

# Hexagonal Difference Prime Labeling of Some Cycle Related Graphs

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**ABSTRACT:** The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. In this paper we developed a new kind of labeling, called hexagonal difference prime labeling. Here the vertices of the graph are labeled with hexagonal numbers and the edges are labeled with absolute difference of the end vertex labels. Here the greatest common incidence number (g c i n) of a vertex of degree greater than one is defined as the g c d of the labels of the incident edges. If the g c i n of each vertex of degree greater than one is 1, then the graph admits hexagonal difference prime labeling. Here we characterize some cycle related graphs for hexagonal difference prime labeling.

**KEYWORDS:** Graph labeling, hexagonal numbers, g c i n, prime labeling, cycle.

## I. INTRODUCTION

In this paper we deal with graphs that are connected, simple, finite and undirected. The symbol  $V$  and  $E$  denote the vertex set and edge set of a graph  $G$ . The graph whose cardinality of the vertex set is called the order of  $G$ , denoted by  $p$  and the cardinality of the edge set is called the size of the graph  $G$ , denoted by  $q$ . A graph with  $p$  vertices and  $q$  edges is called a  $(p,q)$ - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated the hexagonal difference prime labeling of some cycle related graphs.

**Definition: 1.1** Let  $G$  be a graph with  $p$  vertices and  $q$  edges. The greatest common incidence number of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

**Definition: 1.2**  $n^{\text{th}}$  hexagonal number is  $n(2n-1)$ , where  $n$  is a positive integer. The hexagonal numbers are 1, 6, 15, 28, 45, 66-----.

## II. MAIN RESULTS

**Definition 2.1** Let  $G$  be a graph with  $p$  vertices and  $q$  edges. Define a bijection

$f : V(G) \rightarrow \{1,6,15,28, \dots, p(2p-1)\}$  by  $f(v_i) = i(2i - 1)$ , for every  $i$  from 1 to  $p$  and define a 1-1 mapping  $f_{hdpl}^* : E(G) \rightarrow$  set of natural numbers  $N$  by  $f_{hdpl}^*(uv) = |f(u)-f(v)|$ . The induced function  $f_{hdpl}^*$  is said to be hexagonal difference prime labeling, if the g c i n of each vertex of degree at least 2, is one.

**Definition 2.2** A graph which admits hexagonal difference prime labeling is called hexagonal difference prime graph.

**Theorem: 2.1** The cycle  $C_n$  admits hexagonal difference prime labeling, when  $n \equiv 0(mod5)$ ,  $(n + 1) \equiv 0(mod5)$  and  $(n + 2) \equiv 0(mod5)$ .

**Proof :** Let  $G = C_n$  and let  $v_1, v_2, \dots, v_n$  are the vertices of  $G$ .

Here  $|V(G)| = n$  and  $|E(G)| = n$ .

Define a function  $f : V \rightarrow \{1,6,15,28, \dots, n(2n-1)\}$  by

$$f(v_i) = i(2i-1), i = 1,2, \dots, n.$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{hdpl}^*$  is defined as follows

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$$\begin{aligned} f_{hdpl}^*(v_i v_{i+1}) &= (4i+1), & i = 1,2,\dots,n-1 \\ f_{hdpl}^*(v_1 v_n) &= 2n^2-n-1. \end{aligned}$$

Clearly  $f_{hdpl}^*$  is an injection.

$$\begin{aligned} \text{g c i n of } (v_{i+1}) &= \text{g c d of } \{ f_{hdpl}^*(v_i v_{i+1}), f_{hdpl}^*(v_{i+1} v_{i+2}) \} \\ &= \text{g c d of } \{ (4i+1), (4i+5) \} \\ &= \text{g c d of } \{ 4, (4i+1) \} = 1, & i = 1,2,\dots,n-2 \\ \text{g c i n of } (v_1) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_2), f_{hdpl}^*(v_1 v_n) \} = \text{g c d of } \{ 5, 2n^2-n-1 \} = 1. \\ \text{g c i n of } (v_n) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_n), f_{hdpl}^*(v_{n-1} v_n) \} = \text{g c d of } \{ 4n-3, 2n^2-n-1 \} = 1. \end{aligned}$$

So, g c i n of each vertex of degree greater than one is 1.

Hence  $C_n$ , admits hexagonal difference prime labeling.

**Theorem 2.2** Two tuple graph of cycle  $C_n$ , admits hexagonal difference prime labeling.

**Proof :** Let  $G = T^2(P_n)$  and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of  $G$ .

Here  $|V(G)| = 2n$  and  $|E(G)| = 3n$ .

Define a function  $f : V \rightarrow \{1,6,15,28, \dots, 2n(4n-1)\}$  by

$$f(v_i) = i(2i-1), \quad i = 1,2,\dots,2n.$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{hdpl}^*$  is defined as follows

$$\begin{aligned} f_{hdpl}^*(v_{2i-1} v_{2i+1}) &= (16i-2), & i = 1,2,\dots,n-1 \\ f_{hdpl}^*(v_{2i-1} v_{2i}) &= (8i-3), & i = 1,2,\dots,n. \\ f_{hdpl}^*(v_{2i} v_{2i+2}) &= (16i+6), & i = 1,2,\dots,n-1 \\ f_{hdpl}^*(v_1 v_{2n-1}) &= 8n^2-10n+2. \\ f_{hdpl}^*(v_2 v_{2n}) &= 8n^2-2n-6. \end{aligned}$$

Clearly  $f_{hdpl}^*$  is an injection.

$$\begin{aligned} \text{g c i n of } (v_2) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_2), f_{hdpl}^*(v_2 v_4) \} = \text{g c d of } \{ 5, 22 \} = 1. \\ \text{g c i n of } (v_{2n-1}) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_{2n-1}), f_{hdpl}^*(v_{2n-1} v_{2n}), f_{hdpl}^*(v_{2n-3} v_{2n-1}) \} \\ &= \text{g c d of } \{ 8n^2-10n+2, 8n-3, 16n-18 \} = 1 \\ \text{g c i n of } (v_{2i-1}) &= \text{g c d of } \{ f_{hdpl}^*(v_{2i-1} v_{2i}), f_{hdpl}^*(v_{2i-1} v_{2i+1}) \} \\ &= \text{g c d of } \{ 8i-3, 16i-2 \} = 1, & i = 1,2,\dots,n-1 \\ \text{g c i n of } (v_{2i+2}) &= \text{g c d of } \{ f_{hdpl}^*(v_{2i+2} v_{2i}), f_{hdpl}^*(v_{2i+1} v_{2i+2}) \} \\ &= \text{g c d of } \{ 8i+5, 16i+6 \} = 1, & i = 1,2,\dots,n-1 \end{aligned}$$

So, g c i n of each vertex of degree greater than one is 1.

Hence  $T^2(C_n)$ , admits hexagonal difference prime labeling.

**Theorem 2.3** Total graph of cycle  $C_n$ , admits hexagonal difference prime labeling.

**Proof :** Let  $G = T(C_n)$  and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of  $G$ .

Here  $|V(G)| = 2n$  and  $|E(G)| = 4n$ .

Define a function  $f : V \rightarrow \{1,6,15,28, \dots, (2n)(4n-1)\}$  by

$$f(v_i) = i(2i-1), \quad i = 1,2,\dots,2n.$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{hdpl}^*$  is defined as follows

$$\begin{aligned} f_{hdpl}^*(v_i v_{i+1}) &= (4i+1), & i = 1,2,\dots,2n-1 \\ f_{hdpl}^*(v_{2i} v_{2i+2}) &= 16i+6, & i = 1,2,\dots,n-1 \\ f_{hdpl}^*(v_{2i-1} v_{2i+1}) &= 16i-2, & i = 1,2,\dots,n-1 \\ f_{hdpl}^*(v_1 v_{2n}) &= 8n^2-2n-1, \\ f_{hdpl}^*(v_2 v_{2n}) &= 8n^2-2n-6, \\ f_{hdpl}^*(v_1 v_{2n-1}) &= 8n^2-10n+2. \end{aligned}$$

Clearly  $f_{hdpl}^*$  is an injection.

$$\text{g c i n of } (v_{i+1}) = 1, \quad i = 1,2,\dots,2n-2$$

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$$\begin{aligned} \text{g c i n of } (v_1) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_2), f_{hdpl}^*(v_1 v_3) \} = \text{g c d of } \{ 5, 14 \} = 1. \\ \text{g c i n of } (v_{2n}) &= \text{g c d of } \{ f_{hdpl}^*(v_1 v_{2n}), f_{hdpl}^*(v_2 v_{2n}), f_{hdpl}^*(v_{2n-1} v_{2n}), f_{hdpl}^*(v_{2n-2} v_{2n}) \} \\ &= 1. \end{aligned}$$

So, g c i n of each vertex of degree greater than one is 1.

Hence  $T(C_n)$ , admits hexagonal difference prime labeling.

**Theorem 2.4** Corona of cycle  $C_n$ , admits hexagonal difference prime labeling.

**Proof :** Let  $G = C_n \odot K_1$  and let  $v_1, v_2, \dots, v_{2n}$  are the vertices of  $G$ .

Here  $|V(G)| = 2n$  and  $|E(G)| = 2n$ .

Define a function  $f : V \rightarrow \{1, 6, 15, 28, \dots, (2n)(4n-1)\}$  by

$$f(v_i) = i(2i-1), \quad i = 1, 2, \dots, 2n.$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{hdpl}^*$  is defined as follows

$$\begin{aligned} f_{hdpl}^*(v_i v_{i+1}) &= (4i+1), & i = 1, 2, \dots, n+1 \\ f_{hdpl}^*(v_{i+2} v_{n+i+2}) &= 2n^2+4ni+7n, & i = 1, 2, \dots, n-2 \\ f_{hdpl}^*(v_2 v_{n+1}) &= 2n^2+3n-5. \end{aligned}$$

Clearly  $f_{hdpl}^*$  is an injection.

$$\text{g c i n of } (v_{i+1}) = 1, \quad i = 1, 2, \dots, n.$$

So, g c i n of each vertex of degree greater than one is 1.

Hence  $C_n \odot K_1$ , admits hexagonal difference prime labeling.

**Theorem 2.5** The graph  $C_n(2P_m)$ , admits hexagonal difference prime labeling.

**Proof :** Let  $G = C_n(2P_m)$  and let  $v_1, v_2, \dots, v_{2m+n-2}$  are the vertices of  $G$ .

Here  $|V(G)| = 2m+n-2$  and  $|E(G)| = 2m+n-2$ .

Define a function  $f : V \rightarrow \{1, 6, 15, 28, \dots, (2m+n-2)(4m+2n-5)\}$  by

$$f(v_i) = i(2i-1), \quad i = 1, 2, \dots, 2m+n-2$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{hdpl}^*$  is defined as follows

$$\begin{aligned} f_{hdpl}^*(v_i v_{i+1}) &= (4i+1), & i = 1, 2, \dots, 2m+n-3 \\ f_{hdpl}^*(v_m v_{m+n-1}) &= 2n^2-5n+3+4mn-4m. \end{aligned}$$

Clearly  $f_{hdpl}^*$  is an injection.

$$\text{g c i n of } (v_{i+1}) = 1, \quad i = 1, 2, \dots, 2m+n-4.$$

So, g c i n of each vertex of degree greater than one is 1.

Hence  $C_n(2P_m)$ , admits hexagonal difference prime labeling.

### III. CONCLUSION

In this paper we have introduced a new concept 'greatest common incidence number' of a vertex and successfully proved that some cycle related graphs admit hexagonal difference prime labeling.

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