d)} + \frac{(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} - \dots \\ &= \frac{\sqrt{\pi}}{2} \frac{\Gamma(1+x)\Gamma(2-d)\Gamma\left(\frac{1}{2}+x-d\right)}{\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{3}{2}-d\right)\Gamma(1+x-d)} \end{aligned} \quad (2.21)$$

Which complete the generalization form of equation (1.5).

Again, we find out generalization of equation (1.6). For this, we take $a = 1$, $b = \frac{3}{2}$, $c = 1 - x$. Putting the values of a, b, c in equation (1.8) and expand the hypergeometric function as a series

$${}_5F_4 \left(\begin{matrix} 1, \frac{3}{2}, \frac{3}{2}, 1-x, d \\ \frac{1}{2}, \frac{1}{2}, 1+x, 2-d \end{matrix}; 1 \right) = \sum_{n=0}^{\infty} \frac{(1)_n \left(\frac{3}{2}\right)_n \left(\frac{3}{2}\right)_n (1-x)_n (d)_n}{\left(\frac{1}{2}\right)_n \left(\frac{1}{2}\right)_n (1+x)_n (2-d)_n n!}$$

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$$\begin{aligned}
 &= \frac{(1)_0 \left(\frac{3}{2}\right)_0 \left(\frac{3}{2}\right)_0 (1-x)_0 (d)_0}{\left(\frac{1}{2}\right)_0 \left(\frac{1}{2}\right)_0 (1+x)_0 (2-d)_0} + \frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1+x)_1 (2-d)_1 1!} \\
 &\quad + \frac{(1)_2 \left(\frac{3}{2}\right)_2 \left(\frac{3}{2}\right)_2 (1-x)_2 (d)_2}{\left(\frac{1}{2}\right)_2 \left(\frac{1}{2}\right)_2 (1+x)_2 (2-d)_2 2!} + \dots
 \end{aligned} \tag{2.22}$$

From equation (1.2) the first term of the equation (2.22) becomes 1. i.e.

$$\frac{(1)_0 \left(\frac{3}{2}\right)_0 \left(\frac{3}{2}\right)_0 (1-x)_0 (d)_0}{\left(\frac{1}{2}\right)_0 \left(\frac{1}{2}\right)_0 (1+x)_0 (2-d)_0} = 1 \tag{2.23}$$

Now taking the second term of the equation (2.22)

$$\frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1+x)_1 (2-d)_1 1!}$$

On solving with the help of the equation (1.2) it becomes

$$\frac{(1)_1 \left(\frac{3}{2}\right)_1 \left(\frac{3}{2}\right)_1 (1-x)_1 (d)_1}{\left(\frac{1}{2}\right)_1 \left(\frac{1}{2}\right)_1 (1+x)_1 (2-d)_1 1!} = -\frac{9(x-1)d}{(x+1)(2-d)} \tag{2.24}$$

Similarly from the special case of (1.2) the third term of the equation (2.22) becomes

$$\frac{(1)_2 \left(\frac{3}{2}\right)_2 \left(\frac{3}{2}\right)_2 (1-x)_2 (d)_2}{\left(\frac{1}{2}\right)_2 \left(\frac{1}{2}\right)_2 (1+x)_2 (2-d)_2 2!} = \frac{25(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} \tag{2.25}$$

Now equation (2.22) becomes due to (2.23), (2.24), (2.25)

$$1 - \frac{9(x-1)d}{(x+1)(2-d)} + \frac{25(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} - \dots \tag{2.26}$$

Putting the values of a, b, c in the right hand side of the equation (1.8) we obtain

$$\begin{aligned}
 &= \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(1+x)\Gamma(2-d)\Gamma\left(x-d-\frac{1}{2}\right)}{\Gamma(2)\Gamma\left(x-\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-d\right)\Gamma(1+x-d)} \\
 &= \frac{\sqrt{\pi}\Gamma(1+x)\Gamma(2-d)\Gamma\left(x-d-\frac{1}{2}\right)}{\Gamma\left(x-\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-d\right)\Gamma(1+x-d)}
 \end{aligned} \tag{2.27}$$

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By virtue of the equation (1.8), an equation (2.26) and (2.27) gives

$$1 - \frac{9(x-1)d}{(x+1)(2-d)} + \frac{25(x-2)(x-1)d(d+1)}{(x+2)(x+1)(3-d)(2-d)} - \dots$$

$$= \frac{\sqrt{\pi}\Gamma(1+x)\Gamma(2-d)\Gamma(x-d-\frac{1}{2})}{\Gamma(x-\frac{1}{2})\Gamma(\frac{1}{2}-d)\Gamma(1+x-d)} \quad (2.28)$$

Which complete the generalization of the equation (1.6).

2. SPECIAL CASES

- 2.1 Putting $d = \frac{1}{2}$ in the equation (2.7), we obtain equation (1.3).
- 2.2 Putting $d = \frac{x}{2}$ in the equation (2.14), we obtain equation (1.4).
- 2.3 Putting $d = \frac{1}{2}$ in the equation (2.21), we obtain equation (1.5).
- 2.4 Putting $d = \frac{1}{2}$ in the equation (2.28), we obtain equation (1.6).

III. CONCLUSION

Equation (2.7) is the generalization form of the equation (1.3), equation (2.14) is the generalization form of the equation (1.4), and equation (2.21) and (2.28) are the generalization form of the equations (1.5) and (1.6) respectively. Also we give summation formulas due to Ramanujan by putting the parameters in the equations (2.7), (2.14), (2.21), (2.27) and define as special cases.

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