

# Bianchi Type-V Cosmological Model of Universe with Polytropic Equation of State

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**ABSTRACT:** This paper deals with Bianchi type V cosmological model in the presence of perfect fluid with polytropic equation of state (EoS)  $p = K\rho^n$ , where  $K$  and  $n$  are constants called as polytropic constant and polytropic index respectively. The physically realistic solutions of the Einstein's field equations for Bianchi type-V space time have been obtained under the assumption of the scalar expansion  $\theta$  is proportional to the shear scalar  $\sigma^2$ . The kinematic & physical properties of the model are also studied.

**KEYWORDS:** Bianchi type-V Space time, Polytropic equation of state, Perfect Fluid.

## I. INTRODUCTION

Recently it has been discovered that our universe is experiencing an accelerated expansion. Observations of type Ia Supernova show that the universe is accelerating and 68% of the total energy density exists in a dark energy component with negative pressure [1,2]. This was further confirmed by measurements of cosmic microwave background (CMB)[3,4], large scale structure (LSS) [5,6].

From observational data, the equation of state (EoS) parameter  $\omega = \frac{p}{\rho}$  is being estimated as  $\omega = -1$  (standard  $\Lambda$

CDM cosmology),  $\omega > -1$  (quintessence dark energy) or  $\omega < -1$  (phantom dark energy).

The study of Bianchi type-V cosmological models are done extensively by researchers as these models isotropize at special cases and are anisotropic otherwise. Presently cosmology is based on FRW model which is completely homogeneous and isotropic. The anisotropy of dark energy within the framework of Bianchi type can be used to generate arbitrary ellipsoidality to the universe. Out of different models, Bianchi type-V universes are generalisation of FRW model which become isotropic at late times.

Yadav [7] and Kumar & Yadav [8] have investigated spatially homogeneous and anisotropic Bianchi type-V dark energy models by considering constant deceleration parameter. The EoS parameter  $\omega$  is considered as a constant by Kujat *et al.* [9], Bartelmann *et al.* [10]. In recent years, Mukhopadhyay *et al.* [11], Setare [12-14] and Setare & Saridakis [15] have used dark energy models with time dependant  $\omega$ . A few years ago, dark energy models with variable EoS parameter,  $\omega$  have been studied by Ray *et al.* [16], Akarsu & Kilinc [17], Yadav *et al.* [18]. The dark energy universe EoS with inhomogeneous, Hubble parameter dependant term is studied by Nojiri and Odintsov [19]. Recently, Pradhan & Hassan [20] investigated accelerating dark energy models in Bianchi type-V space-time. K.S. Adhav, *et al.* [21] had studied cosmological models of the universe with the help of polytropic EoS in the absence of dark energy. Adhav *et al.* [22,23] have investigated anisotropic and homogeneous cosmological models with polytropic EoS in Bianchi space-time using hybrid expansion law.

The general form of polytropic EoS is given by

$$p = (\alpha\rho + k\rho^{1+1/n})c^2, \quad (1.1)$$

where  $p = \alpha\rho$  is the linear component and  $p = k\rho^{1+1/n}$  is the polytropic component. For the positive index  $n > 0$ , the polytropic component dominates the linear component when the density is high in the early universe. Conversely, for  $n < 0$ , the polytropic component dominates the linear component when the density is quite low. This polytropic

# International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Special Issue 11, May 2017

EoS may correspond to self gravitating Bose-Einstein condensates with repulsive ( $K > 0$ ) or attractive ( $K < 0$ ) self interaction or have another origin. This EoS also describes radiation ( $\alpha = 1/3$ ), pressureless matter ( $\alpha = 0$ ) and vacuum energy ( $\alpha = -1$ ). Chavanis [24,25,26,27] has studied many aspects of Robertson Walker universes using the equation of state given by (1.1). Amanda and Bruni [28] studied different behaviours of FRW and Bianchi I universe using dynamical systems theory and quadratic equation of state in the form

$$p \equiv f(\rho) = p_0 + \omega\rho + \alpha\rho^2 \quad (1.2)$$

This has motivated us to investigate Bianchi type-V cosmological model of the universe with polytropic EoS :

$$p = k\rho^n, \quad (1.3)$$

where  $k$  and  $n$  are polytropic constant and polytropic index respectively.

The paper is organised as follows : metric and field equations in Sect. 2, physical behaviour of the model in Sect. 3, Discussions & Conclusions are given in Sect. 4 and References in Sect. 5.

## II. FIELD EQUATIONS & THEIR SOLUTIONS

We consider the Bianchi type V line element as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2ax} dy^2 + C^2 e^{-2ax} dz^2, \quad (2.1)$$

where  $A$ ,  $B$  and  $C$  are functions of cosmic time  $t$  and  $a$  is a non-zero constant.

The Einstein field equations are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij}, \quad (2.2)$$

where  $R_{ij}$  is the Ricci tensor,  $R$  is the Ricci scalar and  $T_{ij}$  is the energy momentum tensor.

Also the conservation equation of the energy momentum tensor is

$$T_{ij}^{;j} = 0 \quad (2.3)$$

The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij}, \quad (2.4)$$

where  $\rho$  is the energy density,  $p$  is the pressure and  $u^i$  is the four velocity vector satisfying  $g_{ij}u^i u^j = 1$ .

We assume an Equation of State  $p = f(\rho)$  in the form as

$$p = k\rho^n, \quad (2.5)$$

where  $k$  and  $n$  are constants called polytropic constants and polytropic index respectively.

In a co-moving coordinate system, the Einstein field equations (2.2) for the metric (2.1) with the help of equation (2.4) reduce to the following set of equations

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3a^2}{A^2} = \rho \quad (2.6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = -k\rho^n \quad (2.7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = -k\rho^n \quad (2.8)$$

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Vol. 6, Special Issue 11, May 2017

$$\frac{\ddot{C}}{C} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} - \frac{a^2}{A^2} = -k\rho^n \quad (2.9)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (2.10)$$

where overhead dot ( $\dot{\phantom{x}}$ ) denote differentiation with respect to time  $t$ .

The energy conservation equation  $T_{;j}^{ij} = 0$  leads to the following simple expression

$$\dot{\rho} + \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) (\rho + p) = 0 \quad (2.11)$$

We define the spatial volume  $V$  for Bianchi type V space time respectively as

$$V = ABC \quad (2.12)$$

The mean Hubble parameter  $H$  is given by

$$H = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (2.13)$$

The scalar expansion  $\theta$  and the shear scalar  $\sigma^2$  are given by

$$\theta = 3H \quad (2.14)$$

$$\sigma^2 = \frac{1}{2} \left( \sum_{i=1}^3 H_i^2 - 3H^2 \right) \quad (2.15)$$

The average anisotropy parameter  $\Delta$  is given by

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 \quad (2.16)$$

where  $H_i, (i=1,2,3)$  represent the directional Hubble parameters in  $x, y, z$  directions respectively and  $\Delta = 0$  corresponds to isotropic expansion.

Integrating equation (2.10) we get

$$A^2 = BC \quad (2.17)$$

Now, the expansion scalar  $\theta$  is proportional to the shear scalar  $\sigma^2$  so we can substitute the following relation in field equations (2.6) to (2.9)

$$B = C^m \quad (2.18)$$

where  $m \neq 0$  is constant.

The field equations reduce to

$$\frac{-a2}{C^{m+1}} + (m+1) \frac{\ddot{C}}{C} + m^2 \frac{\dot{C}^2}{C^2} = -k\rho^n \quad (2.19)$$

$$\frac{-a2}{C^{m+1}} + \left( \frac{m+3}{2} \right) \frac{\ddot{C}}{C} + \left( \frac{m+1}{2} \right)^2 \frac{\dot{C}^2}{C^2} = -k\rho^n \quad (2.20)$$

$$\frac{-a2}{C^{m+1}} + \left( \frac{3m+1}{2} \right) \frac{\ddot{C}}{C} + \left( \frac{7m^2 - 2m - 1}{4} \right) \frac{\dot{C}^2}{C^2} = -k\rho^n \quad (2.21)$$

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$$\frac{-3a2}{C^{m+1}} + \left(\frac{m^2 + 4m + 1}{2}\right) \frac{\dot{C}^2}{C^2} = -\rho \tag{2.22}$$

Adding equation (2.19) and (2.20) and then subtracting from 2(equation (2.21)) we get

$$\frac{\ddot{C}}{C} + \left(\frac{3m+1}{2}\right) \frac{\dot{C}^2}{C^2} = 0 \tag{2.23}$$

$$\ddot{C} = -\left(\frac{3m+1}{2}\right) \frac{\dot{C}^2}{C} \tag{2.24}$$

Since the above equations are highly non-linear we use proper substitution and separating variables, we solve the above differential equation and we get :

$$C = (\alpha t + \beta)^{2/3(m+1)} \tag{2.25}$$

Where  $\alpha \neq 0$  and  $\beta$  is constant of integration.

From equations 2.18 and 2.25 we get

$$B = (\alpha t + \beta)^{2m/3(m+1)} \tag{2.26}$$

$$A = (\alpha t + \beta)^{1/3} \tag{2.27}$$

Substituting in equation (2.13) we get mean Hubble parameter as

$$H = \frac{\alpha}{3(\alpha t + \beta)} \tag{2.28}$$

The expansion scalar  $\theta$  and the shear scalar  $\sigma^2$  are given by

$$\theta = 3H = \frac{\alpha}{(\alpha t + \beta)} \tag{2.29}$$

$$\sigma^2 = \frac{(m-1)^2}{3(m+1)^2 t^2} \tag{2.30}$$

The average anisotropy parameter  $\Delta$  from equation (2.16) is given by

$$\Delta = \frac{2(m-1)^2}{3(m+1)^2} \tag{2.31}$$

And  $\Delta = 0$  when  $m = 1$

### III. PHYSICAL DISCUSSION

Now, from equation (2.11) and (2.12) we obtain the energy density as

$$\rho = [(\alpha t + \beta)^{(n-1)} - k]^{\frac{1}{n-1}} \tag{3.1}$$

Also, from equation (2.5), we obtain the pressure as

$$p = k [(\alpha t + \beta)^{(n-1)} - k]^{\frac{n}{n-1}} \tag{3.2}$$

The energy density parameter is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{[(\alpha t + \beta)^{(n-1)} - k]^{\frac{1}{n-1}} t^2}{(\alpha t + \beta)^2} \tag{3.3}$$

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The parameter  $\omega = \frac{p}{\rho}$  is obtained as

$$\omega = k[(\alpha t + \beta)^{(n-1)} - k] \quad (3.4)$$

## IV. CONCLUSION

A study of Bianchi type V cosmological model with a polytropic equation of state has been done. The physical and kinematic parameters of cosmological models are obtained.

- i. From equation (2.12), (2.25), (2.26), (2.27) it is observed that spatial volume of the universe is zero at  $t = 0$  and increases infinitely as  $t \rightarrow \infty$ .
- ii. It is also noted that, for  $n > 0$  energy density and pressure decrease as  $t \rightarrow \infty$ .
- iii. Equation (2.31) shows that anisotropy parameter shows early inflation and late time acceleration at  $m \neq 1$ . The model obtained represents accelerating model of the universe which is consistent with the recent observations of type-Ia supernovae. The proposed model in this paper presents a new cosmological model in General theory of relativity with polytropic EoS.
- iv. For  $n > 0$  and  $b = 0$ , it is noted that at  $t = 0$ , the equation of state parameter  $\omega \leq 0$ .

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