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## Some new Results on Bipartite Graph in terms of Topological Separation Axioms

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**ABSTRACT:** Let  $G$  be bipartite graph with  $V$  vertices and  $E$  edges. A topological space  $X$  is a topologized graph if it satisfies the following conditions:

- ❖ Every singleton is open or closed and
- ❖ For all  $x \in X$ ,  $|\partial(x)| \leq 2$ , where  $\partial(x)$  is the boundary of a point  $x$ .

This work examines some results about the topological approach of the Bipartite graphs, Complete Bipartite graphs, separation axioms and its results in the complete bipartite graph. The results are discussed and generalized here.

**KEYWORDS:** Topology, Hausdorff space, Normal space, regular Space, Topologized graph, labeling, Bipartite graph, complete bipartite graph.

### I. INTRODUCTION

In this paper discussed about topologized graphs referred to Antoine Vella [5]. In 2005 Antoine Vella [5] tried to express combinatorial concepts in topological language. As a part the investigation classical topology, pre path, path [6], pre cycle, compact space, cycle space and bond space [6], locally connectedness and ferns were defined. Given a hypergraph  $H$  [4], the classical topology on  $V_H \cup E_H$  is the collection of all sets  $U$  such that, if  $U$  contains a vertex  $v$ , then it also contains all hyperedges incident with  $v$ . It is interesting to note that all these topologies are either defined on the vertex set, or on the union of vertex set and edge set. For Topological graph theory, more important application can be found in printing electronic circuits where the aim is to print (embed) a circuit on a circuit board without two connections crossing each other and resulting in a short circuit. And the Bipartite graphs are extensively used in modern coding theory, especially to decode code words received from the channel. Factor graphs and Tanner graphs are examples of this. A Tanner graph is a bipartite graph in which the vertices on one side of the bipartition represent digits of a codeword, and the vertices on the other side represent combinations of digits that are expected to sum to zero in a codeword without errors. In this paper discussed about some new methods of topology for bipartite graphs [19].

All graphs are finite, simple, undirected graphs with no loops and multiple edges and planar. Every topological space considered here are finite. And the topology is defined on the set  $X$  which is the union of vertices ( $V$ ) and edges ( $E$ ) of the graph  $G$ .

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## II. BASIC DEFINITIONS IN TOPOLOGICAL GRAPH THEORY

**2.1 Definition [17]:** A topology on a set  $X$  is a collection  $\tau$  of a subset of  $X$  with the following properties:

- i.  $\Phi$  and  $X$  are in  $\tau$ .
- ii. The union of the element of any sub collection of  $\tau$  is in  $\tau$  (arbitrary union).
- iii. The intersection of the element of any finite sub collection of  $\tau$  is in  $\tau$ .

The set  $X$  for which a topology  $\tau$  has been specified is called a topological space. The element of  $\tau$  are called open set in  $X$ . A subset of  $X$  is said to be closed if its complement is in  $\tau$ . (i.e., its complement is open) A subset of  $X$  may be open, closed both (clopen set), or neither. The empty set and  $X$  itself are always both closed and open.

**2.2 Definition [5]:** A topologized graph is a topological space  $X$  such that

- Every singleton is open or closed.
- $\forall x \in X, |\partial(x)| \leq 2$ . since  $\partial(x)$  is denoted by the boundary of a point  $x$

**2.3 Definition: [5]**

A topological space is weakly Hausdorff if, for any distinct points  $x, y$ , there exist neighborhoods  $U_x, U_y$  of  $x, y$  respectively such that  $|U_x \cap U_y|$  is finite.

**2.4 Definition: [5]**

A topological space is almost Hausdorff if, for any distinct points  $x, y$ , there exist neighborhoods  $U_x, U_y$  of  $x, y$  respectively such that  $|U_x \cap U_y|$  is finite, and which contains at most one element.

**2.5 Definition: [5]**

A topological space is finitely adjacent if, for all distinct points  $x, y$ , we have  $x^\diamond \cap y^\diamond$  is finite.

**2.6 Definition: [5]**

A topological space is uniquely adjacent if, for all distinct points  $x, y$ , we have  $x^\diamond \cap y^\diamond$  is finite and which intersection is always consist of at most one element.

**2.7 Definition: [5]**

Let  $X$  be a topological space. Then,  $X$  is feebly Hausdorff if for any two distinct points  $x, y$  there exist open sets  $U_x, U_y$  with  $x \in U_x, y \in U_y$  and  $U_x \cap U_y \subseteq x^\diamond$

**2.8 Definition: [5]**

Let  $X$  be a topological space. Then,  $X$  is feebly regular if for any point  $x$  and any closed set  $C$  not containing  $x$ , there exist open sets  $U_x, U_C$  with  $x \in U_x, y \in U_C$  and  $(U_x \cap U_C) \subseteq x^\diamond \cap C^\diamond$

**2.9 Definition: [5]**

Let  $X$  be a topological space. Then,  $X$  is feebly normal if for any two disjoint closed sets  $C, D$  there exist open sets  $C \subseteq U_C, D \subseteq U_D$  and  $U_C \cap U_D \subseteq C^\diamond$

**2.10 Definition: [5]**

Let  $X$  be a topological space. Then,  $X$  is weakly normal if for every two distinct closed sets  $C, D$ , there exist neighborhoods  $U_C, U_D$  of  $C, D$  respectively  $U_C \cap U_D$  is finite.

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## 2.11 Definition: [5]

Given a subset of  $A$  of a topological space, the surrounding set of  $A$  denote by  $A^\circ$ , is the intersection of all the open sets containing  $A$ .

## 2.12 Definition: [18]

A bipartite graph ' $G$ ',  $G = (V, E)$  with partition  $V = \{V, W\}$  is said to be a complete bipartite graph if every vertex in  $V$  is connected to every vertex of  $W$ .

In general, a complete bipartite graph connects each vertex from set  $V$  to each vertex from set  $W$ .

## 2.1 Theorem:[19]

$K_{1,n}$  is a topologized graph (for  $0 < n \leq 2$ )

## 2.2 Theorem:[19]

Every bipartite graph is topologized if degree of every vertex is at most two

## III. RESULTS ON SEPARATION AXIOMS

### 3.1 Theorem:

If  $G$  is a Complete Bipartite graph  $K_{m,n}$  with discrete topology, then

- i). Feebly Hausdorff.
- ii). Weakly Hausdorff if and only if finitely adjacent.
- iii). Almost Hausdorff if and only if uniquely adjacent.
- iv). Feebly normal and Weakly normal.
- v). Feebly regular.

### Proof:

Let  $G$  be a complete Bipartite graph  $K_{m,n}$  with topology defined on  $V \cup E$ . Let  $(X, \tau)$  be a discrete topological space.

(i) Given any two distinct points  $x, y$  there exists corresponding open neighborhoods  $U, V$  with finite intersection. For any point  $z \in U \cap V$  not in  $x^\circ$ , there exists an open set  $A_z$  which contains  $x$  but not  $z$ ; now the set  $U \bigcap_{z \in (U \cap V) \setminus x^\circ} A_z$  is

contained in  $x^\circ$  and, since  $U \cap V$  is finite, is open and so can be taken as the neighborhood of  $x$  together with  $V$  as the neighborhood of  $y$ .

Hence  $X$  is feebly Hausdorff and weakly Hausdorff.

(ii) The necessary part has been proved already, so now to prove the converse part only.

Let  $X$  be a weakly Hausdorff space. For any two distinct points  $u, v$  there exists two neighborhoods  $U, V$  such that  $U \cap V$  is finite.

For any point  $z \in U \cap V$  not in  $u^\circ \cap v^\circ$ , then there exists an open set  $A$  which contains  $u$  and  $v$  but not  $z$ . Then,  $U \cap \bigcap A$  is contained in  $u^\circ$  and also  $V \cap \bigcap A$  is contained in  $v^\circ$ . Hence  $u^\circ \cap v^\circ$  is finite.

Hence  $X$  is finitely adjacent.

(iii) Let  $X$  be almost Hausdorff space. For any two distinct points  $x, y$  there exists neighborhoods  $U$  and  $V$  such that  $U \cap V$  contains at most one element.

For any fixed point  $x, x \in x^\circ$  and for any point  $y, y \in y^\circ$ , so there exists an open set containing  $y$  but not  $x$ , which yields  $X \setminus \{x\}$  is open. That is  $\{x\}$  is closed.

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Hence  $y \in x^\circ \cap y^\circ$ .

Hence  $X$  is uniquely adjacent.

Conversely,

Suppose  $X$  is uniquely adjacent space.

To prove  $X$  is almost Hausdorff.

For any two distinct points  $x, y$  there exists two neighborhoods  $U, V$  respectively. We have any point  $z \in x^\circ \cap y^\circ$  such that  $z \in ((\cap Ux) \cap (\cap Vy))$ . Therefore,  $z \in U \cap V$ .

Hence  $X$  is almost Hausdorff.

(iv) Let  $(X, \tau)$  be a discrete topological space.

For any two distinct closed sets  $C, D$  there exist neighborhoods  $U, V$  such that  $U \cap V$  is finite. Now  $z \in U \cap V$  not in  $C^\circ$ , and the set  $U \cap (\cap Y)$  is contained in  $C^\circ$  and since  $U \cap V$  is finite, is open and so can be taken as the neighborhood of  $X$  together with  $V$  as the neighborhood of  $D$ . This is true for all disjoint pair of closed sets in the topological space.

Hence  $X$  is feebly normal and weakly normal.

(v) Let  $(X, \tau)$  be a discrete topological space.

For any closed set  $C$  and any point  $x$  which is not contained in  $C$ , then there exists two disjoint neighborhoods  $U$  and  $V$  respectively. There exists a point  $y \in C$  with neighborhood  $Z$  such that  $y$  is different from  $x$ . But  $y \in x^\circ$ , since  $y$  may be contained in some neighborhood of  $x$ .

Hence  $y \in x^\circ \cap C^\circ$ , so that  $X$  is feebly regular.

### 3.2 Theorem:

If  $G$  is a complete Bipartite graph  $K_{m,n}$  with connected topology, then

i). Feebly Hausdorff and Weakly Hausdorff.

ii). Almost Hausdorff.

iii). Feebly regular.

iv). Feebly normal and Weakly normal.

#### Proof:

Let  $G$  be a complete graph  $K_n$  with topology defined on  $V \cup E$ . Let  $(X, \tau)$  be a connected topological space.

Given any two distinct points  $x, y$  there exists corresponding open neighborhoods  $U, V$  with finite intersection. For any point  $z \in U \cap V$  not in  $x^\circ$ , there exists an open set  $A_z$  which contains  $x$  but not  $z$ ; now the set  $U \cap \bigcap A_z$  is contained in  $x^\circ$  and, since  $U \cap V$  is finite, is open and so can be taken as the neighborhood of  $X$  together with  $V$  as the neighborhood of  $y$ .

Hence  $X$  is feebly Hausdorff and weakly Hausdorff.

(ii) Let  $(X, \tau)$  be connected topological space.

To prove  $X$  is almost Hausdorff.

For any two distinct points  $x, y$  there exists two neighborhoods  $U, V$  respectively. We have any point  $z \in x^\circ \cap y^\circ$  such that  $z \in ((\cap Ux) \cap (\cap Vy))$ . Therefore,  $z \in U \cap V$ .

Hence  $X$  is almost Hausdorff.

(iii) Let  $(X, \tau)$  be a connected topological space.

For any closed set  $C$  and any point  $x$  which is not contained in  $C$ , then there exists two disjoint neighborhoods  $U$  and  $V$  respectively. There exists a point  $y \in C$  with neighborhood  $Z$  such that  $y$  is different from  $x$ . But  $y \in x^\circ$ , since  $y$  may be contained in some neighborhood of  $x$ .

Hence  $y \in x^\circ \cap C^\circ$ , so that  $X$  is feebly regular.

(iv) Let  $(X, \tau)$  be a connected topological space.

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For any two distinct closed sets  $C, D$  there exist neighborhoods  $U, V$  such that  $U \cap V$  is finite. Now  $z \in U \cap V$  not in  $C^\circ$ , and the set  $U \cap (\cap Y)$  is contained in  $C^\circ$  and since  $U \cap V$  is finite, is open and so can be taken as the neighborhood of  $X$  together with  $V$  as the neighborhood of  $D$ . Hence  $X$  is feebly normal space.

Now prove that  $X$  is weakly normal. For that to show all pair of disjoint closed sets in a topological space is having the neighborhoods with finite intersection. But the only closed disjoint from  $C$  and  $D$  in a connected topological space is the empty set  $\phi$ . Clearly,  $X$  is a weakly normal.

### 3.2.1 Corollary:

If  $G$  is a Complete Bipartite graph  $K_{m,n}$  with topology, then

- Feebly Hausdorff and feebly regular.
- Weakly Hausdorff if and only if finitely adjacent.
- Almost Hausdorff if and only if uniquely adjacent.
- Feebly normal and Weakly normal

#### 3.2.1.(a) Example:

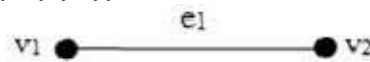
Let  $G$  be a complete Bipartite graph  $K_{1,1}$ . The vertex set and the edge sets are defined as

$V = \{v_1\}; W = \{v_2\}, E = \{e_1\}$ , the edges are labeled as  $f_G(e_1) = \{v_1, v_2\}$ .

Let  $X = \{v_1, v_2, e_1\}$  be a discrete topological space and a topology defined by,

$\tau = \{X, \phi, \{v_1\}, \{v_2\}, \{e_1\}, \{v_1, v_2\}, \{e_1, v_2\}, \{v_1, e_1\}\}$

$\tau^c = \{\phi, X, \{v_2, e_1\}, \{v_1, e_1\}, \{v_1, v_2\}, \{e_1\}, \{v_1\}, \{v_2\}\}$



Let  $v_1, v_2$  be two distinct points in  $X$ .

For  $v_1$  there exists a neighborhoods  $\{v_1\}, \{v_1, e_1\}, \{v_1, v_2\}$  and  $X$ .

For  $v_2$  there exists a neighborhoods  $\{v_2\}, \{v_1, v_2\}, \{e_1, v_2\}$  and  $X$ .

$v_1^\circ = \{v_1\}$ .

$(U_{v_1}) \cap (U_{v_2}) = \{v_1\} \cap \{v_2\} = \phi \subseteq v_1^\circ$

$(U_{v_1}) \cap (U_{v_2}) = \{v_1\} \cap \{v_1, v_2\} = \{v_1\} \subseteq v_1^\circ$

$(U_{v_1}) \cap (U_{v_2}) = \{v_1, v_2\} \cap \{v_1, v_2\} = \{v_1, v_2\}$ . Clearly,  $|(U_{v_1}) \cap (U_{v_2})|$  is finite and which also contains at most one element.

Hence  $X$  is feebly Hausdorff, weakly Hausdorff and almost Hausdorff.

Let  $\{e_1\}$  and  $\{v_1, v_2\}$  be two disjoint closed sets of  $X$ .

For  $\{e_1\}$  there exists a neighborhoods  $\{e_1\}, \{e_1, v_2\}, \{v_1, e_1\}$  and  $X$ .

For  $\{v_1, v_2\}$  there exists a neighborhoods  $\{v_1, v_2\}$  and  $X$ .

$\{e_1\}^\circ = \{e_1\}$ .

$\{e_1\} \cap \{v_1, v_2\} = \phi \subseteq \{e_1\}^\circ$

$\{e_1, v_2\} \cap \{v_1, v_2\} = \{v_2\}$ .

Hence  $X$  is feebly normal.

Here,  $\{\phi, X\}, [\{v_2, e_1\}, \{v_1\}], [\{v_1, e_1\}, \{v_2\}], [\{v_1, v_2\}, \{e_1\}], [\{v_1\}, \{v_2\}], [\{v_2\}, \{e_1\}], [\{v_1\}, \{e_1\}]$  are the all disjoint pairs of closed sets in  $X$ .

For  $\phi$  there exists a neighborhoods  $X, \phi, \{v_1\}, \{v_2\}, \{e_1\}, \{v_1, v_2\}, \{e_1, v_2\}, \{v_1, e_1\}$ .

For  $X$  there exists a neighborhood  $X$ .

$X \cap X = X$ .

$X \cap \phi = \phi$

$X \cap \{v_1\} = \{v_1\}$

For  $\{v_2, e_1\}$  there exists a neighborhoods  $\{v_2, e_1\}$  and  $X$ .

For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}, \{v_1, v_2\}, \{v_1, e_1\}$  and  $X$ .

$\{v_2, e_1\} \cap \{v_1\} = \phi$

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$$\{v_2, e_1\} \cap \{v_1, v_2\} = \{v_2\}$$

For  $\{v_1, e_1\}$  there exists a neighborhoods  $\{v_1, e_1\}$  and X.

For  $\{v_2\}$  there exists a neighborhoods  $\{v_2\}, \{v_1, v_2\}, \{v_2, e_1\}$  and X.

$$\{v_1, e_1\} \cap \{v_2\} = \phi$$

$$\{v_1, e_1\} \cap \{v_1, v_2\} = \{v_1\}$$

For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}, \{v_1, v_2\}, \{v_1, e_1\}$  and X.

For  $\{v_2\}$  there exists a neighborhoods  $\{v_2\}, \{v_1, v_2\}, \{v_2, e_1\}$  and X.

$$\{v_1, v_2\} \cap \{v_1, v_2\} = \{v_1, v_2\}$$

$$\{v_1\} \cap \{v_1, v_2\} = \{v_1\}$$

For  $\{v_2\}$  there exists a neighborhoods  $\{v_2\}, \{v_1, v_2\}, \{v_2, e_1\}$  and X.

For  $\{e_1\}$  there exists a neighborhoods  $\{e_1\}, \{e_1, v_2\}, \{v_1, e_1\}$  and X.

$$\{v_2\} \cap \{e_1\} = \phi$$

$$\{v_2\} \cap \{e_1, v_2\} = \{v_2\}$$

For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}, \{v_1, v_2\}, \{v_1, e_1\}$  and X.

For  $\{e_1\}$  there exists a neighborhoods  $\{e_1\}, \{e_1, v_2\}, \{v_1, e_1\}$  and X.

$$\{v_1\} \cap \{e_1\} = \phi$$

$$\{v_1\} \cap \{e_1, v_1\} = \{v_1\}$$

Here for all pair of disjoint closed set having the neighborhoods with finite intersection. Hence X is weakly normal.

Let  $\{v_2\}$  be any closed set and  $e_1$  be any point which is not contained in the closed set.

For  $v_2 \in \{v_2\}$  there exists a neighborhoods  $\{v_2\}, \{v_1, v_2\}, \{v_2, e_1\}$  and X.

For  $e_1$  there exists a neighborhoods  $\{e_1\}, \{e_1, v_2\}, \{v_1, e_1\}$  and X.

$$\{v_2\}^\diamond = \{v_2\}$$

$$e_1^\diamond = \{e_1\}$$

$$\{v_2\}^\diamond \cap e_1^\diamond = \phi$$

$$\{v_2\} \cap \{e_1\} = \phi \subseteq \{v_2\}^\diamond \cap e_1^\diamond$$

$$\{v_2\} \cap \{e_1, v_2\} = \{v_2\}$$

Hence X is feebly regular.

### 3.3 Theorem

If G is Complement graph  $\overline{K_{m,n}}$  with discrete topology, then

i). Feebly Hausdorff and Weakly Hausdorff.

ii). Almost Hausdorff.

iii). Feebly normal and Weakly normal.

iv). Feebly regular.

#### Proof:

The proof is obvious from Theorem 3.1 and Theorem 3.2.

#### 3.3.1.(a) Example:

Let G be a  $\overline{K_{1,2}}$  graph. The vertex set and the edge sets are defined as  $V = \{v_1\}; W = \{w_1, w_2\};$

Let  $X = \{v_1, w_1, w_2\}$  be a discrete topological space and a topology defined by,  $\tau = \{X, \phi, \{v_1\}, \{w_1\}, \{w_2\}, \{v_1, w_1\}, \{w_2, w_1\}, \{v_1, w_2\}\}$

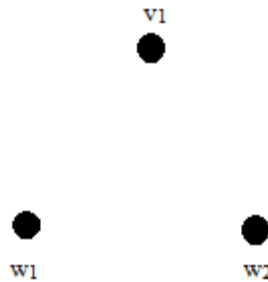
$$\tau^c = \{\phi, X, \{v_2, w_2\}, \{v_1, w_2\}, \{v_1, w_1\}, \{w_2\}, \{v_1\}, \{w_1\}\}$$

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Let  $v_1, w_1$  be two distinct points in  $X$ .

For  $v_1$  there exists a neighborhoods  $\{v_1\}, \{v_1, w_2\}, \{v_1, w_1\}$  and  $X$ .

For  $w_1$  there exists a neighborhoods  $\{w_1\}, \{v_1, w_1\}, \{w_2, w_1\}$  and  $X$ .

$$v_1^\circ = \{v_1\}.$$

$$(U_{v_1}) \cap (U_{w_1}) = \{v_1\} \cap \{w_1\} = \phi \subseteq v_1^\circ$$

$$(U_{v_1}) \cap (U_{w_1}) = \{v_1\} \cap \{v_1, w_1\} = \{v_1\} \subseteq v_1^\circ$$

$$(U_{v_1}) \cap (U_{w_1}) = \{v_1, w_1\} \cap \{v_1, w_1\} = \{v_1, w_1\}.$$

Clearly,  $|(U_{v_1}) \cap (U_{w_1})|$  is finite and which also contains atmost one element.

Hence  $X$  is feebly Hausdorff, weakly Hausdorff and almost Hausdorff.

Let  $\{w_2\}$  and  $\{v_1, w_1\}$  be two disjoint closed sets of  $X$ .

For  $\{w_2\}$  there exists a neighborhoods  $\{w_2\}, \{w_2, w_1\}, \{v_1, w_2\}$  and  $X$ .

For  $\{v_1, w_1\}$  there exists a neighborhoods  $\{v_1, w_1\}$  and  $X$ .

$$\{w_2\}^\circ = \{w_2\}.$$

$$\{w_2\} \cap \{v_1, w_1\} = \phi \subseteq \{w_2\}^\circ$$

$$\{w_2, w_1\} \cap \{v_1, w_1\} = \{w_1\}.$$

Hence  $X$  is feebly normal.

Here,  $[\phi, X], [\{w_1, w_2\}, \{v_1\}], [\{v_1, w_2\}, \{w_1\}], [\{v_1, w_1\}, \{w_2\}], [\{v_1\}, \{w_1\}],$

$[\{w_1\}, \{w_2\}], [\{v_1\}, \{w_2\}]$  are the all disjoint pairs of closed sets in  $X$ .

For  $\phi$  there exists a neighborhoods  $X, \phi, \{v_1\}, \{w_1\}, \{w_2\}, \{v_1, w_1\}, \{w_2, w_1\}, \{v_1, w_2\}$ .

For  $X$  there exists a neighborhood  $X$ .

$$X \cap X = X.$$

$$X \cap \phi = \phi$$

$$X \cap \{v_1\} = \{v_1\}$$

For  $\{w_1, w_2\}$  there exists a neighborhoods  $\{w_1, w_2\}$  and  $X$ .

For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}, \{v_1, w_1\}, \{v_1, w_2\}$  and  $X$ .

$$\{w_1, w_2\} \cap \{v_1\} = \phi$$

$$\{w_1, w_2\} \cap \{v_1, w_1\} = \{w_1\}$$

For  $\{v_1, w_2\}$  there exists a neighborhoods  $\{v_1, w_2\}$  and  $X$ .

For  $\{w_1\}$  there exists a neighborhoods  $\{w_1\}, \{v_1, w_1\}, \{w_1, w_2\}$  and  $X$ .

$$\{v_1, w_2\} \cap \{w_1\} = \phi$$

$$\{v_1, w_2\} \cap \{v_1, w_1\} = \{v_1\}$$

For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}, \{v_1, w_1\}, \{v_1, w_2\}$  and  $X$ .

For  $\{w_1\}$  there exists a neighborhoods  $\{w_1\}, \{v_1, w_1\}, \{w_1, w_2\}$  and  $X$ .

$$\{v_1, w_1\} \cap \{v_1, w_1\} = \{v_1, w_1\}$$

$$\{v_1\} \cap \{v_1, w_1\} = \{v_1\}$$

For  $\{w_1\}$  there exists a neighborhoods  $\{w_1\}, \{v_1, w_1\}, \{w_1, w_2\}$  and  $X$ .

For  $\{w_2\}$  there exists a neighborhoods  $\{w_2\}, \{w_2, w_1\}, \{v_1, w_2\}$  and  $X$ .

$$\{w_1\} \cap \{w_2\} = \phi$$

$$\{w_1\} \cap \{w_2, w_1\} = \{w_1\}$$



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For  $\{v_1\}$  there exists a neighborhoods  $\{v_1\}$ ,  $\{v_1, w_1\}$ ,  $\{v_1, w_2\}$  and  $X$ .  
For  $\{w_2\}$  there exists a neighborhoods  $\{w_2\}$ ,  $\{w_2, w_1\}$ ,  $\{v_1, w_2\}$  and  $X$ .

$$\{v_1\} \cap \{w_2\} = \phi$$
$$\{v_1\} \cap \{w_2, v_1\} = \{v_1\}$$

Here for all pair of disjoint closed set having the neighborhoods with finite intersection. Hence  $X$  is weakly normal.

Let  $\{w_1\}$  be any closed set and  $e_1$  be any point which is not contained in the closed set.

For  $w_1 \in \{w_1\}$  there exists a neighborhoods  $\{w_1\}$ ,  $\{v_1, w_1\}$ ,  $\{w_1, w_2\}$  and  $X$ .

For  $w_2$  there exists a neighborhoods  $\{w_2\}$ ,  $\{w_2, w_1\}$ ,  $\{v_1, w_2\}$  and  $X$ .

$$\{w_1\}^\diamond = \{w_1\}$$
$$w_2^\diamond = \{w_2\}$$
$$\{w_1\}^\diamond \cap w_2^\diamond = \phi$$
$$\{w_1\} \cap \{w_2\} = \phi \subseteq \{w_1\}^\diamond \cap w_2^\diamond$$
$$\{w_1\} \cap \{w_2, w_1\} = \{w_1\}$$

Hence  $X$  is feebly regular.

## IV. CONCLUSION

The above results we discussed about some new results on bipartite graph. We have applied and verified some new separation axioms on topologized graph for bipartite graphs, Complete Bipartite Graphs. Further the work is extended to matching and coloring of a bipartite graph.

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