

An Ingenious Mode for Reducing the Entries of a Matrix

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ABSTRACT: Matrix decomposition is a fundamental theme in linear algebra and applied statistics which has both scientific and engineering significance. Now suppose we are given a matrix A and we have to transform it into a product of two matrices ,say we want to write $A=BC$,this operation is called the decomposition of the matrix A ,or the factorization of the matrix A . In this paper we proposed a mode for factorization of a matrix ,such that this factorization reduces the entries of this matrix.,So our goal here is to minimize the number of the entries besides factorization of the matrix.

In this paper we will introduce a very simple algorithm with which we can factorize any matrix exactly to a product of any numbers of matrices by factorizing again each matrix of matrices which form the product .

KEYWORDS: Factor analysis, Matrix factorization, matrix rank ,column rank, rows operations.

I. INTRODUCTION

In numerical analysis In the mathematical discipline of linear algebra there are many operations defined on matrices ,and there are different types of matrices ,some of them based on the entries of the matrix such a zero matrix ,identity matrix ,,spars matrix and some of them depends on the shape of the matrix like square and rectangular one. Actually we note that in the real world it is not feasible for most of the matrix computations to be calculated in an optimal explicit way, such as matrix inversion . In linear algebra different methods of matrix decomposition are used to implement efficient matrix algorithm. For instance, when solving a system of linear equations ,In matrix factorization , the matrix A can be decomposed via the LU decompositions or any other approach. In factor analysis matrix factorization techniques are usually more effective because they allow us to discover the latent features underlying the interactions between users and items. Of course, matrix factorization is simply a mathematical tool for playing around with matrices, and is therefore applicable in many scenarios where one would like to find out something hidden under the data.

This paper was focused on the development of a new method based

On decomposing a matrix , such that this decomposition reduces the number of the entries of the decomposed matrix by writing it as a product of two matrices . In fact, we can mention that the reduction of this entries has two benefits, the first one if the number of entries is so large, and we want to save these data in our computer ,then these entries will occupy a big region of the memory of the computer, the second one if the amount of the entities is so big ,we will need more time to inter these data , so in this case the reduction of these data will save our time.

II. RELATED WORK

The matrix factorization concept has been presented and a new method base the column space of a matrix for exact decomposition of a matrix has been proposed. The performance of the proposed new method has been measured on numerical examples. The short time steps and the small number of steps involved in the process of the result showed that the new method is certainly a better choice for reducing the amount of entities of a matrix problems using matrix factorization. Since we can viewed multiplying a matrix B from left by a matrix A as a linear combination of B 's columns using coefficients from A , or another way to look at it is that it's a linear combination of the rows of A using coefficients from B , so we can take the columns of B equal to the basis of the column space of matrix M .In our work if

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we suppose that we have an $m \times n$ matrix M which has column rank r such that $r < n$, so M can be written as a product of two matrices B and A , i.e. $BA=M$ where B is an $m \times r$ matrix with columns as column basis of M , and A is an $r \times n$ matrix. The total amount of the entries of the two matrices B, A will be equal to $mr + rn = r(m + n)$.

Now we want the sum of the entries of the two matrices A, B to be less than the amount of the entities of matrix M , which means

$$mr + rn = r(m + n) < mn \Rightarrow r < \frac{mn}{m + n}, \text{ so the amount of the reduced data is equal to } mn - r(m + n)$$

From the equation $BA = M$ we can get A by multiplying both sides of this equation from left with B^T to get :

$$BA = M \Rightarrow B^T BA = B^T M$$

Multiplying both sides of this equation with $(B^T B)^{-1}$, we get

$$A = (B^T B)^{-1} B^T M$$

Note that when $r \ll m$ the amount of the reduced entries will be big e.g. If $m=1000, n=7, r=2$ then the amount of the reduced entities is

$$mn - r(m + n) = 7000 - 2 \times 1007 = 49986$$

III. NUMERICAL EXAMPLES

Example (1)

Now suppose we want to reduce the entities of the matrix

$$M = \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 2 & 1 & 3 & 0 & 1 & 1 \\ 3 & 3 & 6 & 3 & 6 & 5 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$$

First we do rows operations to find the rank of the matrix M .

Then we can use this rows operation to get the column basis of M as follows:

$$R_2 - 2R_1 \rightarrow R_2 \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 3 & 3 & 6 & 3 & 6 & 5 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$$

$$R_3 - 3R_1 \rightarrow R_3 \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$$

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$$R_4 - R_1 \rightarrow R_4 \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 - R_2 \rightarrow R_3 \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank } M = \text{Rank} \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 0 & -3 & -3 & -6 & -9 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = 2$$

We know that $r < \frac{mn}{m+n}$ since we have $2 < \frac{24}{10}$, then reduction is available and the amount of the reduction is $mn - r(m+n) = 24 - 20 = 4$

Now we can take the first two columns of M to get $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix}$

Then we have $BA = M \Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 2 & 1 & 3 & 0 & 1 & 1 \\ 3 & 3 & 6 & 3 & 6 & 5 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$

Multiplying both sides with B^T , we get

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 2 & 1 & 3 & 0 & 1 & 1 \\ 3 & 3 & 6 & 3 & 6 & 5 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$$

Multiplying again both sides of this equation with

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$$\left(\begin{matrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \end{bmatrix} \end{matrix} \right)^{-1} = \begin{bmatrix} 15 & 15 \\ 15 & 18 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/3 \\ -1/3 & 1/3 \end{bmatrix}$$

Then we have

$$A = \begin{bmatrix} 2/5 & -1/3 \\ -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 3 & 6 & 3 \\ 1 & 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 3 & 5 & 4 \\ 2 & 1 & 3 & 0 & 1 & 1 \\ 3 & 3 & 6 & 3 & 6 & 5 \\ 1 & 2 & 3 & 3 & 5 & 4 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 1 & -1 & -1 & -2/3 \\ 0 & 1 & 1 & 2 & 3 & 7/3 \end{bmatrix}$$

So we can write M as two matrices product

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & -1 & -1 & -\frac{2}{3} \\ 0 & 1 & 1 & 2 & 3 & \frac{7}{3} \end{bmatrix}$$

Example (2)

Let

$$A = \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$$R_2 - \left(\frac{7}{5}\right) \times R_1 \rightarrow R_2$$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

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$$R_3 - \left(\frac{8}{5}\right) \times R_1 \rightarrow R_3$$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$$R_4 - \left(\frac{3}{5}\right) \times R_1 \rightarrow R_4$$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$$R_5 - \left(\frac{4}{5}\right) \times R_1 \rightarrow R_5$$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{9}{5} \end{bmatrix}$$

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$$R_3 + R_2 \rightarrow R_3 \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{9}{5} \end{bmatrix}$$

$$R_4 + R_2 \rightarrow R_4 \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{9}{5} \end{bmatrix}$$

$$R_5 + 3R_2 \rightarrow R_5 \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank} \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix} = 2$$

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We know that $r < \frac{mn}{m+n}$, and we have $r=2$, $mn=5 \times 5=25$ and $m+n=5+5=10$, since $2 < \frac{25}{10}$, so reduction could be, and the amount of the reduction is $mn - r(m+n) = 25 - 20 = 5$. Now we can take the first two column

of A as the basis of the column space of A, so we take the matrix $B = \begin{bmatrix} 5 & 7 \\ 7 & 10 \\ 8 & 11 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$,

Recall that .

$$BA = M \Rightarrow B^T BA = B^T M$$

$$\Rightarrow \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \end{bmatrix} \cdot A = \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 163 & 225 \\ 225 & 311 \end{bmatrix} \cdot A = \begin{bmatrix} 163 & 225 & 264 & 101 & 140 \\ 225 & 311 & 364 & 139 & 192 \end{bmatrix}$$

Now multiplying both sides of this equation by

$$\begin{bmatrix} 163 & 225 \\ 225 & 311 \end{bmatrix}^{(-1)} = \begin{bmatrix} \frac{311}{68} & \frac{-225}{68} \\ \frac{-225}{68} & \frac{163}{68} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{311}{68} & \frac{-225}{68} \\ \frac{-225}{68} & \frac{163}{68} \end{bmatrix} \begin{bmatrix} 163 & 225 \\ 225 & 311 \end{bmatrix} \cdot A = D^{-1} B^T M$$

$$A = \begin{bmatrix} \frac{311}{68} & \frac{-225}{68} \\ \frac{-225}{68} & \frac{163}{68} \end{bmatrix} \begin{bmatrix} 163 & 225 & 264 & 101 & 140 \\ 225 & 311 & 364 & 139 & 192 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 3 & 2 & 5 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix}$$

Verification

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$$\begin{aligned}
 BA &= \begin{bmatrix} 5 & 7 \\ 7 & 10 \\ 8 & 11 \\ 3 & 4 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 2 & 5 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix} = M
 \end{aligned}$$

IV. RESULTS

As we see the numerical example which we used showed us a courageous and good results ,so this new ingenious mode is very efficient, effective, and accurate for factorizing a matrix besides reducing the entries of a matrix.

V. CONCLUSION

Since this new method is effective and accurate ,so it could be used in neural net work program to modify the performance of neural net work for factor analysis or any other mathematical branch, such as statistic or engineering fields .

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