

Mean Time to Recruitment for a Multigrade Manpower System When Wastages Are Correlated

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ABSTRACT: In this paper a multi graded organization in which depletion of man powers occur due to its policy decisions taken by the organization is considered. Four cases are constructed by taking exponential thresholds for the loss of man powers in each grade, where the loss of man powers (wastages) are correlated and inter decision times form i) an ordinary renewal process ii) an order statistics iii) a geometric process iv) correlated. Mean time to recruitment is obtained using an univariate CUM policy of recruitment (i.e)“The organization survives iff at least r , ($1 \leq r \leq n$)out of n-grades survives in the sense that threshold crossing has not take place in these grades”,. The influence of the nodal parameters on the system characteristics is studied and relevant conclusions are presented.

KEYWORDS: loss of man powers, inter decision time, univariate, CUM policy, exponential thresholds

I. INTRODUCTION

Exits of personal which is in other words known as wastage, is an important aspects in the study of manpower planning. Many models have been discussed using different types of wastages and also different types of distribution for the loss of man powers, the thresholds and inter decision times. Such models are seen in [1] and [2]. In [3],[4],[5] and [6] the authors have obtained the mean time to recruitment in a two grade manpower system based on order statistics by assuming different distribution for thresholds. In [8] for a two grade manpower system with two types of decisions when the wastages form a geometric process is obtained. The problem of time to recruitment is studied by several authors for the organizations consisting of single grade/two grade/ three grades. More specifically for a two grade system, in all the earlier work, the threshold for the organization is minimum or maximum or sum of the thresholds for the loss of manpower in each grades, no attempt has been made so far to design a comprehensive recruitment policy for a system with two or three grades. In [10],[11]&[12] a new design for a comprehensive univariate CUM recruitment policy of manpower system is used with n grades in order to bring results proved independently for maximum, minimum model as a special case. In all previous work, the problem of time to recruitment is studied for only an organization consisting of atmost three grades. In[11],[12] author has worked on this comprehensive univariate policy when wastages form ordinary renewal process and inter decision time form geometric and order statistics. In this paper an organization with n-grades is considered and the mean time to recruitment are obtained using an appropriate univariate CUM policy of recruitment (i.e)“The organization survives iff atleast r , ($1 \leq r \leq n$)out of n-grades survives in the sense that threshold crossing has not take place in these grades”, when wastages form an order statistics.

II. MODEL DESCRIPTION AND ASSUMPTIONS

- 1) An organization having n grades in which decisions are taken at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man

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- hours to the organization if a person quits.
- 2) It is assumed that the loss of man hours is linear and cumulative.
 - 3) The loss of manpower at any decision epoch forms a sequence of independent and identically distributed random variables which form order statistics.
 - 4) The inter-decision times are independent and identically distributed random variables.
 - 5) The loss of manpower process and the process of inter-decision times are statistically independent.
 - 6) The thresholds for the n-grades are independent and identically distributed exponential random variable.
 - 7) Univariate CUM policy of recruitment: "The organization survives iff atleast r , ($1 \leq r \leq n$) out of n-grades survives in the sense that threshold crossing has not take place in these grades"

Notations

X_i : The continuous random variable denoting the amount of depletion caused to the organization due to the exit of persons corresponding to the j^{th} decision, $j=1,2,3,\dots$ and X_i 's are exchangeable and constantly correlated exponential random variables

$G(x)$: distribution function of x such that $G(x) = 1 - e^{-cx}$, $g(x)$: probability density function.

$G_k(\cdot)$: The distribution function of $S_k = \sum_{i=1}^k X_i$

$g_k(\cdot)$ its probability density function.

R : be the correlation between X_i and X_j where $i \neq j$, $\rho = \frac{1-R}{c}$

U_i : $i = 1,2,3,\dots$ the inter decision time between $(i-1)^{th}$ and i^{th} decision.

$F(\cdot)$ [$f(\cdot)$]: distribution (density) function.

$F_k(\cdot)$, $f_k(\cdot)$: The distribution (density) function of $\sum_{i=1}^k U_i$.

$V_k(t)$: The probability that there are exactly k decision making epoch in $(0, t]$.

$N(t)$: the number of policy decisions.

$V_k(t)$: probability that there are exactly k decisions taken in $(0, t]$.

Y_j : The continuous random variable denoting the thresholds for the j^{th} grade.

Y : The continuous random variable denoting the thresholds for the organization.

$H(\cdot)$: the distribution function of Y .

T_j : Time taken for threshold crossing in the j^{th} grade, $j=1,2,3,\dots,n$.

T : Time to recruitment of the organization

$E(T)$: mean time to recruitment.

III. MAIN RESULT

The survival function of the time to recruitment is given by

$$P(T > t) = \sum_{k=0}^{\infty} P(\text{Exactly } k \text{ decision epoch}(0, t] \text{ and the threshold and level } Y \text{ is not crossed}$$

by the total loss of manhours in these k decisions in atleast r grades)

$$i. e. P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^k X_i < Y\right)$$

(1)

Conditioning upon Y and using law of total probability

$$\begin{aligned} P\left(\sum_{i=1}^k X_i < Y\right) &= \int_0^{\infty} P\left[\sum_{i=1}^k x_i < Y / Y = y\right] h(y) dy \\ &= \int_0^{\infty} P\left(\sum_{i=1}^k x_i < y\right) h(y) dy = \int_0^{\infty} G_k(y) h(y) dy \end{aligned}$$

(2)

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The cumulative distribution function $G_k(\cdot)$ is given by Gurland(1955) as

$$G_k(y) = \left(\frac{1-R}{1-R+kR} \right) \sum_{j=0}^{\infty} \left(\frac{kR}{1-R+kR} \right)^j \frac{\varphi(k+j, \frac{y}{v})}{(k+j-1)!}$$

where $v = \frac{1-R}{c}$ and $\varphi(k+j, \frac{y}{v}) = \int_0^{\frac{y}{v}} e^{-\epsilon} \epsilon^{k+j-1} d\epsilon$.

(3)

$$1 - H(y) = \sum_{i=r}^n nC_i [e^{-\theta y}]^i [1 - e^{-\theta y}]^{n-i}$$

$$H(y) = 1 - \sum_{i=r}^n nC_i [y]^i [1 - e^{-\theta y}]^{n-i}$$

$$= 1 - \sum_{i=r}^n nC_i e^{-i\theta y} [1 - (n-i)C_1 e^{-\theta y} + (n-i)C_2 e^{-(i+2)\theta y} + \dots + (-1)^{n-i} e^{-n\theta y}]$$

$$= 1 - \sum_{i=r}^n nC_i [e^{-i\theta y} - (n-i)C_1 e^{-(i+1)\theta y} + (n-i)C_2 e^{-(i+2)\theta y} + \dots - (-1)^{n-i} e^{-n\theta y}]$$

$$h(y) = - \sum_{i=r}^n nC_i [-i\theta e^{-i\theta y} + (n-i)C_1 (i+1)\theta e^{-(i+1)\theta y} - (n-i)C_2 (i+2)\theta e^{-(i+2)\theta y} + \dots - (-1)^{n-i+1} n\theta e^{-n\theta y}]$$

$$h(y) = \sum_{i=r}^n nC_i [i\theta e^{-i\theta y} - (n-i)C_1 (i+1)\theta e^{-(i+1)\theta y} + (n-i)C_2 (i+2)\theta e^{-(i+2)\theta y} - \dots - (-1)^{n-i} n\theta e^{-n\theta y}]$$

(4)

Using (4) in (2)

$$P\left(\sum_{i=1}^k X_i < Y\right) = \int_0^{\infty} G_k(y) h(y) dy$$

$$= \int_0^{\infty} G_k(y) \sum_{i=r}^n nC_i [i\theta e^{-i\theta y} - (n-i)C_1 (i+1)\theta e^{-(i+1)\theta y} + (n-i)C_2 (i+2)\theta e^{-(i+2)\theta y} - \dots - (-1)^{n-i} n\theta e^{-n\theta y}] dy$$

(5)

Consider

$$\int_0^{\infty} G_k(y) e^{-\theta y} dy = \frac{1}{\theta} \left(\frac{1-R}{1-R+kR} \right) \sum_{j=0}^{\infty} \left(\frac{kR}{1-R+kR} \right)^j \frac{1}{(k+j-1)!} \int_0^{\infty} \varphi(k+j, \frac{y}{v}) e^{-\theta y} dy$$

Simplifying

$$= \frac{1}{\theta} \frac{1}{(1+v\theta)^k} \left[1 + \frac{kRv\theta}{(1-R)(1+v\theta)} \right]^{-1}$$

$$= \frac{1}{\theta} \left[(1+v\theta)^k \left[1 + \frac{kRv\theta}{(1-R)(1+v\theta)} \right] \right]^{-1} = \frac{1}{\theta} W(k, \theta)$$

(6)

Where $W(k, \theta) = \left[(1+v\theta)^k \left[1 + \frac{kRv\theta}{(1-R)(1+v\theta)} \right] \right]^{-1}$

(7)

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Using (6) in (5)

$$P\left(\sum_{i=1}^k X_i < Y\right) = \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (8)$$

From renewal theory $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$

$$P(T > t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]$$

$$L(t) = 1 - P(T > t)$$

$$L(t) = 1 - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (9)$$

$$l(t) = - \sum_{k=0}^{\infty} [f_k(t) - f_{k+1}(t)] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (10)$$

$$\bar{l}(s) = - \sum_{k=0}^{\infty} [\bar{f}_k(s) - \bar{f}_{k+1}(s)] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (11)$$

We now obtain the analytical result for variance of time to recruitment in closed form for four different cases on inter-decision times $\{U_i\}_1^{\infty}$

Case(i) $\{U_i\}_{i=1}^{\infty}$ form an ordinary renewal process.

The inter decision times are assumed to be independent and identically distributed hyper exponential random variable with probability density function $f(t) = pe^{-\lambda_h t} + qe^{-\lambda_l t}$, $p + q = 1$. Where λ_h, λ_l are high and low attrition rate, p, q are the proportion of decisions having high and low attrition.

$$\begin{aligned} \bar{f}(s) &= \frac{p\lambda_h}{\lambda_h + s} + \frac{q\lambda_l}{\lambda_l + s}, \bar{f}(0) = \frac{p\lambda_h}{\lambda_h} + \frac{q\lambda_l}{\lambda_l} = p + q = 1, \\ \frac{d}{ds}(\bar{f}(s)) &= \frac{-p\lambda_h}{(\lambda_h + s)^2} + \frac{-q\lambda_l}{(\lambda_l + s)^2} \\ \left(\frac{d}{ds}(\bar{f}(s))\right)_{s=0} &= -\left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l}\right), \bar{f}(0) = 1 \end{aligned} \quad (12)$$

From equation (11)

$$\frac{d}{ds}(\bar{l}(s)) = - \sum_{k=0}^{\infty} [k[\bar{f}(s)]^{k-1} - (k+1)[\bar{f}(s)]^k] \frac{d}{ds}(\bar{f}(s)) \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]$$

Since $\bar{f}(0) = 1$

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$$\left[\frac{d}{ds} \bar{l}(s) \right]_{s=0} = \sum_{k=0}^{\infty} \left[\frac{d}{ds} (\bar{f}(s)) \right]_{s=0} \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (13)$$

Using (12) in (13)

$$\left[\frac{d}{ds} (\bar{l}(s)) \right]_{s=0} = - \sum_{k=0}^{\infty} \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]$$

The mean time to recruitment is

$$E(T) = - \left[\frac{d}{ds} (\bar{l}(s)) \right]_{s=0} = \sum_{k=0}^{\infty} \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (14)$$

Equation (14) gives the mean time to recruitment together with (7)

Case(ii) $\{U_i\}_{i=1}^{\infty}$ form an order statistics

Consider the population $\{U_i\}_{i=1}^{\infty}$ of independent and identically distributed interdecision times with hyper exponential cumulative distribution $F(t) = 1 - pe^{-\lambda_h t} - qe^{-\lambda_l t}$ and the corresponding density function $f(t)$. Assume that $\{U_i\}_{i=1}^m$ be a sample of size m selected from this population. Let U_1, U_2, \dots, U_m be the order statistics corresponding to this sample with respective probability density function $f_{U_1}, f_{U_2}, \dots, f_{U_m}$. U_1 is the first order statistics and U_m is the m^{th} order statistics such that $U_1 \leq U_2 \leq \dots \leq U_m$ and hence not independent.

The probability density function of j th order statistics is given by [Sheldon M. Ross 2005]

$$f_{U_j}(t) = j \binom{m}{j} [F(t)]^{j-1} f(t) [1 - F(t)]^{m-j}, j = 1, 2, 3 \dots m \quad (15)$$

Therefore the probability density function of U_1 and U_m are given by

$$f_{U_1}(t) = m f(t) [1 - F(t)]^{m-1} \quad (16)$$

$$f_{U_m}(t) = m f(t) [F(t)]^{m-1} \quad (17)$$

SubCase(i) If $f(t) = f_{U_1}(t)$

$$\bar{l}(s) = - \sum_{k=0}^{\infty} \left[[\bar{f}_{U(1)}(s)]^k - [\bar{f}_{U(1)}(s)]^{k+1} \right] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (18)$$

$$\frac{d}{ds} (\bar{l}(s)) = - \sum_{k=0}^{\infty} \left[k [\bar{f}_{U(1)}(s)]^{k-1} - (k+1) [\bar{f}_{U(1)}(s)]^k \right] \frac{d}{ds} (\bar{f}(s)) \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (19)$$

Since $\bar{f}(0) = 1$

$$\left[\frac{d}{ds} (\bar{l}(s)) \right]_{s=0} = \sum_{k=0}^{\infty} \left[\frac{d}{ds} (\bar{f}_{U(1)}(s)) \right]_{s=0} \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \quad (20)$$

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$$\begin{aligned}\bar{f}(s) &= \bar{f}_{U_1}(s) = \int_0^\infty e^{-st} mf(t)[1 - F(t)]^{m-1} dt \\ &= \int_0^\infty e^{-st} [-d(1 - F(t))]^m \\ \bar{f}(s) &= \int_0^\infty e^{-st} [-d(pe^{-\lambda_h t} + qe^{-\lambda_l t})^m]\end{aligned}$$

By using binomial expansion

$$\begin{aligned}\bar{f}(s) &= \int_0^\infty e^{-st} [-d(\sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} e^{-(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)t})] \\ &= \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} (\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m) \left[\frac{1}{(s + \lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right] \\ \frac{d}{ds}(\bar{f}(s)) &= \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} (\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m) \left[\frac{-1}{(s + \lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)^2} \right] \\ \left[\frac{d}{ds}(\bar{f}(s)) \right]_{s=0} &= - \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right]\end{aligned}\tag{21}$$

Using (20) and (21)

$$\begin{aligned}E(T) &= - \left[\frac{d}{ds}(\bar{l}(s)) \right]_{s=0} \\ &= \sum_{k=0}^\infty \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right] \times \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) \\ &\quad + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]\end{aligned}\tag{22}$$

Equation (22) along with (7), gives the mean time to recruitment.

Subcase(ii)

$$\begin{aligned}f(t) &= f_{U_m}(t) = mf(t)[F(t)]^{m-1} \\ \bar{l}(s) &= - \sum_{k=0}^\infty \left[[\bar{f}_{U(m)}(s)]^k - [\bar{f}_{U(m)}(s)]^{k+1} \right] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + \\ &\quad (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]\end{aligned}\tag{23}$$

$$\begin{aligned}\frac{d}{ds}(\bar{l}(s)) &= - \sum_{k=0}^\infty \left[k[\bar{f}_{U(m)}(s)]^{k-1} - (k+1)[\bar{f}_{U(m)}(s)]^k \right] \frac{d}{ds}(\bar{f}_{U(m)}(s)) \times \\ &\quad \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]\end{aligned}\tag{24}$$

Since $\bar{f}(0) = 1$

$$\begin{aligned}\left[\frac{d}{ds}(\bar{l}(s)) \right]_{s=0} &= \sum_{k=0}^\infty \left[\frac{d}{ds}(\bar{f}_{U(m)}(s)) \right]_{s=0} \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) \\ &\quad - \dots (-1)^{n-i} W(k, n\theta)]\end{aligned}\tag{25}$$

$$\bar{f}(s) = \bar{f}_{U_m}(s) = \int_0^\infty e^{-st} mf(t)[F(t)]^{m-1} dt$$

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$$\begin{aligned}
 &= \int_0^\infty e^{-st} d(F(t))^m = \int_0^\infty e^{-st} [d(1 - pe^{-\lambda_h t} - qe^{-\lambda_l t})]^m \\
 &= \int_0^\infty e^{-st} d\left(\sum_{r_1=0}^m \binom{m}{r_1} (-1)^{m-r_1} 1^{r_1} (pe^{-\lambda_h t} + qe^{-\lambda_l t})^{m-r_1}\right)
 \end{aligned} \tag{26}$$

$$\bar{f}(s) = \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \times \frac{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}{(s + \lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)} \tag{27}$$

$$\begin{aligned}
 \left[\frac{d}{ds}(\bar{f}(s))\right]_{s=0} &= - \left[\sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}{(s + \lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)^2} \right]_{s=0} \\
 \left[\frac{d}{ds}(\bar{f}(s))\right]_{s=0} &= \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)}
 \end{aligned} \tag{28}$$

Using (25) and (28)

$$\begin{aligned}
 E(T) &= - \left[\frac{d}{ds}(\bar{l}(s))\right]_{s=0} \\
 &= \sum_{k=0}^\infty \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_l m)} \sum_{i=r}^n nC_i [W(k, i\theta) \\
 &\quad - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^n W(k, n\theta)]
 \end{aligned} \tag{29}$$

Equation (29) gives the mean time to recruitment together with (7)

Case(iii) $\{U_i\}_{i=1}^\infty$ form an geometric process

Assume that the inter decision times $U_i, i = 1, 2, 3 \dots$ form a geometric process with rate $b, (b > 0)$. It is assumed that the probability density function of U_1 is hyper exponential density function $f(t) = p\lambda_h e^{-\lambda_h t} + q\lambda_l e^{-\lambda_l t}, p + q = 1$.

$$\left. \begin{aligned}
 \bar{f}(s) &= \frac{p\lambda_h}{\lambda_h + s} + \frac{q\lambda_l}{\lambda_l + s} \\
 \bar{f}(0) &= \frac{p\lambda_h}{\lambda_h} + \frac{q\lambda_l}{\lambda_l} = p + q = 1 \\
 \frac{d}{ds}(\bar{f}(s)) &= \frac{-p\lambda_h}{(\lambda_h + s)^2} + \frac{-q\lambda_l}{(\lambda_l + s)^2} \\
 \left(\frac{d}{ds}(\bar{f}(s))\right)_{s=0} &= - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l}\right)
 \end{aligned} \right\} \tag{30}$$

$$\begin{aligned}
 \left[\frac{d}{ds}(\bar{l}(s))\right]_{s=0} &= - \sum_{k=0}^\infty \left[\left[\frac{d}{ds}(\bar{f}_k(s))\right]_{s=0} - \left[\frac{d}{ds}(\bar{f}_{k+1}(s))\right]_{s=0} \right] \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) \\
 &\quad + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]
 \end{aligned} \tag{31}$$

$$\bar{f}_k(s) = \prod_{i=1}^k \bar{f}\left(\frac{s}{b^{i-1}}\right)$$

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$$\begin{aligned} \frac{d}{ds}(\bar{f}_k(s)) &= \frac{d}{ds} \left(\prod_{i=1}^k \bar{f} \left(\frac{s}{b^{i-1}} \right) \right) = \frac{d}{ds} \left(\bar{f}(s) \times \bar{f} \left(\frac{s}{b} \right) \times \bar{f} \left(\frac{s}{b^2} \right) \times \dots \times \bar{f} \left(\frac{s}{b^{k-1}} \right) \right) \\ \left(\frac{d}{ds}(\bar{f}_k(s)) \right)_{s=0} &= \left(\frac{d}{ds} \left(\prod_{i=1}^k \bar{f} \left(\frac{s}{b^{i-1}} \right) \right) \right)_{s=0} \\ &= \left(\frac{d}{ds}(\bar{f}(s)) \right)_{s=0} + \frac{1}{b} \left(\frac{d}{ds}(\bar{f}(s)) \right)_{s=0} + \frac{1}{b^2} \left(\frac{d}{ds}(\bar{f}(s)) \right)_{s=0} + \dots + \frac{1}{b^{k-1}} \left(\frac{d}{ds}(\bar{f}(s)) \right)_{s=0} \\ &= \sum_{i=1}^k \frac{1}{b^{i-1}} \times \left(\frac{d}{ds}(\bar{f}(s)) \right)_{s=0} = - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{i=1}^k \frac{1}{b^{i-1}} \end{aligned} \tag{32}$$

Consider

$$\begin{aligned} \sum_{k=0}^{\infty} \left[\left[\frac{d}{ds}(\bar{f}_k(s)) \right]_{s=0} - \left[\frac{d}{ds}(\bar{f}_{k+1}(s)) \right]_{s=0} \right] &= \sum_{k=0}^{\infty} - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \left[\sum_{i=1}^k \frac{1}{b^{i-1}} - \sum_{i=1}^{k+1} \frac{1}{b^{i-1}} \right] \\ &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{k=0}^{\infty} \frac{1}{b^k} \\ \left[\frac{d}{ds}(\bar{l}(s)) \right]_{s=0} &= - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{k=0}^{\infty} \frac{1}{b^k} \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) \\ &\quad + (n-i)C_2 W(k, (i+2)\theta) - \dots - (-1)^{n-i} W(k, n\theta)] \end{aligned} \tag{33}$$

$$\begin{aligned} E(T) &= - \left[\frac{d}{ds}(\bar{l}(s)) \right]_{s=0} \\ &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{k=0}^{\infty} \frac{1}{b^k} \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) \\ &\quad - \dots - (-1)^{n-i} W(k, n\theta)] \end{aligned} \tag{34}$$

Equation (34) gives the mean time to recruitment together with (7)

Case(iv) When U_i 's are correlated.

The inter decision times are assumed to be exchangeable and constantly correlated exponential random variables with mean $\frac{1}{\mu}$ ($\mu > 0$). Let ρ be the constant correlation between U_i and $U_j, i \neq j$.

By taking Laplace Stieljes transform both side using

$$\begin{aligned} \bar{L}(s) &= - \sum_{k=1}^{\infty} [\bar{F}_k(s) - \bar{F}_{k+1}(s)] \times \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) \\ &\quad - \dots - (-1)^{n-i} W(k, n\theta)] \end{aligned} \tag{35}$$

The cumulative distribution function of the partial sum $U_1 + U_2 + \dots + U_k$ is given by Gurland(1955) as

$$F_k(t) = \left(\frac{1-\rho}{1-\rho+k\rho} \right) \sum_{j=0}^{\infty} \left(\frac{k\rho}{1-\rho+k\rho} \right)^j \frac{\varphi(k+j, \frac{t}{u})}{(k+j-1)!}$$

where $u = \frac{1-\rho}{\mu}$ and $\varphi(k+j, \frac{t}{u}) = \int_0^{\frac{t}{u}} e^{-\epsilon} \epsilon^{k+j-1} d\epsilon$.

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$$\bar{F}_k(s) = \left(\frac{1-\rho}{1-\rho+k\rho}\right) \sum_{j=0}^{\infty} \left(\frac{k\rho}{1-\rho+k\rho}\right)^j \frac{1}{(k+j-1)!} \int_0^{\infty} \varphi\left(k+j, \frac{t}{u}\right) e^{-st} dt$$

simplifying

$$\bar{F}_k(s) = \frac{(1-\rho)m^k}{1-\rho+k\rho-k\rho m} \text{ where } m = \frac{1}{1+us}$$

$$\left[\frac{d}{ds}[\bar{F}_k(s)]\right]_{s=0} = (1-\rho) \left[\frac{(1-\rho)(-ku) - k\rho u}{(1-\rho)^2}\right] = \frac{-ku}{(1-\rho)}$$

$$\left[\frac{d}{ds}[\bar{F}_k(s)]\right]_{s=0} - \left[\frac{d}{ds}[\bar{F}_{k+1}(s)]\right]_{s=0} = \frac{-ku}{(1-\rho)} + \frac{(k+1)u}{(1-\rho)} = \frac{u}{(1-\rho)} \tag{36}$$

The mean time to recruitment is

$$E(T) = \sum_{k=0}^{\infty} \left[\left[\frac{d}{ds}[\bar{F}_k(s)]\right]_{s=0} - \left[\frac{d}{ds}[\bar{F}_{k+1}(s)]\right]_{s=0} \right] \times \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)]$$

$$\sum_{k=0}^{\infty} \left[\frac{u}{1-\rho} \right] \times \sum_{i=r}^n nC_i [W(k, i\theta) - (n-i)C_1 W(k, (i+1)\theta) + (n-i)C_2 W(k, (i+2)\theta) - \dots (-1)^{n-i} W(k, n\theta)] \tag{37}$$

Equation (37) gives the mean time to recruitment together with (7)

NUMERICAL ILLUSTRATION

The behavior of the performance measure due to the change in parameter is analyzed numerically for different values of n and r.

Case(i)

Sub Case (i) n=3,r=1

From (14) the mean time to recruitment is given by,

$$E(T) = \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l}\right) \sum_{k=0}^{\infty} [w(k, 3\theta) - 3w(k, 2\theta) + 3w(k, \theta)]$$

Sub Case (ii) n=3,r=2

From (14) the mean time to recruitment is given by

$$E(T) = \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l}\right) \sum_{k=0}^{\infty} [3w(k, 2\theta) - 2w(k, 3\theta)]$$

Sub Case (ii) n=3,r=3

From (14) the mean time to recruitment is given by

$$E(T) = \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l}\right) \sum_{k=0}^{\infty} [w(k, 3\theta)]$$

Case(ii) $f(t) = f_{U_1}(t)$

Sub Case (i) n=3,r=1

From (22) the mean time to recruitment is given by,

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$$E(T) = \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right] \sum_{k=0}^{\infty} [w(k, 3\theta) - 3w(k, 2\theta) + 3w(k, \theta)]$$

Sub Case (ii) n=3,r=2

From (22) the mean time to recruitment is given by

$$E(T) = \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right] \sum_{k=0}^{\infty} [3w(k, 2\theta) - 2w(k, 3\theta)]$$

Sub Case (iii) n=3,r=3

From (22) the mean time to recruitment is given by

$$E(T) = \sum_{r_1=0}^m \binom{m}{r_1} p^{r_1} q^{m-r_1} \left[\frac{1}{(\lambda_h r_1 - \lambda_l r_1 + \lambda_1 m)} \right] \sum_{k=0}^{\infty} [w(k, 3\theta)]$$

$f(t) = f_{U_m}(t)$

Sub Case (i) n=3,r=1

From (29) the mean time to recruitment is given by,

$$E(T) = \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_1 m)} \sum_{k=0}^{\infty} [w(k, 3\theta) - 3w(k, 2\theta) + 3w(k, \theta)]$$

Sub Case (ii) n=3,r=2

From (29) the mean time to recruitment is given by

$$E(T) = \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_1 m)} \sum_{k=0}^{\infty} [3w(k, 2\theta) - 2w(k, 3\theta)]$$

Sub Case (iii) n=3,r=3

From (29) the mean time to recruitment is given by

$$E(T) = \sum_{r_1=0}^m \sum_{r_2=0}^{m-r_1} (-1)^{m-r_1+1} \frac{m!}{r_1! r_2! (m-r_1-r_2)!} p^{r_2} q^{m-r_1-r_2} \frac{1}{(\lambda_h r_2 - \lambda_l r_2 - \lambda_l r_1 + \lambda_1 m)} \sum_{k=0}^{\infty} [w(k, 3\theta)]$$

The influence of parameters on the performance measures namely the mean time for recruitment is studied numerically. In the following tables these performance measures are calculated by varying the parameter 'ρ', 'b' and 'R' one at a time and taking the parameters p=0.3,q=0.7, m=3, v=0.5, θ = 0.6 λ_h=0.5 λ_l=0.6.

Case(i) Table.1 Effect of 'R' on the performance measures E[T]

R	SubCase(i)	SubCase(ii)	SubCase(iii)
0.3	7.5652	5.0502	3.1579
0.4	7.1322	4.7124	2.9931
0.5	6.6344	4.3537	2.8233
0.6	6.0545	3.9660	2.6456
0.7	5.3635	3.5374	2.4564

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Case(ii) Table 2 : Effect of ‘R’ on the performance measures E[T]

$f(t) = f_{U(1)}(t)$			
R	SubCase(i)	SubCase(ii)	SubCase(iii)
0.3	2.5098	1.6754	1.0476
0.4	2.3661	1.5633	0.9930
0.5	2.2010	1.4443	0.9366
0.6	2.0086	1.3157	0.8777
0.7	1.7793	1.1735	0.8149
$f(t) = f_{U(m)}(t)$			
0.3	13.8985	9.2780	5.8016
0.4	13.1029	8.6574	5.4988
0.5	12.1885	7.9984	5.1869
0.6	11.1231	7.2861	4.8604
0.7	9.8536	6.4988	4.5128

- It is observed from tables 1 to 2, that if ‘R’, the correlation factor for wastage increases, the mean time to recruitment decreases
Similarly for other cases , numerical illustrations and observation can be found.

IV. CONCLUSION

From the present work we can study about two grade and three grade manpower system. This work also can be extended in two sources of depletion. Numerical values can be obtained by varying n,r.

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