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## Operators in Mathematics

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**ABSTRACT:** In mathematics, an operator is generally a mapping or function that acts on elements of a space to produce elements of another space (possibly and sometimes required to be the same space). There is no general definition of an operator, but the term is often used in place of function when the domain is a set of functions or other structured objects. Also, the domain of an operator is often difficult to characterize explicitly (for example in the case of an integral operator), and may be extended so as to act on related objects (an operator that acts on functions may act also on differential equations whose solutions are functions that satisfy the equation).

**KEYWORDS**-operator, mathematics, function, equations, domain

### I.INTRODUCTION

The most basic operators are linear maps, which act on vector spaces. Linear operators refer to linear maps whose domain and range are the same space, for example from  $\mathbb{R}^n$  to  $\mathbb{R}^{n^{[1][2][a]}}$ . Such operators often preserve properties, such as continuity. For example, differentiation and indefinite integration are linear operators; operators that are built from them are called differential operators, integral operators or integro-differential operators.

Operator is also used for denoting the symbol of a mathematical operation. This is related with the meaning of "operator" in computer programming; see Operator (computer programming).

The important concepts directly related to operators between finite-dimensional vector spaces are the ones of rank, determinant, inverse operator, and eigenspace.

Linear operators also play a great role in the infinite-dimensional case. The concepts of rank and determinant cannot be extended to infinite-dimensional matrices. This is why very different techniques are employed when studying linear operators (and operators in general) in the infinite-dimensional case. The study of linear operators in the infinite-dimensional case is known as functional analysis (so called because various classes of functions form interesting examples of infinite-dimensional vector spaces).[1,2,3]

The space of sequences of real numbers, or more generally sequences of vectors in any vector space, themselves form an infinite-dimensional vector space. The most important cases are sequences of real or complex numbers, and these spaces, together with linear subspaces, are known as sequence spaces. Operators on these spaces are known as sequence transformations.

Bounded linear operators over a Banach space form a Banach algebra in respect to the standard operator norm. The theory of Banach algebras develops a very general concept of spectra that elegantly generalizes the theory of eigenspaces.

Three operators are key to vector calculus:

- Grad (gradient), (with operator symbol  $\nabla$ ) assigns a vector at every point in a scalar field that points in the direction of greatest rate of change of that field and whose norm measures the absolute value of that greatest rate of change.
- Div (divergence), (with operator symbol  $\nabla \cdot$ ) is a vector operator that measures a vector field's divergence from or convergence towards a given point.

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- Curl, (with operator symbol  $\nabla \times$ ) is a vector operator that measures a vector field's curling (winding around, rotating around) trend about a given point.

As an extension of vector calculus operators to physics, engineering and tensor spaces, grad, div and curl operators also are often associated with tensor calculus as well as vector calculus.<sup>[3]</sup>

## II.DISCUSSION

In geometry, additional structures on vector spaces are sometimes studied. Operators that map such vector spaces to themselves bijectively are very useful in these studies, they naturally form groups by composition.

For example, bijective operators preserving the structure of a vector space are precisely the invertible linear operators. They form the general linear group under composition. However, they do not form a vector space under operator addition; since, for example, both the identity and  $-$ identity are invertible (bijective), but their sum, 0, is not.

Operators preserving the Euclidean metric on such a space form the isometry group, and those that fix the origin form a subgroup known as the orthogonal group. Operators in the orthogonal group that also preserve the orientation of vector tuples form the special orthogonal group, or the group of rotations.<sup>[3,4,5]</sup>

Operators are also involved in probability theory, such as expectation, variance, and covariance, which are used to name both number statistics and the operators which produce them. Indeed, every covariance is basically a dot product: Every variance is a dot product of a vector with itself, and thus is a quadratic norm; every standard deviation is a norm (square root of the quadratic norm); the corresponding cosine to this dot product is the Pearson correlation coefficient; expected value is basically an integral operator (used to measure weighted shapes in the space).

### Fourier series and Fourier transform

The Fourier transform is useful in applied mathematics, particularly physics and signal processing. It is another integral operator; it is useful mainly because it converts a function on one (temporal) domain to a function on another (frequency) domain, in a way effectively invertible. No information is lost, as there is an inverse transform operator. In the simple case of periodic functions, this result is based on the theorem that any continuous periodic function can be represented as the sum of a series of sine waves and cosine waves

## III.RESULTS

In functional analysis, a branch of mathematics, an operator algebra is an algebra of continuous linear operators on a topological vector space, with the multiplication given by the composition of mappings.

The results obtained in the study of operator algebras are often phrased in algebraic terms, while the techniques used are often highly analytic.<sup>[1]</sup> Although the study of operator algebras is usually classified as a branch of functional analysis, it has direct applications to representation theory, differential geometry, quantum statistical mechanics, quantum information, and quantum field theory.

Operator algebras can be used to study arbitrary sets of operators with little algebraic relation simultaneously. From this point of view, operator algebras can be regarded as a generalization of spectral theory of a single operator. In general operator algebras are non-commutative rings.

An operator algebra is typically required to be closed in a specified operator topology inside the whole algebra of continuous linear operators. In particular, it is a set of operators with both algebraic and topological closure properties. In some disciplines such properties are axiomized and algebras with certain topological structure become the subject of the research.

Though algebras of operators are studied in various contexts (for example, algebras of pseudo-differential operators acting on spaces of distributions), the term operator algebra is usually used in reference to algebras of bounded operators on a Banach space or, even more specially in reference to algebras of operators on a separable Hilbert space, endowed with the operator norm topology.<sup>[4,5,6]</sup>

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In the case of operators on a Hilbert space, the Hermitian adjoint map on operators gives a natural involution, which provides an additional algebraic structure that can be imposed on the algebra. In this context, the best studied examples are self-adjoint operator algebras, meaning that they are closed under taking adjoints. These include C\*-algebras, von Neumann algebras, and AW\*-algebra. C\*-algebras can be easily characterized abstractly by a condition relating the norm, involution and multiplication. Such abstractly defined C\*-algebras can be identified to a certain closed subalgebra of the algebra of the continuous linear operators on a suitable Hilbert space. A similar result holds for von Neumann algebras.

Commutative self-adjoint operator algebras can be regarded as the algebra of complex-valued continuous functions on a locally compact space, or that of measurable functions on a standard measurable space. Thus, general operator algebras are often regarded as a noncommutative generalizations of these algebras, or the structure of the base space on which the functions are defined. This point of view is elaborated as the philosophy of noncommutative geometry, which tries to study various non-classical and/or pathological objects by noncommutative operator algebras.

Examples of operator algebras that are not self-adjoint include:

- nest algebras,
- many commutative subspace lattice algebras,
- many limit algebras.

The Mathematical Operators block has sixteen variation sequences defined for standardized variants.<sup>[3][4]</sup> They use U+FE00 VARIATION SELECTOR-1 (VS01) to denote variant symbols (depending on the font):

Variation sequences			
Base character	Base	+VS01	Description
U+2205 EMPTY SET	∅	∅ <sup>VS01</sup>	zero with long diagonal stroke overlay form
U+2229 INTERSECTION	∩	∩ <sup>VS01</sup>	with serifs
U+222A UNION	∪	∪ <sup>VS01</sup>	with serifs
U+2268 LESS-THAN BUT NOT EQUAL TO	≠	≠ <sup>VS01</sup>	with vertical stroke
U+2269 GREATER-THAN BUT NOT EQUAL TO	≠	≠ <sup>VS01</sup>	with vertical stroke
U+2272 LESS-THAN OR EQUIVALENT TO	≲	≲ <sup>VS01</sup>	following the slant of the lower leg
U+2273 GREATER-THAN OR EQUIVALENT TO	≳	≳ <sup>VS01</sup>	following the slant of the lower leg
U+228A SUBSET OF WITH NOT EQUAL TO	⊄	⊄ <sup>VS01</sup>	with stroke through bottom members
U+228B SUPERSET OF WITH NOT EQUAL TO	⊅	⊅ <sup>VS01</sup>	with stroke through bottom members
U+2293 SQUARE CAP	⊐	⊐ <sup>VS01</sup>	with serifs
U+2294 SQUARE CUP	⊑	⊑ <sup>VS01</sup>	with serifs
U+2295 CIRCLED PLUS	⊕	⊕ <sup>VS01</sup>	with white rim
U+2297 CIRCLED TIMES	⊗	⊗ <sup>VS01</sup>	with white rim
U+229C CIRCLED EQUALS	⊖	⊖ <sup>VS01</sup>	with equal sign touching the circle
U+22DA LESS-THAN EQUAL TO OR GREATER-THAN	≧	≧ <sup>VS01</sup>	with slanted equal
U+22DB GREATER-THAN EQUAL TO OR LESS-THAN	≦	≦ <sup>VS01</sup>	with slanted equal

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Supplemental Mathematical Operators is a Unicode block containing various mathematical symbols, including N-ary operators, summations and integrals, intersections and unions, logical and relational operators, and subset/superset relations

Supplemental Mathematical Operators <sup>[1]</sup>		Official Unicode Consortium code chart (PDF)														
	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
U+2A0x	⊙	⊕	⊗	⊔	⊓	∏	∏	∞	∞	×	∑	∑	∫	∫	∫	∫
U+2A1x	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫	∫
U+2A2x	≫	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	×
U+2A3x	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
U+2A4x	∩	∪	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2A5x	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2A6x	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2A7x	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2A8x	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2A9x	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2AAx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2ABx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2ACx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2ADx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2AEx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
U+2AFx	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩

## IV.CONCLUSION

Mathematical Operators is a Unicode block containing characters for mathematical, logical, and set notation. Notably absent are the plus sign (+), greater than sign (>) and less than sign (<), due to them already appearing in the Basic Latin Unicode block, and the plus-or-minus sign (±), multiplication sign (×) and obelus (÷), due to them already appearing in the Latin-1 Supplement block, although a distinct minus sign (−) is included, semantically different from the Basic Latin hyphen-minus (-).[7]

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