

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 6, June 2017

On PFSE Domination and PFSE-t Domination

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ABSTRACT: Let $T \subseteq E$ be an edge dominating set of a fuzzy soft graph (FSG) $G_{A,V}$. Then T is said to be a *Perfect fuzzy soft edge dominating (PFSED) set* if for every edge in $E - T$ is adjacent to exactly one edge in T for each parameter $e \in A$. The minimum cardinality of a minimal perfect fuzzy soft edge dominating set is called a *perfect fuzzy soft edge domination number* and is denoted by $\gamma_{pfs}^1(G_{A,V})$.

KEYWORDS:- Perfect fuzzy soft edge domination , perfect fuzzy soft edge domination number, perfect fuzzy soft t-edge domination and perfect fuzzy soft t-edge domination number.

I.INTRODUCTION

The concept of domination in fuzzy graphs was first introduced by A Somasundaram and S Somasundaram [10]. C.Y Ponnappan, S.Basheer Ahamed and P Surulinathan [6] discussed edge domination in fuzzy graphs. S.Ramya and S. Lavanya[5] discussed perfect vertex(edge) domination in fuzzy graphs. The notion of fuzzy soft graphs and some operations in fuzzy soft graphs was introduced by Sumit Mohinta and Samanta[9] . Muhammed Akram and Saira Nawas[8] introduced different types of fuzzy soft graphs and their properties and later on T K Mathew varkey and Rani Rajeevan introduced the concept of edge domination in fuzzy soft graphs[3] and perfect vertex domination in fuzzy soft graphs[4].

In this paper we introduce the concepts of perfect edge domination in fuzzy soft graphs ,perfect fuzzy soft edge domination number, perfect fuzzy soft t-edge domination and perfect fuzzy soft t-edge domination number.

II.PRELIMINARIES

Definition 2.1[7] Let G be a graph with V be the set of all vertices and E be the set of all edges. If there exist two functions $\rho : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(x, y) \leq \rho(x) \wedge \rho(y) \forall x, y \in V$. Then $G(\rho, \mu)$ is called a fuzzy graph.

Definition 2.2 [8] Let $V = \{x_1, x_2, x_3, \dots, x_n\}$ (non empty set) E (parameters set) and $A \subseteq E$. Also let $\rho_e : V \rightarrow [0, 1]$ such that each $x_i \in V$ mapped to $\rho_e(x_i)$, where $\rho_e = \rho(e)$ is the ρ image of $e \in A$ and $\mu_e : V \times V \rightarrow [0, 1]$ such that each $(x_i, x_j) \in E$ mapped to $\mu_e(x_i, x_j)$, where $\mu_e = \mu(e)$ is the μ image of $e \in A$, then the pair $((A, \rho), (A, \mu))$ is called a fuzzy soft graph if and only if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A$ and $\forall i, j = 1, 2, 3, \dots, n$ is denoted by $G_{A,V}$.

Definition 2.3 [8] The underlying crisp graph of a fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is denoted by

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$G^* = (\rho^*, \mu^*)$, where $\rho^* = \{x_i \in V; \rho_e(x_i) > 0 \text{ for some } e \in E\}$ and
 $\mu^* = \{(x_i, x_j) \in V \times V; \mu_e(x_i, x_j) > 0 \text{ for some } e \in E\}$.

Definition 2.4 [7] A fuzzy soft graph $G_{A,V}$ is called a **complete fuzzy soft graph** if $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall x_i, x_j \in \rho^*, e \in A$ and is denoted by $K_{G_{A,V}}$.

Definition 2.5 [7] Let $G_{A,V}$ be a fuzzy soft graph. Then the **order** of $G_{A,V}$ is defined as

$O(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i \in V} \rho_e(x_i) \right)$ and **size** of $G_{A,V}$ is defined as $S(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i, x_j \in V} \mu_e(x_i, x_j) \right)$ and the **degree** of a

vertex x_i is defined as $d_{(G_{A,V})}(x_i) = \sum_{e \in A} \left(\sum_{x_j \in V, x_j \neq x_i} \mu_e(x_i, x_j) \right)$.

Definition 2.6[3] **Degree** of a fuzzy soft graph $G_{A,V}$ is defined as $D_{G_{A,V}} = \max \{d_{G_{A,V}}(x_i); \forall x_i \in V\}$.

Definition 2.8[3] A fuzzy soft graph $G_{A,V}$ is said to be **regular fuzzy soft graph** if the fuzzy graph corresponding to each parameter $e \in A$ is a regular fuzzy graph.

Definition 2.9[3] A fuzzy soft graph $G_{A,V}$ without fuzzy soft loops and fuzzy soft multiple edges is called a **fuzzy soft simple graph**.

Definition 2.10 [3] An FSG is said to be **isolated** if $\mu_e(x_i, x_j) = 0 \forall (x_i, x_j) \in E, e \in A$.

Definition 2.11[3] Two fuzzy soft vertices are said to be adjacent if they are connecting by a fuzzy soft edge and two fuzzy soft edges are said to be adjacent if they are incident on a common fuzzy soft vertex.

Definition 2.12[3] The **complement** of a FSG $G_{A,V}$ is defined as $\bar{G}_{A,V} = ((A, \rho), (\bar{A}, \bar{\mu}))$ where $\bar{\mu}_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) - \mu_e(x_i, x_j) \forall (x_i, x_j) \in E \& e \in A$.

Definition 2.13[3] **Neighborhood of a vertex** is defined as $N(x_i) = \{x_j \in V \mid \mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A\}$.

Definition 2.14[3] Let $G_{A,V}$ be an FSG. $\delta^1(G_{A,V}) = \min \{d_{(G_{A,V})}(x_i) \mid \forall x_i \in V\}$ &

$\Delta^1(G_{A,V}) = \max \{d_{(G_{A,V})}(x_i) \mid \forall x_i \in V\}$.

Definition 2.15[3] The **edge degree** of a FSG is defined as $d_{(G_{A,V})}^1(x_i, x_j) = d_{(G_{A,V})}(x_i) + d_{(G_{A,V})}(x_j)$.

$N(x_i, x_j)$ is the set of all edges incident with the vertices x_i and x_j .

Definition 2.16 [3] Let $G_{A,V}$ be a fuzzy soft graph. A subset T of E is said to be a **fuzzy soft edge dominating (FSED) set** if for every edge in $E - T$ is adjacent to at least one edge in T for each parameter $e \in A$.

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The minimum cardinality of a fuzzy soft edge dominating set is called *fuzzy soft edge domination number* and is denoted by $\gamma_{fs}^1(G_{A,V})$.

Definition 2.17[3] Suppose $G_{A,V}$ is a FSG and T be an FSED set. Then T is said to be *minimal FSED* set if deletion of an edge from T is not a FSED set of $G_{A,V}$.

Definition 2.18[3] An FSED set T of an FSG $G_{A,V}$ is said to be a *fuzzy soft t-edge dominating (FSED-t) set* if for every edge in $E - T$ is adjacent to at least t -edges in T for each parameter $e \in A$. The minimum cardinality of a minimal FSED-t set is called *fuzzy soft t-edge domination number* and is denoted by $\gamma_{fs-t}^1(G_{A,V})$.

III.PERFECT FUZZY SOFT EDGE DOMINATING (PFSED) SET & PERFECT FUZZY SOFT T-EDGE DOMINATING (PFSED-T) SET

Definition 3.1 Let $G_{A,V}$ be a fuzzy soft graph. A subset T of E is said to be a *perfect fuzzy soft edge dominating(PFSED) set* if for every edge in $E - T$ is adjacent to exactly one edge in T for each parameter $e \in A$. The minimum cardinality of a minimal perfect fuzzy soft edge dominating set is called *perfect fuzzy soft edge domination number* and is denoted by $\gamma_{pfs}^1(G_{A,V})$.

Definition 3.2 Suppose $G_{A,V}$ is a FSG and T be an PFSED set. Then T is said to be *minimal PFSED* set if deletion of an edge from T is not a FSED set of $G_{A,V}$.

Definition 3.3 An FSED set T of an FSG $G_{A,V}$ is said to be a *perfect fuzzy soft t-edge dominating (PFSED-t) set* if for every edge in $E - T$ is adjacent to exactly t -edges in T for each parameter $e \in A$. The minimum cardinality of a minimal PFSED-t set is called *perfect fuzzy soft t-edge domination number* and is denoted by $\gamma_{pfs-t}^1(G_{A,V})$.

Theorem 3.4 Let $G_{A,V}$ be a fuzzy soft graph and T is minimal PFSED set then for each edge (x_i, x_j) in T satisfies any one of the following two conditions.

- (a) $N(x_i, x_j) \cap T = \phi$.
- (b) There exist an edge $(x_m, x_n) \in E - T$ such that $N(x_m, x_n) \cap T = \{(x_i, x_j)\}$.

Proof: Suppose T is a minimal PFSED set and $(x_i, x_j) \in T$. Then $T - \{(x_i, x_j)\}$ is not a PFSED set and hence there exist $(x_m, x_n) \in E - [T - \{(x_i, x_j)\}]$ such that (x_m, x_n) is not adjacent to any element of $T - \{(x_i, x_j)\}$. If $\{(x_m, x_n)\} = \{(x_i, x_j)\}$ then we get $N(x_i, x_j) \cap T = \phi$ and if $\{(x_m, x_n)\} \neq \{(x_i, x_j)\}$, then we get $N(x_m, x_n) \cap T = \{(x_i, x_j)\}$.

Definition 3.5 Let $G_{A,V}$ be a fuzzy soft graph. An edge (x_i, x_j) is said to be an isolated fuzzy soft edge if no edge incident on x_i and x_j for each parameter $e \in A$.

Theorem 3.6 If T_1 and T_2 are any two disjoint PFSED sets in a FSG and $T_1 \cup T_2 = E$, then $|T_1| = |T_2|$.

Proof: Let (x_i, x_j) be an edge in T_1 , then from the definition of PFSED set, there exist exactly one edge in the complement of T_1 which is adjacent to (x_i, x_j) . Thus for each edge in T_1 , there exist exactly one edge in T_2 , which is

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adjacent to (x_i, x_j) . Also for each edge (x_m, x_n) in T_2 , there exist an exactly one edge in T_1 , which is adjacent to (x_m, x_n) . Thus there exist a one to one correspondence between T_1 and T_2 .

Hence $|T_1| = |T_2|$.

Theorem 3.7 Let G^* be a cycle of length n . Then any set of $n-1$ edges always form a perfect fuzzy soft 2-edge dominating set.

Proof: Let G^* be a cycle of length n and $G_{A,V}$ be a fuzzy soft graph and let T be any set of $n-1$ edges. Then its complement contains only one vertex and which is adjacent to exactly two edges in T . There fore T is a perfect fuzzy soft 2-edge dominating set.

Theorem 3.8 Let G^* is an even cycle and $G_{A,V}$ be a fuzzy soft graph. Then the set of all alternating edges in it is always form a perfect fuzzy soft 2-edge dominating set.

Proof: Let $G_{A,V}$ be a fuzzy soft graph in an even cycle G^* and T be the set of all alternating edges. Then any edge in $E-T$ is adjacent to exactly 2-edges in T . Hence T is a perfect fuzzy soft 2-edge dominating set.

Remark: If $G_{A,V}$ be a fuzzy soft graph without isolated edges then for every minimal PFSED set T , its complement need not be a PFSED set.

Theorem 3.9[3] If $G_{A,V}$ be a complete fuzzy soft graph with even number of vertices, then there must exist FSED-t set and the minimum number of elements in the FSED-t set is $\frac{n}{2}$, where $n \geq 4$.

Theorem 3.10 If $G_{A,V}$ be a complete fuzzy soft graph with even number of vertices, then there must always exist PFSED-2 set.

Proof: By above theorem we get a FSED-t set and the minimum number of elements in the FSED-t set is $\frac{n}{2}$. So each element in the complement set is adjacent to exactly 2 edges in the dominating set. So the FSED set becomes a PFSED-2 set and hence the theorem.

Theorem 3.11 Suppose $G_{A,V}$ is an FSG without isolated edges then $\frac{S(G_{A,V})}{\Delta^1(G_{A,V}) + 1} \geq \gamma_{pfs}^1(G_{A,V})$.

Proof: Let P be a PFSED set. We have $|P| \Delta^1(G_{A,V}) \leq \sum_{(x_i, x_j) \in P} d(x_i, x_j)$

$$= \sum_{(x_i, x_j) \in P} |N(x_i, x_j)|$$

$$\leq \left| \bigcup_{(x_i, x_j) \in P} N(x_i, x_j) \right|$$

$$\leq S(G_{A,V}) - |P|$$

$$\therefore |P| \Delta^1(G_{A,V}) + |P| \leq S(G_{A,V})$$

$$\Rightarrow |P| \leq \frac{S(G_{A,V})}{\Delta^1(G_{A,V}) + 1}$$

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$$\Rightarrow \gamma_{pfs}^1(G_{A,V}) \leq \frac{S(G_{A,V})}{\Delta^1(G_{A,V}) + 1} .$$

Hence the theorem.

IV.CONCLUSION

In this paper we defined new concepts such as perfect fuzzy soft edge domination (PFSED), perfect fuzzy soft edge domination number, perfect fuzzy soft t-edge domination and perfect fuzzy soft t-edge domination number and proved some theorems related to this. The perfect fuzzy soft edge domination and perfect fuzzy soft edge domination number are very useful for solving wide range of problems.

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