

On \check{g} -closed Sets in Ideal Topological Spaces

T.Indira¹, S.Vijayalakshmi²

Associate Professor, PG & Research Department of Mathematics, Seethalakshmi Ramaswami College, Trichy, India¹

Research Scholar, PG & Research Department of Mathematics, Seethalakshmi Ramaswami College, Trichy, India²

ABSTRACT: The purpose of this paper is to introduce \check{g} -closed sets, $I_{\check{g}}$ -continuous, $I_{\check{g}}$ -irresolute, strongly $I_{\check{g}}$ -continuous maps and perfectly $I_{\check{g}}$ -continuous maps in ideal topological spaces and to present its characteristics. Also it aims to introduce $T_{I_{\check{g}}}$ -space and $T_{I_{\check{g}}}^*$ -space in ideal topological spaces.

KEYWORDS: \check{g} -closed, $I_{\check{g}}$ -closed, $I_{\check{g}}$ -continuous, $I_{\check{g}}$ -irresolute, $T_{I_{\check{g}}}$ -space, $T_{I_{\check{g}}}^*$ -space, strongly $I_{\check{g}}$ -continuous maps and perfectly $I_{\check{g}}$ -continuous maps.

I. INTRODUCTION AND PRELIMINARIES

An ideal is triplets (X, τ, I) where τ is topology and I is ideal which satisfies (i) $A \in I$ and $B \subset A \Rightarrow B \in I$ and (ii) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$. An ideal topological space (X, τ, I) with an ideal I on X and it is denoted by (X, τ, I) . For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \notin I \text{ for every neighbourhood } U \text{ of } x\}$ is called the local function of A with respect to I and τ [4]. We simply write A^* instead of $A^*(I)$ to be brief. For every ideal topological space (X, τ, I) , there exists a topology $\tau^*(I)$, finer than τ , generated by $\beta(I, \tau) = \{U - i : U \in \tau \text{ and } i \in I\}$, but in general $\beta(I, \tau)$ is not always a topology. Additionally, $cl^*(A) = A \cup A^*$ defines a kuratowski closure operator for $\tau^*(I)$. Given a space (X, τ, I) and $A \subset X$, A is called I -open if $A \subset int(A^*)$ and a subset K is called I -closed if its complement is I -open.[4] A subset A of an ideal space (X, τ, I) is $*$ -closed[3] (resp. $*$ -dense in itself[2]) if $A^* \subseteq A$ (resp. $A \subseteq A^*$). An ideal I is said to be codense [2] or τ -boundry if $\tau \cap I = \{\emptyset\}$. An ideal I is said to be completely codense[2] if $PO(X) \cap I = \{\emptyset\}$, where $PO(X)$ is the family of all pre-open sets in (X, τ) .

In this paper we extend some basic concepts of topological spaces to ideal topological spaces.

Definition 1.1: A subset A of a space (X, τ) is said to

- (1) g -closed[6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of g -closed set is called g -open.
- (2) \hat{g} -closed set[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X . The complement of \hat{g} -closed set is called \hat{g} -open.
- (3) sg -closed set[1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X . The complement of sg -closed set is called sg -open.
- (4) gs -closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of gs -closed set is called gs -closed.
- (5) \check{g} -closed set[8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg -open in X . The complement of \check{g} -closed set is called \check{g} -open.

Lemma 1.2: Let (X, τ, I) be an ideal topological space and A, B subsets of X . Then the following properties hold:[3]

1. $A \subseteq B \Rightarrow A^* \subseteq B^*$,
2. $A^* = cl(A^*) \subseteq cl(A)$,
3. $(A^*)^* \subseteq A^*$,
4. $(A \cup B)^* = A^* \cup B^*$,
5. $(A \cap B)^* \subseteq A^* \cap B^*$.

Lemma 1.3: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. If $A \subseteq A^*$, then $A^* = cl(A^*) = cl(A) = cl^*(A)$. [3]

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Lemma 1.4: If (X, τ) is a topological space, then every closed set is sg-closed but not conversely.[6]

Lemma 1.5: Every g-closed set is $I_{\tilde{g}}$ -closed but not conversely.[6]

Definition 1.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) Strongly continuous [5] if $f^{-1}(V)$ is both open and closed in X for each subset V in Y.
- (2) Perfectly continuous [5] if $f^{-1}(V)$ is both open and closed in X for each open set V in Y.

Definition 1.4: A topological space X is called $T_{1/2}$ -space [5] if every g-closed set of X is closed in X.

II. $I_{\tilde{g}}$ -CLOSED SETS

Definition 2.1: A subset A of an ideal topological space (X, τ, I) is said to be

- (1) $I_{\tilde{g}}$ -closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is sg-open.
- (2) $I_{\tilde{g}}$ -open if its complement is $I_{\tilde{g}}$ -closed.

Theorem 2.2: Every closed set in (X, τ, I) is $I_{\tilde{g}}$ -closed set.

Example 2.3: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$ and $I = \{\emptyset\}$. Then $I_{\tilde{g}}$ -closed sets are $X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}$ and closed sets are $X, \emptyset, \{c\}, \{a, b\}$. It is clear that $\{a\}$ is $I_{\tilde{g}}$ -closed but it is not closed.

Theorem 2.4: Union of two closed sets is $I_{\tilde{g}}$ -closed.

Theorem 2.5: If (X, τ, I) is any ideal topological space and $A \subseteq X$, then every $I_{\tilde{g}}$ -closed set is $I_{\tilde{g}}$ -closed set but not conversely.

Example 2.6: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}\}$ and $I = \{\emptyset\}$. It is clear that $\{a\}$ is $I_{\tilde{g}}$ -closed but it is not $I_{\tilde{g}}$ -closed.

Theorem 2.7: If (X, τ, I) is any ideal topological space and $A \subseteq X$, then the following are equivalent.

- (1) A is $I_{\tilde{g}}$ -closed.
- (2) $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in X.

Theorem 2.8: Every *-closed set is $I_{\tilde{g}}$ -closed but not conversely.

Example 2.9: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $I = \{\emptyset\}$. It is clear that $\{a, c\}$ is $I_{\tilde{g}}$ -closed but not *-closed.

Theorem 2.10: Let (X, τ, I) be an ideal topological space. For every $A \in I$, A is $I_{\tilde{g}}$ -closed.

Theorem 2.11: If (X, τ, I) is an ideal topological space, then A^* is always $I_{\tilde{g}}$ -closed for every subset A of X.

Theorem 2.12: Let (X, τ, I) be an ideal topological space. Then every $I_{\tilde{g}}$ -closed and sg-open set is *-closed.

Theorem 2.13: Let (X, τ, I) be an ideal topological space. Then every \tilde{g} -closed set is an $I_{\tilde{g}}$ -closed set but not conversely.

Example 2.14: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$. It is clear that $\{a\}$ is $I_{\tilde{g}}$ -closed but no \tilde{g} -closed.

Theorem 2.15: If (X, τ, I) is any ideal space and $A \subseteq X$, then the following are equivalent.

- (1) A is $I_{\tilde{g}}$ -closed,
- (2) $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in X,
- (3) For all $x \in cl^*(A)$, $sg-cl(\{x\}) \cap A \neq \emptyset$.
- (4) $cl^*(A) - A$ contains no nonempty sg-closed set,
- (5) $A^* - A$ contains no nonempty sg-closed set.

Corollary 2.16: Let (X, τ, I) be an ideal space and A be an $I_{\tilde{g}}$ -closed set. Then the following are equivalent.

- (1) A is *-closed set,
- (2) $cl^*(A) - A$ is a sg-closed set,
- (3) $A^* - A$ is a sg-closed set.

Theorem 2.17: If (X, τ, I) is an ideal topological space and A is a *-dense in itself, $I_{\tilde{g}}$ -closed subset of X, then A is \tilde{g} -closed.

Corollary 2.18: If (X, τ, I) is an ideal topological space where $I = \{\emptyset\}$ then A is $I_{\tilde{g}}$ -closed iff A is \tilde{g} -closed.

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Corollary 2.19: In an ideal topological space (X, τ, I) where I is codense, if A is a semi open and $I_{\tilde{g}}$ -closed subset of X , then A is \tilde{g} -closed.

Example 2.20: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}, \{b\}$ and $I = \{\emptyset\}$. It is clear that $\{a, c\}$ is \tilde{g} -closed but not $I_{\tilde{g}}$ -closed.

Example 2.21: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. It is clear that $\{b\}$ is $I_{\tilde{g}}$ -closed but not \tilde{g} -closed.

Example 2.22: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$. It is clear that $\{a\}$ is \tilde{g} closed but not \tilde{g} -closed.

Theorem 2.23: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. If $A \subseteq B \subseteq A^*$, then $A^* = B^*$ and B is $*$ -dense in itself.

Proof: Since $A \subseteq B$ then $A^* \subseteq B^*$ and since $B \subseteq A^*$, then $B^* \subseteq (A^*)^* \subseteq A^*$. Therefore $B^* \subseteq A^*$. Hence $A^* = B^*$. Since $B \subseteq A^* \subseteq B^*$, then $B \subseteq B^*$. Therefore B is $*$ -dense in itself.

Theorem 2.24: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then A is $I_{\tilde{g}}$ -closed iff $A = F - N$ where F is $*$ -closed and N contains no nonempty \tilde{g} -closed set.

Theorem 2.25: Let (X, τ, I) be an ideal topological space. If A and B are subsets of X such that $A \subseteq B \subseteq cl^*(A)$ and A is $I_{\tilde{g}}$ -closed then B is $I_{\tilde{g}}$ -closed.

Corollary 2.26: Let (X, τ, I) be an ideal space. If A and B are subsets of X such that $A \subseteq B \subseteq A^*$ and A is $I_{\tilde{g}}$ -closed, then A and B are \tilde{g} -closed sets.

Theorem 2.27: Let (X, τ, I) be an ideal topological space. Then every subset of X is $I_{\tilde{g}}$ -closed iff every \tilde{g} -open is $*$ -closed.

Theorem 2.28: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then A is $I_{\tilde{g}}$ -open iff $F \subseteq int^*(A)$ whenever F is \tilde{g} -closed and $F \subseteq A$.

Corollary 2.29: Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $I_{\tilde{g}}$ -open, then $F \subseteq int^*(A)$ whenever F is closed and $F \subseteq A$.

Proof: Since every closed is \tilde{g} -closed. This result follows from theorem 2.28.

Theorem 2.30: Let (X, τ, I) be an ideal space and $A \subseteq X$. If A is $I_{\tilde{g}}$ -open and $int^*(A) \subseteq A \subseteq B$, then B is $I_{\tilde{g}}$ -open.

Theorem 2.31: Let (X, τ, I) be an ideal space and $A \subseteq X$. Then the following are equivalent.

- (1) A is $I_{\tilde{g}}$ -closed,
- (2) $A \cup (X - A^*)$ is $I_{\tilde{g}}$ -closed,
- (3) $A^* - A$ is $I_{\tilde{g}}$ -open.

III. $I_{\tilde{g}}$ -CONTINUOUS

Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be a function from an ideal topological space (X, τ, I) into a topological space (Y, σ) .

Definition 3.1: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is called $I_{\tilde{g}}$ -continuous ($I_{\tilde{g}}$ -continuous) if the inverse image of every closed set in Y is $I_{\tilde{g}}$ -closed ($I_{\tilde{g}}$ -closed) in X .

Theorem 3.2: If a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is continuous, then it is $I_{\tilde{g}}$ -continuous but not conversely.

Proof: Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be continuous. Let V be any closed set in Y . Then the inverse image $f^{-1}(V)$ is closed set in Y . By theorem 2.2, $f^{-1}(V)$ is $I_{\tilde{g}}$ -closed in X . Therefore f is $I_{\tilde{g}}$ -continuous.

The converse of the above theorem need not to be true as seen from the following example.

Example 3.3: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}\}$, $\sigma = \{\emptyset, X, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$. $I_{\tilde{g}}$ -closed sets are $X, \emptyset, \{a\}, \{b, c\}$. Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is $I_{\tilde{g}}$ -continuous but not continuous, since for the closed set $\{c\}$ in Y , $f^{-1}(\{c\}) = \{a\}$ is not closed set in X .

Theorem 3.4: If a map $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $I_{\tilde{g}}$ -continuous, then it is $I_{\tilde{g}}$ -continuous but not conversely.

Proof: Proof follows from theorem 2.5.

The converse of the above theorem need not to be true as seen from the following example.

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Example 3.5: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\sigma = \{\emptyset, X, \{a\}\}$ and $I = \{\emptyset\}$. I_g -closed sets are $X, \emptyset, \{b\}, \{a, b\}, \{b, c\}$ and $I_{\check{g}}$ -closed sets are $X, \emptyset, \{b\}, \{b, c\}$. Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be a map defined by $f(a)=b, f(b)=c, f(c)=b$. Then f is not $I_{\check{g}}$ -continuous. Since for the closed set $\{b, c\}$ in Y , $f^{-1}(\{b, c\}) = \{a, b\}$ is not $I_{\check{g}}$ -closed in X .

Remark 3.6: The composition of two $I_{\check{g}}$ -continuous functions need not be $I_{\check{g}}$ -continuous as seen from the following example.

Theorem 3.7: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g: (Y, \sigma, J) \rightarrow (Z, \eta)$ be two functions. Then $g \circ f$ is $I_{\check{g}}$ -continuous if g is continuous and f is $I_{\check{g}}$ -continuous.

Proof: Let F be closed in (Z, η) . Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ, J) . As f is $I_{\check{g}}$ -continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $I_{\check{g}}$ -closed in (X, τ, I) . Hence $g \circ f$ is $I_{\check{g}}$ -continuous.

Definition 3.8: A function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called $I_{\check{g}}$ -irresolute if the inverse image of every $I_{\check{g}}$ -closed set in Y is $I_{\check{g}}$ -closed in X .

Theorem 3.9: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g: (Y, \sigma, J) \rightarrow (Z, \eta, K)$ be two functions. Then

- $g \circ f$ is $I_{\check{g}}$ -continuous if g is $I_{\check{g}}$ -continuous and f is $I_{\check{g}}$ -irresolute
- $g \circ f$ is $I_{\check{g}}$ -irresolute if g is $I_{\check{g}}$ -irresolute and f is $I_{\check{g}}$ -irresolute.

Theorem 3.10: If a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $I_{\check{g}}$ -irresolute, then it is $I_{\check{g}}$ -continuous but not conversely. The converse of the above theorem need not to be true as seen from the following example.

Example 3.11: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\sigma = \{\emptyset, X, \{a\}\}$, $I = \{\emptyset\}$ and $J = \{\emptyset, \{a\}\}$. $I_{\check{g}}$ -closed sets are $X, \emptyset, \{b\}, \{b, c\}$ and $I_{\check{g}}$ -closed sets are $Y, \emptyset, \{a\}, \{b, c\}$. Let $f: (X, \tau, I) \rightarrow (Y, \sigma)$ be a map defined by $f(a)=a, f(b)=b, f(c)=c$. Then f is not $I_{\check{g}}$ -irresolute, since for the $I_{\check{g}}$ -closed set $\{a\}$ in Y , $f^{-1}(\{a\}) = \{a\}$ is not $I_{\check{g}}$ -closed in X . But f is $I_{\check{g}}$ -continuous.

IV. SEPARATION AXIOMS IN IDEAL TOPOLOGICAL SPACE

Definition 4.1: An ideal topological space (X, τ, I) is called a $T_{I_{\check{g}}}$ -space if every $I_{\check{g}}$ -closed set of X is closed in X .

Definition 4.2: An ideal topological space (X, τ, I) is called a T_{I_g} -space if every I_g -closed set of X is closed in X .

Definition 4.3: An ideal topological space (X, τ, I) is called a $T_{I_{\check{g}}}^*$ -space if every $I_{\check{g}}$ -closed set of X is $I_{\check{g}}$ -closed in X .

Theorem 4.4: If X is T_{I_g} then it is $T_{I_{\check{g}}}$ but not conversely.

Proof: Let X be a T_{I_g} -space and A be a $I_{\check{g}}$ -closed set in X . Since every $I_{\check{g}}$ -closed set is I_g -closed and X is T_{I_g} -space (by defn 3.2), A is closed in X . Hence X is $T_{I_{\check{g}}}$. The converse need not be true.

Theorem 4.5: If X is $T_{I_{\check{g}}}^*$ then it is T_{I_g} but not conversely.

The converse of the above theorem need not to be true as seen from the following example.

Example 4.6: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$. $I_{\check{g}}$, I_g -closed sets of X are $\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}$. Then (X, τ, I) is $T_{I_{\check{g}}}^*$ -space but not T_{I_g} -space. Since the I_g -closed set $\{a\}$ is not closed in X .

Theorem 4.7: If a function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is continuous and Y is a $T_{I_{\check{g}}}$ -space then f is $I_{\check{g}}$ -irresolute.

Proof: Assume that f is continuous. Let G be any $I_{\check{g}}$ -closed set in Y . Since Y is $T_{I_{\check{g}}}$ -space, G is closed in Y . (By defn 3.1), then $f^{-1}(G)$ is closed in X . Since every closed set is $I_{\check{g}}$ -closed, $f^{-1}(G)$ is $I_{\check{g}}$ -closed set in X . Hence f is $I_{\check{g}}$ -irresolute.

V. STRONGLY \check{g} -CONTINUOUS AND PERFECTLY \check{g} -CONTINUOUS MAPS IN IDEAL TOPOLOGICAL SPACES.

Definition 5.1: A function $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is said to be strongly $I_{\check{g}}$ -continuous if the inverse image of every $I_{\check{g}}$ -closed set in Y is closed in X .

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Remark: When Y is $T_{I_{\tilde{g}}}$ -space, Strongly $I_{\tilde{g}}$ -continuous coincides with continuity.

Theorem 5.2: If a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ from a topological space into an ideal topological space is strongly $I_{\tilde{g}}$ -continuous then it is continuous but not conversely.

Proof: Assume that f is strongly $I_{\tilde{g}}$ -continuous. Let G be open in Y . Since every open set is $I_{\tilde{g}}$ -open, G is $I_{\tilde{g}}$ -open in Y . Since f is strongly $I_{\tilde{g}}$ -continuous, $f^{-1}(G)$ is open in X . Therefore f is continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 5.3: Let $X=Y=\{a,b,c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$,
 $\sigma = \{\emptyset, X, \{a\}, \{a, b\}$ and $J = \{\emptyset, \{b\}\}$. $I_{\tilde{g}}$ -closed sets are $Y, \emptyset, \{b\}, \{c\}, \{b, c\}$. Define a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ by $f(a)=b$, $f(b)=a$, $f(c)=c$ then f is continuous. But f is not strongly $I_{\tilde{g}}$ -continuous. Since $f^{-1}(\{b\}) = \{a\}$ is not closed in X , where $\{b\}$ is $I_{\tilde{g}}$ -closed in Y .

Theorem 5.4: A function $f: (X, \tau) \rightarrow (Y, \sigma, J)$ from a topological space (X, τ) into an ideal topological space (Y, σ, J) is strongly $I_{\tilde{g}}$ -continuous if and only if the inverse image of every $I_{\tilde{g}}$ -open set in Y is open in X .

Theorem 5.5: If a function $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is strongly continuous, then it is strongly $I_{\tilde{g}}$ -continuous but not conversely.

Converse of the above theorem need not to be true as seen from the following example.

Example 5.6: Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$,
 $\sigma = \{\emptyset, X, \{a\}, \{a, b\}$ and $J = \{\emptyset, \{b\}\}$. $I_{\tilde{g}}$ -closed sets are $Y, \emptyset, \{b\}, \{c\}, \{b, c\}$. Define a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ by $f(a)=a$, $f(b)=c$, $f(c)=b$, then f is strongly $I_{\tilde{g}}$ -continuous. But f is not strongly continuous. Since for the set $\{a\}$ in Y , $f^{-1}(\{a\}) = \{a\}$ is open but not closed in X .

Theorem 5.7: If a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is strongly $I_{\tilde{g}}$ -continuous and a map $g: (Y, \sigma, J) \rightarrow (Z, \gamma)$ is $I_{\tilde{g}}$ -continuous, then the composition $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is continuous.

Proof: Let G be any open set in Z . Since g is $I_{\tilde{g}}$ -continuous, $g^{-1}(G)$ is $I_{\tilde{g}}$ -open in Y . Since f is strongly $I_{\tilde{g}}$ -continuous, $f^{-1}(g^{-1}(G))$ is open in X . But $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$. Therefore $g \circ f$ is continuous.

Theorem 5.8: If a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is strongly $I_{\tilde{g}}$ -continuous and a map $g: (Y, \sigma, J) \rightarrow (Z, \gamma, K)$ is strongly $I_{\tilde{g}}$ -continuous, then the composition $g \circ f: (X, \tau) \rightarrow (Z, \gamma, K)$ is strongly $I_{\tilde{g}}$ -continuous.

(ie) composition of two strongly $I_{\tilde{g}}$ -continuous function is strongly $I_{\tilde{g}}$ -continuous.

Theorem 5.9: If a map $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is strongly $I_{\tilde{g}}$ -continuous and a map $g: (Y, \sigma, J) \rightarrow (Z, \gamma, K)$ is strongly $I_{\tilde{g}}$ -continuous, then the composition $g \circ f: (X, \tau, I) \rightarrow (Z, \gamma, K)$ is $I_{\tilde{g}}$ -irresolute.

Theorem 5.10: If a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is strongly $I_{\tilde{g}}$ -continuous and a map $g: (Y, \sigma, J) \rightarrow (Z, \gamma, K)$ is $I_{\tilde{g}}$ -irresolute, then the composition $g \circ f: (X, \tau) \rightarrow (Z, \gamma, K)$ is continuous.

Theorem 5.11: If a function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is continuous and Y is $T_{I_{\tilde{g}}}$ -space then f is strongly $I_{\tilde{g}}$ -continuous.

Definition 5.12: A map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ is said to be perfectly $I_{\tilde{g}}$ -continuous if the inverse image of every $I_{\tilde{g}}$ -open set in (Y, σ, J) is both open and closed in (X, τ) .

Theorem 5.13: A map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ from a topological space (X, τ) into an ideal topological space (Y, σ, J) is perfectly $I_{\tilde{g}}$ -continuous, then it is strongly $I_{\tilde{g}}$ -continuous but not conversely.

Proof: Assume that f is perfectly $I_{\tilde{g}}$ -continuous. Let G be $I_{\tilde{g}}$ -open in (Y, σ, J) . Since f is perfectly $I_{\tilde{g}}$ -continuous, $f^{-1}(G)$ is open in (X, τ) . Therefore f is strongly $I_{\tilde{g}}$ -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 5.14: Let $X=Y=\{a,b,c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$,
 $\sigma = \{\emptyset, X, \{a\}, \{a, b\}$ and $J = \{\emptyset, \{b\}\}$. $I_{\tilde{g}}$ -closed sets are $Y, \emptyset, \{b\}, \{c\}, \{b, c\}$.

Define a map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ as the identity map. Then f is strongly $I_{\tilde{g}}$ -continuous but not perfectly $I_{\tilde{g}}$ -continuous. For the $I_{\tilde{g}}$ -open set $\{a\}$ in Y , $f^{-1}(\{a\}) = \{a\}$ which is open but not closed in X .

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Theorem 5.15: A map $f: (X, \tau) \rightarrow (Y, \sigma, J)$ from a topological space (X, τ) into an ideal topological space (Y, σ, J) is perfectly $I_{\tilde{g}}$ -continuous if and only if $f^{-1}(G)$ is both open and closed for every $I_{\tilde{g}}$ -closed set in (Y, σ, J) .

VI. CONCLUSION

Topology as a field of mathematics is concerned with all questions directly or indirectly related to open / closed sets. Therefore, generalization of open / closed sets is one of the most important subjects in topology. Topology plays a significant role in quantum physics, high energy physics and superstring theory. In this paper, new type of generalization of closed set called $I_{\tilde{g}}$ -closed set was introduced and its properties were investigated, \tilde{g} -continuous, strongly \tilde{g} -continuous and perfectly \tilde{g} -continuous maps in ideal topological spaces were introduced and their properties were studied.

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