

Condition for 3-Degeneracy of Planar Graph

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ABSTRACT: In this paper, we will investigate the condition for 3-degeneracy of planar graph.

KEYWORDS: Planar Graphs, Degeneracy.

I. INTRODUCTION

In graph theory, a k -degenerate graph is an undirected graph in which every subgraph has a vertex of degree at most k : that is, some vertex in the subgraph touches k or fewer of the subgraph's edges. The degeneracy of a graph is the smallest value of k for which it is k -degenerate.

Degeneracy is also known as the k -core number, width, and linkage and is essentially the same as the coloring number. k -degenerate graphs have also been called k -inductive graphs. The degeneracy of a graph may be computed in linear time by an algorithm that repeatedly removes minimum-degree vertices. The connected components that are left after all vertices of degree less than k have been removed are called the k -cores of the graph and the degeneracy of a graph is the largest value k such that it has a k -core.

A planar graph without 3-cycles is 3-degenerate because it always contains a vertex of degree at most 3 as otherwise it would contradict to Lemma 2. It has been proved earlier that a planar graph without 5-cycles is 3-degenerate. Note that an icosidodecahedron, that is, the line graph of a dodecahedron, is a 4-regular planar graph without 4-cycles. Hence the lack of 4-cycles does not imply 3-degeneracy of a planar graph. Choudum constructed 4-regular 3-connected planar graph without k -cycles for each $k \geq 7$. Thus a planar graph without k -cycles for each $k \geq 7$ is not 3-degenerate. The case $k=6$ remains unsettled.

II. RELATED WORKS

We will find the following definitions useful:

1. **Plane graph:** A graph $G(V,E)$ is called plane if V is a set of points in the plane and E is a set of curves in the plane such that
 - a) every curve contains at most two vertices and these vertices are the ends of the curve;
 - b) the intersection of every two curves is either empty, or one, or two vertices of the graph.
2. **Planar graph:** A graph is called planar, if it is isomorphic to a plane graph.
3. **k -degenerate graph:** A graph G is said to be k -degenerate if every induced subgraph of G contains a vertex of degree at most k .
4. **Fat 3 face:** A 3-face $f \in F(G)$ is said to be fat if $|T^*(f)| = 2$ where $T^*(f)$ is the set of 3-faces adjacent to the face f .
5. **4-fat 3-face:** A fat 3-face $f \in F(G)$ is said to be 4-fat if $d(v) = 4$ for every vertex $v \in V(f)$.
6. **Major face (or Major vertex):** A 8^+ - face $f \in F(G)$ (or a 5^+ - vertex $v \in V(G)$) is called a major face (vertex).

Also, the following lemmas and theorems are necessary for establishing the result:

Lemmas

1. If G is a planar graph with n vertices, m edges, f faces and k components then $n - m + f = k + 1$.
2. If G is a triangle-free connected planar graph with n vertices, m edges and f faces then $m \leq 2n - 4$.
3. If G be a planar connected graph without 6-cycles $\delta(G) \geq 3$, then there is an edge $uv \in E(G)$, such that $d(u) + d(v) \leq 8$
4. If G is a planar connected graph without 6-cycles, then $\delta(G) \leq 4$.

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5. The sum of the degrees of all faces of a plane graph G is twice the number of edges in G , i.e. $\sum_{f \in F(G)} \lambda(f) = 2m$, where m is the number of edges in $E(G)$.

Theorem

1. Let G be a plane graph without 6-cycles. If G does not contains two adjacent triangles, then G is 3-degenerate.
2. Every planar graph without 4-cycles and 6-cycles is 3-degenerate.
3. If G is a plane graph without 6-cycles and 7-cycles then G is 3-degenerate.

III. CONCLUSION

Using the above lemmas and theorem, we conclude that every planar graph G without 6-cycles is 3-degenerate holds if graph G does not contains two adjacent triangles.

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