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The Variance of Busy Period Analysis of M/M/2/ ∞ Interdependent Queueing Model with Controllable Arrival Rates and Feedback

G. Rani¹, A. Thiposiny²

Associate Professor, PG and Research Department of Mathematics, Seethalakshmi Ramaswami College,
Tiruchirappalli, Tamilnadu, India¹

Research Scholar, PG and Research Department of Mathematics, Seethalakshmi Ramaswami College, Tiruchirappalli,
Tamilnadu, India²

ABSTRACT: In this paper the variance of busy period analysis of M/M/2/ ∞ interdependent *queueing model* with controllable arrival rates and feedback is considered. Distribution of busy period, mean length of busy period and variance of busy period carried out for this model. The analytical results are numerically illustrated and the effect of the nodal parameters on the expected length of busy period is studied and relevant conclusions are presented.

KEYWORDS: Two server Markovian queueing model, Controllable arrival rates, Feedback infinite capacity, Variance of busy period analysis.

I. INTRODUCTION

In the earlier work, Rani and Srinivasan [9] have analysed the busy period analysis of interdependent queueing model with controllable arrival rates and feedback are employed and obtained the average length of the busy period for the model. Takagi and Tarabia [8] have studied a explicit probability density function for the length of a busy period for a finite capacity. Artalejo and Lopez Herrero [1] have analysed the busy period of the retrial queue with general service time distribution. Gopalan [2] has analysed the busy period analysis of a two-stage multi-product system. Soren Asmussen [6] has studied a busy period analysis, rare Events and Transient behaviour in fluid flow Models. In this paper, the variance of busy period analysis of two server M/M/2/ ∞ interdependent queueing model with controllable arrival rates and feedback is considered. In section II, the description of the queueing model is given stating the relevant postulates. In section III, differential-difference equations are derived. In section IV, busy period analysis of the model and average length of the busy period of the model is obtained for faster rate of arrivals, variance of the busy period of the model is obtained for faster rate of arrivals. In section V, the analytical results are numerically illustrated and relevant conclusion is presented based upon a hypothetical data for faster rate of arrivals, VI, busy period analysis of the model and average length of the busy period of the model is obtained for slower rate of arrivals, variance of the busy period of the model is obtained for slower rate of arrivals, VII, the analytical results are numerically illustrated and relevant conclusion is presented based upon a hypothetical data for faster rate of arrivals.

II. DESCRIPTION OF THE MODEL

Consider two server infinite capacity queueing system with controllable arrival rate and feedback. Customer arrive at the service station one by one according to a bivariate Poisson stream with arrival rates $(\lambda_0 - \epsilon)$, $(\lambda_1 - \epsilon)$ (> 0). There are two

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servers which provide service to all the arriving customers. Service times are identically and independently distributed exponential random variables with mean $(\mu - \varepsilon)$ (> 0). After the completion of each service, the customers can either join at the end of the queue with probability p or customers can leave the system with probability q , $p + q = 1$. The customer both newly arrived and those who opted for feedback are served in the order in which they join the tail of the original queue. It is assumed that there is no difference between regular arrivals and feedback arrivals. The customers are served according to the first come first served rule with a busy period as the interval of time from the instant customers arrive either with feedback or without feedback at an empty system and their service begins to the instant when the server becomes free for the first time. It is assumed that, the arrival process $\{X_1(t)\}$ and the service process $\{X_2(t)\}$ of the system are correlated and follow a bivariate Poisson distribution is given by

$$P[X_1(t) = x_1, X_2(t) = x_2] = \frac{e^{-(\lambda_i + \mu - \varepsilon)t} \sum_{j=0}^{\min(x_1, x_2)} (\varepsilon t)^j [(\lambda_i - \varepsilon)t]^{x_1 - j} [(\mu - \varepsilon)t]^{x_2 - j}}{j! (x_1 - j)! (x_2 - j)!}$$

Where $x_1, x_2 = 1, 2, \dots$, and $\lambda_0, \lambda_1, \mu > 0, 0 \leq \varepsilon < \min(\lambda_i, \mu)$, ($i = 0, 1$) with parameters $\lambda_0, \lambda_1, \mu$ and ε for mean faster rate of arrivals, mean slower rate of arrivals either with feedback or without feedback, identical mean service rate for first server, second server and mean dependence rate respectively. Also the state space of the system is

$$S = \{1, 2, 3, \dots, r - 1, r, r + 1, \dots, R - 1, R, R + 1, \dots, K - 1, K, K + 1, \dots\}$$

Postulates of the model are

1. Probability that there is no arrival and no service completion during a small interval of time h , when the system is in faster rate of arrivals either with feedback or without feedback, is $1 - [(\lambda_0 - \varepsilon) + 2p(\mu - \varepsilon) + 2q(\mu - \varepsilon)]h + o(h)$
2. Probability that there is one arrival and no service completion during a small interval of time h , when the system is in faster rate of arrivals either with feedback or without feedback, is $(\lambda_0 - \varepsilon)h + o(h)$
3. Probability that there is no arrival and no service completion during a small interval of time h , when the system is in slower rate of arrivals either with feedback or without feedback, is $1 - [(\lambda_1 - \varepsilon) + 2p(\mu - \varepsilon) + 2q(\mu - \varepsilon)]h + o(h)$
4. Probability that there is one arrival and no service completion during a small interval of time h , when the system is in slower rate of arrivals either with feedback or without feedback, is $(\lambda_1 - \varepsilon)h + o(h)$
5. Probability that there is no arrival and one service completion during a small interval of time h , when the system is either in faster or is in slower rate of arrivals either with feedback or without feedback, is $[2p(\mu - \varepsilon) + 2q(\mu - \varepsilon)]h + o(h)$
6. Probability that there is one arrival and one service completion during a small interval of time h , when the system is either in faster or slower rate of arrivals either with feedback or without feedback, is $[(\lambda_0 - \varepsilon) + (\lambda_1 - \varepsilon) + 2p(\mu - \varepsilon) + 2q(\mu - \varepsilon)]h + o(h)$

III. DIFFERENTIAL – DIFFERENCE EQUATIONS

$P_n(t_0)$: The probability that there are n customers in the system at time t_0 when the system is in faster rate of arrival either with feedback or without feedback.

$P_n(t_1)$: The probability that there are n customers in the system at time t_1 when the system is in slower rate of arrival either with feedback or without feedback.

We observe that $P_n(t_0)$ exists when $n = 0, 1, 2, 3, \dots, c-1, c, r-1, r$; both $P_n(t_0)$ and $P_n(t_1)$ exist when $n = r+1, r+2, r+3, \dots, R-2, R-1$; only $P_n(t_1)$ exist when $n = R, R+1, R+2, \dots$. Assume that the initial system size when the system is in faster rate of arrivals is 1 either with feedback or without feedback and that of when the system is in slower rate of arrivals is $r+1$.

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Let $P_0'(t_0)$ be the busy period density when the system is in faster rate of arrival either with feedback or without feedback and $P_{r+1}'(t_1)$ be the busy period density when the system is in slower rate of arrivals either with feedback or without feedback.

The differential – difference equations governing the system size with an absorbing barrier imposed at zero system size when the system is in faster rate of arrivals either with feedback or without feedback and with an absorbing barrier imposed at (r+1) system size when the system is in slower rate of arrivals either with feedback or without feedback are

$$P_0'(t_0) = [2q(\mu - \varepsilon)]P_1(t_0) \quad \dots(1)$$

(because of the absorbing barrier)

$$P_1'(t_0) = -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]P_1(t_0) + 2q(\mu - \varepsilon)P_2(t_0) \quad \dots(2)$$

(because of the absorbing barrier)

$$P_n'(t_0) = -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]P_n(t_0) + 2q(\mu - \varepsilon)P_{n+1}(t_0) + (\lambda_0 - \varepsilon)P_{n-1}(t_0) \quad n = 2, 3, 4, \dots, r-1 \quad \dots(3)$$

$$P_r'(t_0) = -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]P_r(t_0) + 2q(\mu - \varepsilon)P_{r+1}(t_0) + (\lambda_0 - \varepsilon)P_{r-1}(t_0) + 2q(\mu - \varepsilon)P_{r+1}(t_0) \quad \dots(4)$$

$$P_n'(t_0) = -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]P_n(t_0) + 2q(\mu - \varepsilon)P_{n+1}(t_0) + (\lambda_0 - \varepsilon)P_{n-1}(t_0) \quad n = r + 1, r + 2, \dots, R - 2 \quad \dots(5)$$

$$P_{R-1}'(t_0) = -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]P_{R-1}(t_0) + (\lambda_0 - \varepsilon)P_{R-2}(t_0) \quad \dots(6)$$

(because of the absorbing barrier)

$$P_{r+1}'(t_1) = [2q(\mu - \varepsilon)]P_{r+2}(t_1) \quad \dots(7)$$

(because of the absorbing barrier)

$$P_{r+2}'(t_1) = -[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]P_{r+2}(t_1) + 2q(\mu - \varepsilon)P_{r+3}(t_1) \quad \dots(8)$$

(because of the absorbing barrier)

$$P_n'(t_1) = -[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]P_n(t_1) + 2q(\mu - \varepsilon)P_{n+1}(t_1) + (\lambda_1 - \varepsilon)P_{n-1}(t_1) \quad n = r + 3, r + 4, \dots, R - 1 \quad \dots(9)$$

$$P_R'(t_1) = -[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]P_R(t_1) + 2q(\mu - \varepsilon)P_{R+1}(t_1) + (\lambda_1 - \varepsilon)P_{R-1}(t_1) + (\lambda_0 - \varepsilon)P_{R-1}(t_0) \quad \dots(10)$$

$$P_n'(t_1) = -[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]P_n(t_1) + 2q(\mu - \varepsilon)P_{n+1}(t_1) + (\lambda_1 - \varepsilon)P_{n-1}(t_1) \quad n = R + 1, R + 2, R + 3, \dots \quad \dots(11)$$

Let $\frac{\rho(0)}{2} = \frac{\lambda_0 - \varepsilon}{2q(\mu - \varepsilon)}$ and $\frac{\rho(1)}{2} = \frac{\lambda_1 - \varepsilon}{2q(\mu - \varepsilon)}$

where $\frac{\rho(0)}{2}$ is faster rate of arrivals intensity and $\frac{\rho(1)}{2}$ is slower rate of arrivals intensity.

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IV. BUSY PERIOD ANALYSIS OF THE MODEL

Define

$$P(z, t_0) = \sum_{n=0}^{R-1} P_n(t_0) z^n \quad \dots (12)$$

$$P(z, t_1) = \sum_{n=r+1}^{\infty} P_n(t_1) z^n \quad \dots (13)$$

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Such that the summation is convergent in and on unit circle. When (1) to (6) are multiplied through by z^n for $n = 0, 1, 2, \dots, R-1$ and summing over from $n = 0, 1, 2, \dots, R-1$, it is found that

$$\begin{aligned} \frac{d}{dt_0} [P(z, t_0)] &= \sum_{n=0}^{R-1} P_n'(t_0) z^n \\ &= P_0'(t_0) + P_1'(t_0)z + \dots + P_{R-1}'(t_0)z^{R-1} \\ &= -[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)] \sum_{n=0}^{R-1} P_n(t_0) z^n + (\lambda_0 - \varepsilon)z \sum_{n=1}^{R-1} P_{n-1}(t_0) z^{n-1} + \frac{2q(\mu - \varepsilon)}{z} \sum_{n=0}^{R-1} P_{n+1}(t_0) z^{n+1} \\ &\quad + 2q(\mu - \varepsilon) P_{r+1}(t_1) z^r + \left\{ [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)] - (\lambda_0 - \varepsilon)z - 2q \frac{(\mu - \varepsilon)}{z} \right\} P_0(t_0) \\ z \frac{d}{dt_0} [P(z, t_0)] &= [(\lambda_0 - \varepsilon)z^2 - [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]z + 2q(\mu - \varepsilon)]P(z, t_0) \\ &\quad + [-(\lambda_0 - \varepsilon)z^2 + [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]z - 2q(\mu - \varepsilon)]P_0(t_0) \\ &\quad + 2q(\mu - \varepsilon)z^{r+1}P_{r+1}(t_1) \quad \dots(14) \end{aligned}$$

Taking Laplace transform on both sides of (14) we get

$$\begin{aligned} z[s_0 \bar{P}(z, s_0) - P(z, 0)] &= [(\lambda_0 - \varepsilon)z^2 - [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]z + 2q(\mu - \varepsilon)]\bar{P}(z, s_0) + 2q(\mu - \varepsilon)z^{r+1}\bar{P}_{r+1}(s_0) \\ &\quad + [-(\lambda_0 - \varepsilon)z^2 + [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon)]z - 2q(\mu - \varepsilon)]\bar{P}_0(s_0) \\ \bar{P}(z, s_0) &= \frac{z^2 - (1-z)[2q(\mu - \varepsilon) - (\lambda_0 - \varepsilon)z]\bar{P}_0(s_0) + 2q(\mu - \varepsilon)z^{r+1}\bar{P}_{r+1}(s_1)}{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0]z - 2q(\mu - \varepsilon) - (\lambda_0 - \varepsilon)z^2} \quad \dots (15) \end{aligned}$$

Where

$$L\{P(z, t_0)\} = \bar{P}(z, s_0)$$

$$L\{P_0(t_0)\} = \bar{P}_0(s_0), \quad L\{P_{r+1}(t_1)\} = \bar{P}_{r+1}(s_1) \quad \text{and} \quad P(z, 0) = z$$

Since the Laplace transform $\bar{P}(z, s_0)$ converges in the region $|z| \leq 1, \text{Real}(s_0) \geq 0$, wherever the denominator of the right hand side of (15) has zeros in that region, so must the numerator, the denominator has two zeros, that is

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$$(\lambda_0 - \varepsilon)z^2 - [(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0]z + 2q(\mu - \varepsilon) = 0$$

$$z_1^{(0)} = \frac{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0] - \sqrt{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0]^2 - 8(\lambda_0 - \varepsilon)q(\mu - \varepsilon)}}{2(\lambda_0 - \varepsilon)} \quad \dots(16)$$

$$z_2^{(0)} = \frac{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0] + \sqrt{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0]^2 - 8(\lambda_0 - \varepsilon)q(\mu - \varepsilon)}}{2(\lambda_0 - \varepsilon)} \quad \dots(16^*)$$

From (16) & (16*),

It is clear that $|z_1^{(0)}| < |z_2^{(0)}|$

$$z_1^{(0)} + z_2^{(0)} = \frac{(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0}{(\lambda_0 - \varepsilon)} \quad \dots (17)$$

$$z_1^{(0)}z_2^{(0)} = \frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \quad \dots (17^*)$$

Using Rouché's theorem in (15) we get

$$\bar{P}_0(s_0) = \frac{z_1^{(0)2} + 2q(\mu - \varepsilon)z_1^{(0)r+1}\bar{P}_{r+1}(s_1)}{s_0z_1^{(0)}} \quad \dots (18)$$

Using the result (18) in (15) we get

$$\bar{P}(z, s_0) = \frac{z^2 - (1-z)[2q(\mu - \varepsilon) - (\lambda_0 - \varepsilon)z] \frac{z_1^{(0)2} + 2q(\mu - \varepsilon)z_1^{(0)r+1}\bar{P}_{r+1}(s_1)}{s_0z_1^{(0)}} + 2q(\mu - \varepsilon)z^{r+1}\bar{P}_{r+1}(s_1)}{[(\lambda_0 - \varepsilon) + 2q(\mu - \varepsilon) + s_0]z - 2q(\mu - \varepsilon) - (\lambda_0 - \varepsilon)z^2}$$

$$= \frac{z^2 - (1-z)[2q(\mu - \varepsilon) - (\lambda_0 - \varepsilon)z] \frac{z_1^{(0)2} + 2q(\mu - \varepsilon)z_1^{(0)r+1}\bar{P}_{r+1}(s_1)}{s_0z_1^{(0)}} + 2q(\mu - \varepsilon)z^{r+1}\bar{P}_{r+1}(s_1)}{(\lambda_0 - \varepsilon)[z_2^{(0)} - z][z - z_1^{(0)}]}$$

$$\bar{P}(0, s_0) = \bar{P}_0(s_0) = \frac{-2q(\mu - \varepsilon) \left[\frac{z_1^{(0)2} + 2q(\mu - \varepsilon)z_1^{(0)r+1}\bar{P}_{r+1}(s_1)}{s_0z_1^{(0)}} \right]}{-(\lambda_0 - \varepsilon)z_1^{(0)}z_2^{(0)}} \quad \dots(19)$$

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Using (17*) in (19) we get

$$s_0 \bar{P}_0(s_0) = \frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)z_2^{(0)}} + 2q(\mu - \varepsilon) \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \frac{\bar{P}_{r+1}(s_1)}{z_2^{(0)r}} \quad \dots(20)$$

From Equation (20) we get

$$\begin{aligned} \frac{d}{ds_0} [s_0 \bar{P}_0(s_0)] &= \frac{d}{ds_0} \left[\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)z_2^{(0)}} + 2q(\mu - \varepsilon) \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \frac{\bar{P}_{r+1}(s_1)}{z_2^{(0)r}} \right] \\ - \frac{d}{ds_0} [s_0 \bar{P}_0(s_0)]_{s_0=0} &= \frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)} \left[\frac{1}{(\lambda_0 - \varepsilon)} + r \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right] \end{aligned} \quad \dots(21)$$

$$\begin{aligned} \frac{d}{ds_0} [s_0 \bar{P}_0(s_0)]_{s_0=0} &= \frac{d}{ds_0} L\{P_0'(t_0)\}_{s_0=0} \\ &= \frac{d}{ds_0} \left[\int_0^\infty e^{-s_0 t_0} P_0'(t_0) dt_0 \right]_{s_0=0} \\ &= \left[\int_0^\infty e^{-s_0 t_0} (-t_0) P_0'(t_0) dt_0 \right]_{s_0=0} \\ &= - \int_0^\infty t_0 P_0'(t_0) dt_0 = -E\{T_{busy}^{(0)}\} \end{aligned}$$

$$\frac{d}{ds_0} [s_0 \bar{P}_0(s_0)]_{s_0=0} = E\{T_{busy}^{(0)}\} \quad \dots(22)$$

From Equation (21) & (22) we get

$$E\{T_{busy}^{(0)}\} = \frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)} \left[\frac{1}{(\lambda_0 - \varepsilon)} + r \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right]$$

∴ The average length of the busy period when the system is in faster rate of arrivals either with feedback or without feedback is given by

$$E\{T_{busy}^{(0)}\} = \frac{1}{\rho^{(0)} - 1} \left[\frac{1}{(\lambda_0 - \varepsilon)} + r \left[\frac{2}{\rho^{(0)}} \right]^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right] \quad \dots(23)$$

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which is consistent with the corresponding result of the conventional model for case $\varepsilon = 0, p = q = 1, \lambda_0 = \lambda_1 = \lambda$ and is also consistent with the corresponding result of the busy period analysis of M/M/1/ ∞ model for the case $p = q = 1$.

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From equation (20) we get

$$\frac{d^2}{ds_0^2} [s_0 \bar{P}_0(s_0)] = \frac{d^2}{ds_0^2} \left[\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)z_2^{(0)}} + 2q(\mu - \varepsilon) \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \frac{\bar{P}_{r+1}(s_1)}{z_2^{(0)r}} \right]$$

$$\frac{d^2}{ds_0^2} [s_0 \bar{P}_0(s_0)]_{s_0=0} = \frac{4q(\mu - \varepsilon)}{[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)]^3} + r2q(\mu - \varepsilon) \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \bar{P}_{r+1}(s_1)$$

$$\left\{ \frac{(r + 1)[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)}{[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \right\}$$

$$\frac{d^2}{ds_0^2} [s_0 \bar{P}_0(s_0)]_{s_0=0} = \frac{2q(\mu - \varepsilon)}{[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \left\{ \begin{array}{l} 2 + r \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1} \\ [(r + 1)[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \end{array} \right\}$$

...(24)

$$\frac{d^2}{ds_0^2} [s_0 \bar{P}_0(s_0)]_{s_0=0} = \frac{d^2}{ds_0^2} L\{P_0'(t_0)\}_{s_0=0}$$

$$= \frac{d^2}{ds_0^2} \left[\int_0^\infty e^{-s_0 t_0} P_0'(t_0) dt_0 \right]_{s_0=0}$$

$$= \left[\int_0^\infty e^{-s_0 t_0} (-t_0)^2 P_0'(t_0) dt_0 \right]_{s_0=0}$$

$$= \int_0^\infty t_0^2 P_0'(t_0) dt = E \{T_{busy}^{(0)2}\}$$

$$\frac{d^2}{ds_0^2} [s_0 \bar{P}_0(s_0)]_{s_0=0} = E \{T_{busy}^{(0)2}\} \quad \dots (25)$$

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From (16) & (17) we get (26)

$$\begin{aligned}
 E\{T_{busy}^{(0)2}\} &= \frac{2q(\mu - \varepsilon)}{[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \left\{ 2 + r \left(\frac{2q(\mu - \varepsilon)}{(\lambda_0 - \varepsilon)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right. \\
 &\quad \left. [(r + 1)[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right\} \\
 E\{T_{busy}^{(0)}\} &= \frac{1}{[2q(\mu - \varepsilon)]^2 \left[\frac{\rho(0)}{2} - 1 \right]^3} \left\{ 2 + r \left(\frac{2}{\rho(0)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right. \\
 &\quad \left. [(r + 1)[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right\} \\
 V\{T_{busy}^{(0)}\} &= E\{T_{busy}^{(0)2}\} - [E\{T_{busy}^{(0)}\}]^2 \\
 &= \frac{1}{[2q(\mu - \varepsilon)]^2 \left[\frac{\rho(0)}{2} - 1 \right]^3} \left\{ 2 + r \left(\frac{2}{\rho(0)} \right)^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right. \\
 &\quad \left. [(r + 1)[(\lambda_0 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right\} \\
 &\quad - \frac{1}{\left[\frac{\rho(0)}{2} - 1 \right]^2} \left[\frac{1}{(\lambda_0 - \varepsilon)} + r \left[\frac{2}{\rho(0)} \right]^r \lim_{s_0 \rightarrow 0} \bar{P}_{r+1}(s_1) \right]^2 \quad \dots(26)
 \end{aligned}$$

V. NUMERICAL ILLUSTRATIONS

For various values of r , λ_0 , μ and ε , $E\{T_{busy}^{(0)}\}$, $E\{T_{busy}^{(0)2}\}$ and $V\{T_{busy}^{(0)}\}$ are computed and tabulated for faster rate of arrivals by taking $p = q = \frac{1}{2}$

Table-1

r	λ_0	M	E	$E\{T_{busy}^{(0)}\}$	$E\{T_{busy}^{(0)2}\}$	$V\{T_{busy}^{(0)}\}$
2	3	4	0	1.3335	5.3333	3.5551
2	3	4	0.5	1.4003	4.2012	2.2404
2	3	4	1	1.4997	2.9991	0.7500
2	6	8	0	0.6672	1.3351	0.8899
2	6	8	0.5	0.6822	1.1939	0.6985
2	6	8	1	0.6994	1.0489	0.5597
3	8	10	0	1.2500	3.1250	1.5625
3	8	10	0.5	1.2666	2.5866	0.9823
3	8	10	1	1.2851	2.5703	0.9188
5	12	14	0	2.3369	8.1802	2.7191
5	12	14	0.5	2.3471	7.6290	2.1200
5	12	14	1	2.3613	7.0824	1.5067

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VI. AVERAGE LENGTH OF THE BUSY PERIOD FOR SLOWER RATE OF ARRIVAL

From (7) to (11) are multiplied through by z^n

$n = r + 1, r + 2, r + 3, \dots, R - 1, R, R + 1, \dots$, and summing on n

from $n = r + 1, r + 2, r + 3, \dots, R - 1, R, R + 1, \dots$, it is found that

$$\frac{d}{dt_1} [P(z, t_1)] = \sum_{n=r+1}^{\infty} P_n'(t_1) z^n \quad \dots (27)$$

Using the procedure as in the case when the system is in faster rate of arrivals either with feedback or without feedback we get

$$\begin{aligned} \frac{d}{dt_1} [P(z, t_1)] &= P_{r+1}'(t_1) z^{r+1} + P_{r+2}'(t_1) z^{r+2} + \dots + P_R'(t_1) z^R + P_{R+1}'(t_1) z^{R+1} + \dots \\ &= \left[-[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)] + \frac{2q(\mu - \varepsilon)}{z} + (\lambda_1 - \varepsilon)z \right] \sum_{n=r+1}^{\infty} P_n(t_1) z^n + (\lambda_0 - \varepsilon) P_{R-1}(t_0) z^R \\ &\quad + \left[[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)] - \frac{2q(\mu - \varepsilon)}{z} - (\lambda_1 - \varepsilon)z \right] P_{r+1}(t_1) z^{r+1} \\ z \frac{d}{dt_1} [P(z, t_1)] &= [-[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]z + 2q(\mu - \varepsilon) + (\lambda_1 - \varepsilon)z^2] P(z, t_1) \\ &\quad + (\lambda_0 - \varepsilon) P_{R-1}(t_0) z^{R+1} \\ &\quad + [[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]z - 2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z^2] P_{r+1}(t_1) z^{r+1} \end{aligned}$$

Taking Laplace transform on both sides we get

$$\begin{aligned} z[s_1 \bar{P}(z, s_1) - P(z, r + 1)] &= [(\lambda_1 - \varepsilon)z^2 - [(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]z + 2q(\mu - \varepsilon)] \bar{P}(z, s_1) + (\lambda_0 - \varepsilon) z^{R+1} \bar{P}_{R-1}(s_0) \\ &\quad + [-(\lambda_1 - \varepsilon)z^2 + [(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon)]z - 2q(\mu - \varepsilon)] \bar{P}_{r+1}(s_1) z^{r+1} \\ &\quad + z^{r+2} - (1 - z)[2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z] z^{r+1} \bar{P}_{r+1}(s_1) \\ &\quad + (\lambda_0 - \varepsilon) z^{R+1} \bar{P}_{R-1}(s_0) \\ \bar{P}(z, s_1) &= \frac{z^{r+2} - (1 - z)[2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z] z^{r+1} \bar{P}_{r+1}(s_1) + (\lambda_0 - \varepsilon) z^{R+1} \bar{P}_{R-1}(s_0)}{[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon) + s_1]z - 2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z^2} \end{aligned} \quad \dots (28)$$

VARIANCE OF THE BUSY PERIOD FOR SLOWER RATE OF ARRIVALS

From Equation (37) we get

$$\begin{aligned} \frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)] &= \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} \frac{d^2}{ds_1^2} \left[\frac{1}{z_2^{(1)}} \right]^{r+1} + (\lambda_0 - \varepsilon) \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \frac{d^2}{ds_1^2} \left[\frac{1}{z_2^{(1)}} \right]^R \\ \frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} &= (r + 1) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} \left\{ \frac{(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)}{[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \right\} \end{aligned}$$

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$$+ R (\lambda_0 - \varepsilon) \lim_{s_1=0} \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu-\varepsilon)}{(\lambda_1-\varepsilon)} \right]^R \left\{ \frac{(R+1)[(\lambda_1-\varepsilon)-2q(\mu-\varepsilon)]+4q(\mu-\varepsilon)}{[(\lambda_1-\varepsilon)-2q(\mu-\varepsilon)]^3} \right\}$$

Where

$$L\{P(z, t_1)\} = \bar{P}(z, s_1)$$

$$L\{P_{R-1}(t_0)\} = \bar{P}_{R-1}(s_0),$$

$$L\{P_{r+1}(t_1)\} = \bar{P}_{r+1}(s_1) \text{ and } P(z, r + 1) = z^{r+1}$$

Since the Laplace transform $\bar{P}(z, s_1)$ converges in the region $|z| \leq 1, \text{Real}(s_1) \geq 0$, wherever the denominator of the right hand side of (25) has zeros in that region, so must the numerator, the denominator has two zeros, that is

$$z^{r+2} - (1-z)[2q(\mu-\varepsilon) - (\lambda_1-\varepsilon)z]z^{r+1}\bar{P}_{r+1}(s_1) + (\lambda_0-\varepsilon)z^{R+1}\bar{P}_{R-1}(s_0) = 0 \quad \dots (29)$$

$$(\lambda_1-\varepsilon)z^2 - [(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1]z + 2q(\mu-\varepsilon) = 0 \quad \dots (30)$$

From (30) we get

$$z_1^{(1)} = \frac{[(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1] - \sqrt{[(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1]^2 - 8(\lambda_1-\varepsilon)q(\mu-\varepsilon)}}{2(\lambda_1-\varepsilon)} \quad \dots (31)$$

$$z_2^{(1)} = \frac{[(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1] + \sqrt{[(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1]^2 - 8(\lambda_1-\varepsilon)q(\mu-\varepsilon)}}{2(\lambda_1-\varepsilon)} \quad \dots (31^*)$$

From (31) & (31*),

It is clear that $|z_1^{(1)}| < |z_2^{(1)}|$

$$z_1^{(1)} + z_2^{(1)} = \frac{(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1}{(\lambda_1-\varepsilon)}$$

$$z_1^{(1)} z_2^{(1)} = \frac{2q(\mu-\varepsilon)}{(\lambda_1-\varepsilon)}$$

and hence

$$z_1^{(1)} + z_2^{(1)} = \frac{(\lambda_1-\varepsilon) + 2q(\mu-\varepsilon) + s_1}{(\lambda_1-\varepsilon)} \quad \dots (32)$$

$$z_1^{(1)} z_2^{(1)} = \frac{2q(\mu-\varepsilon)}{(\lambda_1-\varepsilon)} \quad \dots (32^*)$$

From (29) we get

$$\bar{P}_{r+1}(s_1) = \frac{z_1^{(1)r+2} + (\lambda_0-\varepsilon)z^{(1)R+1}\bar{P}_{R-1}(s_0)}{(1-z_1^{(1)})[2q(\mu-\varepsilon) - (\lambda_1-\varepsilon)z_1^{(1)}]z_1^{(1)r+1}} \quad \dots (33)$$

Using Rouché's theorem in (25) we get

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$$\bar{P}_{r+1}(s_1) = \frac{z_1^{(1)r+1} + (\lambda_0 - \varepsilon)z_1^{(1)R}\bar{P}_{R-1}(s_0)}{s_1} \quad \dots (34)$$

Using the result (33) , (34) in (28) we get

$$\begin{aligned} \bar{P}(z, s_1) &= \frac{z^{r+2} - (1-z)[2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z] \left[\frac{z_1^{(1)r+2} + (\lambda_0 - \varepsilon)z_1^{(1)R+1}\bar{P}_{R-1}(s_0)}{(1-z_1^{(1)})[2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z_1^{(1)}]} \right] + (\lambda_0 - \varepsilon)z^{R+1}\bar{P}_{R-1}(s_0)}{[(\lambda_1 - \varepsilon) + 2q(\mu - \varepsilon) + s_1]z - 2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z^2} \\ &= \frac{z^{r+2} - (1-z)[2q(\mu - \varepsilon) - (\lambda_1 - \varepsilon)z] \left[\frac{z_1^{(1)r+1} + (\lambda_0 - \varepsilon)z_1^{(1)R}\bar{P}_{R-1}(s_0)}{s_1} \right] + (\lambda_0 - \varepsilon)z^{R+1}\bar{P}_{R-1}(s_0)}{(\lambda_1 - \varepsilon)[z_2^{(1)} - z][z - z_1^{(1)}]} \\ \bar{P}(0, s_1) = \bar{P}_{r+1}(s_1) &= \frac{-2q(\mu - \varepsilon) [z_1^{(1)r+1} + (\lambda_0 - \varepsilon)z_1^{(1)R}\bar{P}_{R-1}(s_0)]}{(\lambda_1 - \varepsilon) - z_1^{(1)}z_2^{(1)}s_1} \quad \dots (35) \end{aligned}$$

Using the result (32*) in (35) we get

$$\bar{P}_{r+1}(s_1) = \frac{[z_1^{(1)r+1} + (\lambda_0 - \varepsilon)z_1^{(1)R}\bar{P}_{R-1}(s_0)]}{s_1} \quad \dots (36)$$

$$s_1 \bar{P}_{r+1}(s_1) = z_1^{(1)r+1} + (\lambda_0 - \varepsilon)z_1^{(1)R}\bar{P}_{R-1}(s_0)$$

$$s_1 \bar{P}_{r+1}(s_1) = \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)z_2^{(1)}} \right]^{r+1} + (\lambda_0 - \varepsilon)\bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)z_2^{(1)}} \right]^R \quad \dots (37)$$

From equation (37) we get

$$\begin{aligned} \frac{d}{ds_1} [s_1 \bar{P}_{r+1}(s_1)] &= \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} \frac{d}{ds_1} \left[\frac{1}{z_2^{(1)}} \right]^{r+1} + (\lambda_0 - \varepsilon)\bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \frac{d}{ds_1} \left[\frac{1}{z_2^{(1)}} \right]^R \\ - \frac{d}{ds_1} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} &= \left[\frac{1}{(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)} \right] \left\{ \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} (r+1) + R \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R (\lambda_0 - \varepsilon) \lim_{s_1 \rightarrow 0} \bar{P}_{R-1}(s_0) \right\} \quad \dots (38) \end{aligned}$$

$$\frac{d}{ds_1} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} = \frac{d}{ds_1} L\{P_{r+1}'t_1\}_{s_1=0}$$

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$$\begin{aligned}
 &= \frac{d}{ds_1} \left[\int_0^\infty e^{-s_1 t_1} P_{r+1}'(t_1) dt_1 \right]_{s_1=0} \\
 &= \left[\int_0^\infty e^{-s_1 t_1} (-t_1) P_{r+1}'(t_1) dt_1 \right]_{s_1=0} \\
 &= - \int_0^\infty t_1 P_{r+1}'(t_1) dt_1 = -E\{T_{busy}^{(1)}\}
 \end{aligned}$$

$$\frac{d}{ds_1} [s_0 \bar{P}_0(s_0)]_{s_0=0} = E\{T_{busy}^{(1)}\} \quad \dots(39)$$

From equation (38) & (39) we get

$$E\{T_{busy}^{(1)}\} = \left[\frac{1}{(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)} \right] \left\{ \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} (r + 1) + R \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R (\lambda_0 - \varepsilon) \lim_{s_0 \rightarrow 0} \bar{P}_{R-1}(s_0) \right\}$$

∴ The average length of the busy period when the system is in slower rate of arrivals either with feedback or without feedback is given by

$$E\{T_{busy}^{(1)}\} = \frac{1}{2q(\mu - \varepsilon) \left[\frac{\rho(1)}{2} - 1 \right]} \left[(r + 1) \left[\frac{2}{\rho(1)} \right]^{r+1} + R(\lambda_0 - \varepsilon) \left[\frac{2}{\rho(1)} \right]^R \lim_{s_0 \rightarrow 0} \bar{P}_{R-1}(s_0) \right] \quad \dots(40)$$

which is consistent with the corresponding result of the conventional model for case $\varepsilon = 0, p = q = 1$ and $\lambda_0 = \lambda_1 = \lambda$ the present model reduces to busy period analysis of M/M/1/∞ interdependent model with controllable arrival rates for the case $p = q = 1$, Two server = single server ($2q\mu = q\mu$).

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From Equation (37) we get

$$\begin{aligned}
 \frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)] &= \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} \frac{d^2}{ds_1^2} \left[\frac{1}{z_2^{(1)}} \right]^{r+1} + (\lambda_0 - \varepsilon) \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \frac{d^2}{ds_1^2} \left[\frac{1}{z_2^{(1)}} \right]^R \\
 \frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} &= (r + 1) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} \left\{ \frac{(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)}{[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \right\} \\
 &+ R(\lambda_0 - \varepsilon) \lim_{s_1=0} \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \left\{ \frac{(R+1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)}{[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \right\} \\
 &= \frac{1}{[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \times
 \end{aligned}$$

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$$\left\{ \begin{aligned} &(r + 1) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} [(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \\ &\quad + R(\lambda_0 - \varepsilon) \lim_{s_1=0} \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \\ &[(R + 1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon) \end{aligned} \right\} \dots(41)$$

$$\begin{aligned} \frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} &= \frac{d^2}{ds_1^2} L\{P_{r+1}'(t_1)\}_{s_1=0} \\ &= \frac{d^2}{ds_1^2} \left[\int_0^\infty e^{-s_1 t_1} P_{r+1}'(t_1) dt_1 \right]_{s_1=0} \\ &= \int_0^\infty t_1^2 P_{r+1}'(t_1) dt_1 = E\{T_{busy}^{(1)2}\} \end{aligned}$$

$$\frac{d^2}{ds_1^2} [s_1 \bar{P}_{r+1}(s_1)]_{s_1=0} = E\{T_{busy}^{(1)2}\} \dots (42)$$

From equation (41) & (42) we get

$$E\{T_{busy}^{(1)2}\} = \frac{1}{[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)]^3} \times$$

$$\left\{ \begin{aligned} &(r + 1) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^{r+1} [(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \\ &\quad + R(\lambda_0 - \varepsilon) \lim_{s_1=0} \bar{P}_{R-1}(s_0) \left[\frac{2q(\mu - \varepsilon)}{(\lambda_1 - \varepsilon)} \right]^R \\ &[(R + 1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon) \end{aligned} \right\}$$

$$E\{T_{busy}^{(1)2}\} = \frac{1}{[2q(\mu - \varepsilon)]^3 \left[\frac{\rho(1)}{2} \right]^3} \times$$

$$\left\{ \begin{aligned} &(r + 1) \left[\frac{2}{\rho(1)} \right]^{r+1} [(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \\ &\quad + R(\lambda_0 - \varepsilon) \lim_{s_1=0} \bar{P}_{R-1}(s_0) \left[\frac{2}{\rho(1)} \right]^R \\ &[(R + 1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon) \end{aligned} \right\} \dots(43)$$

$$V\{T_{busy}^{(1)}\} = E\{T_{busy}^{(1)2}\} - [E\{T_{busy}^{(1)}\}]^2$$

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$$\begin{aligned}
 &= \frac{1}{[2q(\mu - \varepsilon)]^3 \left[\frac{\rho(1)}{2}\right]^3} \times \\
 &\left\{ (r + 1) \left[\frac{2}{\rho(1)}\right]^{r+1} [(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right. \\
 &\quad \left. + R(\lambda_0 - \varepsilon) \lim_{s_1 \rightarrow 0} \bar{P}_{R-1}(s_0) \left[\frac{2}{\rho(1)}\right]^R \right. \\
 &\quad \left. [(R + 1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right\} \\
 &- \frac{1}{[2q(\mu - \varepsilon)]^2 \left[\frac{\rho(1)}{2} - 1\right]^2} \left[(r + 1) \left[\frac{2}{\rho(1)}\right]^{r+1} + R(\lambda_0 - \varepsilon) \left[\frac{2}{\rho(1)}\right]^R \lim_{s_0 \rightarrow 0} \bar{P}_{R-1}(s_0) \right]^2
 \end{aligned}$$

$$V\{T_{busy}^{(1)}\} = \frac{1}{[2q(\mu - \varepsilon)]^2 \left[\frac{\rho(1)}{2} - 1\right]^2} \times$$

$$\left\{ \frac{1}{2q(\mu - \varepsilon) \left[\frac{\rho(1)}{2} - 1\right]} \right. \\
 \left. \left\{ (r + 1) \left[\frac{2}{\rho(1)}\right]^{r+1} [(r + 2)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right. \right. \\
 \left. \left. + R(\lambda_0 - \varepsilon) \lim_{s_1 \rightarrow 0} \bar{P}_{R-1}(s_0) \left[\frac{2}{\rho(1)}\right]^R \right. \right. \\
 \left. \left. [(R + 1)[(\lambda_1 - \varepsilon) - 2q(\mu - \varepsilon)] + 4q(\mu - \varepsilon)] \right\} \right. \\
 \left. - \left[(r + 1) \left[\frac{2}{\rho(1)}\right]^{r+1} + R(\lambda_0 - \varepsilon) \left[\frac{2}{\rho(1)}\right]^R \lim_{s_0 \rightarrow 0} \bar{P}_{R-1}(s_0) \right]^2 \right\}$$

...(44)

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VII. NUMERICAL ILLUSTRATIONS

For various values of $r, R, \lambda_0, \lambda_1, \mu$ and $\epsilon, E\{T_{busy}^{(1)}\}, E\{T_{busy}^{(1)2}\}$ and $V\{T_{busy}^{(1)}\}$ are computed and tabulated for slower rate of arrival by taking $p = q = \frac{1}{2}$

Table-2

R	R	λ_0	μ	λ_1	E	$E\{T_{busy}^{(1)}\}$	$V\{T_{busy}^{(1)}\}$
1	3	4	5	4	0	1.5625	3.8086
1	3	4	5	4	0.5	1.6526	2.2258
1	3	4	5	4	1	1.7777	0.3951
3	5	8	9	8	0	1.6010	10.2416
3	5	8	9	8	0.5	1.6465	8.8012
3	5	8	9	8	1	1.7060	7.3256
9	11	14	15	14	0	2.0046	12.1283
9	11	14	15	14	0.5	2.0546	10.2727
9	11	14	15	14	1	2.0890	8.0782

- (a). It is observed that from the Table-1&Table-2 the average length of the busy period is increase and the variance is increase when the system is either in faster or slower rate of arrivals by increasing the value of r while the other parameters were fixed.
- (b). The average length of the busy period rapidly increases and the variance decreases when the system is in faster rate of arrivals by increasing the value of λ_0 and keeping the other parameters were fixed.
- (c). The average length of the busy period rapidly decreases and the variance rapidly decreases by increasing the value of μ and keeping the other parameters were fixed while the system is in faster rate of arrivals.
- (d). The average length of the busy period rapidly increases and the variance rapidly increases by increasing the value of μ and keeping the other parameters were fixed while the system is in slower rate of arrivals.
- (e). The average length of the busy period rapidly increases and the variance decreases while system is in faster rate of arrivals by increasing the value of ϵ keeping the other parameters were fixed.
- (f). The average length of the busy period rapidly increases and the variance decreases while system is in slower rate of arrivals by increasing the value of ϵ and keeping the other parameters were fixed.
- (g). The average length of the busy period rapidly increases and the variance increases while system is in slower rate of arrivals by increasing the value of R keeping the other parameters were fixed.

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