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## Total Resolving Number of Cactus – Some Characterizations

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**ABSTRACT:** Let  $G = (V, E)$  be a simple connected graph. An ordered subset  $W$  of  $V$  is said to be a *resolving set* of  $G$  if every vertex is uniquely determined by its vector of distances to the vertices in  $W$ . The minimum cardinality taken over all resolving sets is called the *resolving number* of  $G$  and is denoted by  $r(G)$ . The *total resolving number* as the minimum cardinality taken over all resolving sets in which  $\langle W \rangle$  has no isolates. It is denoted by  $tr(G)$ . A *cactus* is a connected graph in which every block is an edge or a cycle. We characterize cactus for which the total resolving number is  $2e_c$  and  $\sigma(G) - ex(G) + \Theta(G)$ .

**KEYWORDS:** Resolving Number, Total Resolving Number, Cactus.

### I. INTRODUCTION

Let  $G = (V, E)$  be a finite, simple, connected and undirected graph. The *degree* of a vertex  $v$  in  $G$  is the number of edges incident to  $v$  and it is denoted by  $d(v)$ . The maximum degree in  $G$  is denoted by  $\Delta(G)$  and the minimum degree is denoted by  $\delta(G)$ . The *distance*  $d(u, v)$  between two vertices  $u$  and  $v$  in  $G$  is the length of a shortest  $u$ - $v$  path in  $G$ .  $P_n$  denote the *path* on  $n$  vertices.  $C_n$  denote the *cycle* on  $n$  vertices. A vertex  $v$  is said to be a cut vertex of  $G$  if  $G \setminus \{v\}$  is disconnected or a trivial graph. A cut vertex  $v$  is called a *path support* if  $G \setminus \{v\}$  contains a path component. A cut vertex  $v$  is called a *simple path support* if  $G \setminus \{v\}$  contains exactly one path component. A cut vertex  $v$  is called a *multi path support* if  $G \setminus \{v\}$  contains more than one path component. A block of a graph  $G$  is a maximal connected subgraph of  $G$  that has no cut vertex. The *girth* of a graph  $G$  with a cycle is the length of its shortest cycle and it is denoted by  $g(G)$ .

The *eccentricity*  $e(v)$  of a vertex  $v$  in  $G$  is defined as  $e(v) = \max\{d(u, v) \mid u \in V(G)\}$ . If  $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$  is an ordered set, then the ordered  $k$ -tuple  $(d(v, w_1), d(v, w_2), \dots, d(v, w_k))$  is called the representation of  $v$  with respect to  $W$  and it is denoted by  $r(v|W)$ . Since the representation for each  $w_i \in W$  contains exactly one 0 in the  $i^{\text{th}}$  position, all the vertices of  $W$  have distinct representations.  $W$  is called a resolving set or  $G$  if all the vertices of  $V \setminus W$  also have distinct representations. The minimum cardinality taken over all resolving sets is called the *resolving number* of  $G$  and it is denoted by  $r(G)$ .

In 1975, Slater [8] introduced these ideas and used *locating set* for what we have called resolving set. He referred to the cardinality of a minimum resolving set in  $G$  as its *location number*. In 1976, Harary and Melter [5] discovered these concepts independently as well but used the term *metric dimension* rather than location number. In 2003, Ping Zhang and Varaporn Saenpholphat [6, 7] studied *connected resolving number* and in 2015, we introduced and studied total resolving number in [3]. In this paper, we use the term *resolving number* to maintain uniformity in the current literature.

If  $W$  is a resolving set and the induced subgraph  $\langle W \rangle$  has no isolates, then  $W$  is called a *total resolving set* of  $G$ . The minimum cardinality taken over all total resolving sets of  $G$  is called the *total resolving number* of  $G$  and is denoted by  $tr(G)$ .

In this paper, we characterize cactus for which the total resolving number is  $2e_c$  and  $\sigma(G) - ex(G) + \Theta(G)$ .

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## II. BUILDING BLOCKS

In this section, we give several definition and results which are used in subsequent sections.

**Definition 2.1.** A cycle  $C_r$  is called an *end cycle* if  $C_r$  contains exactly one vertex of degree at least 3.

**Notation 2.2.** Let  $e_c$  denote the number of end cycles of  $G$ .

**Lemma 2.3.** [4] Let  $G$  be a graph with  $e_c \geq 1$ , and each end cycle is of size at least 4. Then  $tr(G) \geq 2e_c$ .

**Definition 2.4.** A block of  $G$  containing exactly one cut vertex of  $G$  is called an *end block* of  $G$ .

**Definition 2.5.** [2] Any end vertex  $u$  of  $G$  is said to be a *terminal vertex* of a major vertex  $v$  of  $G$  if  $d(u, v) < d(u, w)$  for every other major vertex  $w$  of  $G$ . The terminal degree  $ter(v)$  of a major vertex  $v$  is the number of terminal vertices of  $v$ . A major vertex  $v$  of  $G$  is an *exterior major vertex* of  $G$  if it has positive terminal degree. Let  $\sigma(G)$  be the sum of the terminal degrees of the major vertices of  $G$  and  $ex(G)$  be the number of exterior major vertices of  $G$ . Let  $\Theta(G)$  be the number of exterior major vertices of  $G$  with terminal degree at least two.

**Lemma 2.6.** [3] If  $G$  is a graph of order at least 3 and  $\delta(G) = 1$ , then  $tr(G) \geq \sigma(G) - ex(G) + \Theta(G)$ .

**Theorem 2.7.** [3] For any tree  $T$  which is not a path,  $tr(T) = \sigma(T) - ex(T) + \Theta(T)$ .

## III. CACTUS

In this section, we characterize cactus for which the total resolving number is  $2e_c$  and  $\sigma(G) - ex(G) + \Theta(G)$ .

**Definition 3.1.** A *cactus* is a connected graph in which every block is an edge or a cycle.

**Theorem 3.2.** Let  $G$  be a cactus with  $g(G) \geq 4$  and with no pendant edge. Then  $tr(G) = 2e_c$  if and only if the following two conditions are satisfied.

- (i)  $G$  has no even cycle  $C_r$  with exactly two vertices in  $C_r$  at a distance having degree at least 3 and
- (ii) there is no two consecutive cycles  $C_s$  and  $C_t$  such that  $u_1 \in V(C_s) \cap V(C_t)$  and  $u_1u_2, u_1u_3 \in E(G)$  with  $d(u_2) \geq 3$  and  $d(u_3) \geq 3$  where  $u_2 \in V(C_s)$ ,  $u_3 \in V(C_t)$  and  $d(x) = 2$  for all  $x \in (V(C_s) \cup V(C_t)) \setminus \{u_1, u_2, u_3\}$ .

**Proof.** Let  $tr(G) = 2e_c$ . Then there exists a minimum total resolving set of  $G$ . Let  $W$  be such set. By proof of Lemma 2.3,  $W$  contains exactly two adjacent vertices in each end cycle of  $G$ . Let  $A_1, A_2, \dots, A_r$  be the end cycle of  $G$ . Let  $V(A_i) = \{v_{i1}, v_{i2}, \dots, v_{i|k_i|}\} / 1 \leq i \leq r$  and  $d(v_{i1}) \geq 3$ .

Suppose  $G$  does not satisfy the condition (i). Let  $C_r: v_1v_2 \dots v_r v_1$  and  $d(v_1) \geq 3$  and  $d(v_{\frac{r}{2}+1}) \geq 3$ . Then  $d(v_2, w) = d(v_r, w)$  for all  $w \in W$ . It follows that  $r(v_2 | W) = r(v_r | W)$ , which is a contradiction. Thus  $G$  satisfies (i). Suppose  $G$  does not satisfy the condition (ii). Let  $a, b \in N(u_1) \cap (V(C_s) \cup V(C_t))$ . Then  $d(a, w) = d(b, w)$  for all  $w \in W$ . It follows that  $r(a | W) = r(b | W)$ , which is a contradiction. Thus  $G$  satisfies (ii).

Conversely, suppose that  $G$  satisfies (i) and (ii). By Lemma 2.3,  $tr(G) \geq 2e_c$ . Next, we claim that  $tr(G) \leq 2e_c$ . Let  $W = \{v_{i\lfloor \frac{k_i}{2} \rfloor}, v_{i\lfloor \frac{k_i}{2} \rfloor + 1} / 1 \leq i \leq r\}$ . Let  $x$  and  $y$  be two distinct vertices of  $V(G) \setminus W$ . Then we consider the following two cases.

**Case 1:**  $x \in V(A_i)$  for some  $1 \leq i \leq r$ .

Without loss of generality let  $x \in V(A_1)$ . If  $y \in V(A_1)$ , then  $r(x | (W \cap V(A_1))) \neq r(y | (W \cap V(A_1)))$ . It follows that  $r(x | W) \neq r(y | W)$ . So we may assume that  $y$  is not in  $V(A_1)$ . If  $d(x, v_{1\lfloor \frac{k_1}{2} \rfloor}) \neq d(y, v_{1\lfloor \frac{k_1}{2} \rfloor})$  or  $d(x, v_{1\lfloor \frac{k_1}{2} \rfloor + 1}) \neq d(y, v_{1\lfloor \frac{k_1}{2} \rfloor + 1})$ , then  $r(x | W) \neq r(y | W)$ . So we may assume that  $d(x, v_{1\lfloor \frac{k_1}{2} \rfloor}) = d(y, v_{1\lfloor \frac{k_1}{2} \rfloor})$  or  $d(x, v_{1\lfloor \frac{k_1}{2} \rfloor + 1}) = d(y, v_{1\lfloor \frac{k_1}{2} \rfloor + 1})$ . Clearly,  $d(x, w) > d(y, w)$  for all  $w \in W \setminus (W \cap V(A_1))$ . It follows that  $r(x | W) \neq r(y | W)$ .

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**Case 2:**  $x$  is not in  $V(A_i)$  for all  $1 \leq i \leq r$ .

If  $y \in V(A_i)$  for some  $1 \leq i \leq r$ , then by Case 1,  $r(x | W) \neq r(x | W)$ . So we may assume that  $y$  is not in  $V(A_i)$  for all  $1 \leq i \leq r$ . If  $x$  lies on the path between  $y$  and a vertex  $w$  of  $W$ , then  $d(x, w) < d(y, w)$ . If  $y$  lies on the path between  $x$  and a vertex  $w$  of  $W$ , then  $d(x, w) > d(y, w)$ . It follows that  $r(x | W) \neq r(x | W)$ . So we may assume that  $x$  does not lie on the path between  $y$  and a vertex  $w$  of  $W$  and  $y$  does not lie on the path between  $x$  and a vertex  $w$  of  $W$ . If  $x$  or  $y$ , say  $x$  does not lie on any cycle of  $G$ , then  $x$  is a cut vertex of  $G$ . Therefore we have at least two branches at  $x$ , say  $B_1$  and  $B_2$ . If  $y \in V(B_1)$ , then  $d(x, w) > d(y, w)$  for all  $w \in (W \cap V(B_1))$ . Otherwise  $d(x, w) < d(y, w)$  for all  $w \in (W \cap V(B_1))$ . It follows that  $r(x | W) \neq r(x | W)$ . So we may assume that  $x$  and  $y$  are in some cycle of  $G$ . Clearly, there exists a major vertex  $a$  such that  $d(x, a) < d(y, a)$ . Then there exists at least one branch at  $a$ , say  $B_1$  such that  $d(x, v) < d(y, v)$  for all  $v \in V(B_1)$  and hence  $d(x, w) < d(y, w)$  for all  $w \in (W \cap V(B_1))$ . It follows that  $r(x | W) \neq r(x | W)$ .

Thus  $tr(G) \leq 2e_c$  and hence  $tr(G) = 2e_c$ .

**Theorem 3.3.** Let  $G$  be a cactus with  $g(G) \geq 4$  and a unique vertex  $v$  such that

$$d(v) \geq 3. \text{ Then } tr(G) = \begin{cases} d(v) - 1, & \text{G contains exactly one pendant} \\ d(v), & \text{otherwise.} \end{cases}$$

**Proof.** Let  $B_1, B_2, \dots, B_r$  be the branches of  $G$  at  $v$ . Let  $V(B_i) = \{v_{i1}, v_{i2}, \dots, v_{ik_i} / 1 \leq i \leq r\}$  and  $v = v_{i1}, 1 \leq i \leq r$ . Then we consider the following two cases.

**Case 1:**  $\delta(G) = 1$ .

Then  $G$  contains at least one branch at  $v$  is a path. Without loss of generality, let  $B_1$  be such branch. If  $G$  is a tree, then by Theorem 2.7,  $tr(G) = \sigma(G) - ex(G) + \Theta(G) = d(v) - 1 + 1 = d(v)$ . If  $G$  is not a tree, then  $G$  contains at least one branch at  $v_1$  is a cycle. Without loss of generality, let  $B_2$  be such branch. Then we consider the following two subcases.

**Sub Case 1:**  $G$  contains exactly one pendant.

Then we have  $B_2, B_3, \dots, B_r$  are cycles. By Lemma 2.3,  $tr(G) \geq 2e_c = 2(s - 1) = d(v) - 1$ . Next, we claim that  $tr(G) \leq d(v) - 1$ . Let  $W = \{v_{i \lfloor \frac{k_i}{2} \rfloor}, v_{i \lfloor \frac{k_i}{2} \rfloor + 1} / 2 \leq i \leq r\}$ . Let  $x$  and  $y$  be two distinct vertices of  $V(G) \setminus W$ . If  $x, y \in$

$V(B_i)$  for some  $2 \leq i \leq s$ , then  $r(x | (W \cap V(B_i))) \neq r(y | (W \cap V(B_i)))$ . It follows that  $r(x | W) \neq r(x | W)$ . If  $x \in V(B_i)$  and  $y \in V(B_j)$  for some  $2 \leq i \neq j \leq r$ , then  $d(x, w) < d(y, w)$  for all  $w \in W \cap V(B_i)$ . It follows that  $r(x | W) \neq r(x | W)$ . Thus  $tr(G) \leq d(v) - 1$  and hence  $tr(G) = d(v) - 1$ .

**Sub Case 2:**  $G$  contains more than one pendant.

Let  $G$  contains  $s \geq 2$  pendant vertices. Then Using proof of Lemmas 2.3 and 2.6,  $tr(G) \geq 2s + \sigma(G) - ex(G) + \Theta(G) = d(v)$ . Next, we claim that  $tr(G) \leq d(v)$ . Let  $W = \{v_{i \lfloor \frac{k_i}{2} \rfloor}, v_{i \lfloor \frac{k_i}{2} \rfloor + 1} / s + 1 \leq i \leq r\} \cup \{v\} \cup \{v_{22}, v_{32}, \dots, v_{s2}\}$ .

Let  $x$  and  $y$  be two distinct vertices of  $V(G) \setminus W$ . If  $x, y \in V(B_i)$  for some  $s + 1 \leq i \leq r$ , then  $r(x | (W \cap V(B_i))) \neq r(y | (W \cap V(B_i)))$ . It follows that  $r(x | W) \neq r(x | W)$ . If  $x \in V(B_i)$ ,  $y$  is not in  $V(B_i)$  for some  $s + 1 \leq i \leq r$ , then  $d(x, w) < d(y, w)$  for all  $w \in (W \cap V(B_i))$ . It follows that  $r(x | W) \neq r(x | W)$ . So we may assume that  $x, y$  are not in  $V(B_i)$  for all  $s + 1 \leq i \leq r$ . If  $x$  and  $y$  are lies on same branch at  $v$ , then  $x$  lies on  $y$ - $v$  path or  $y$  lies on  $x$ - $v$  path. Without loss of generality, let  $x$  lies on  $y$ - $v$  path. Then  $d(x, v) < d(y, v)$ . It follows that  $r(x | W) \neq r(x | W)$ . If  $x$  and  $y$  are lies on distinct branches at  $v$ , then without loss of generality, let  $x \in V(B_1) \setminus \{v\}$  or  $x \in V(B_i) \setminus \{v\}$  for some  $2 \leq i \leq s$ . Let  $x \in V(B_1) \setminus \{v\}$ . Then  $y \in V(B_i) \setminus \{v\}$  for some  $2 \leq i \leq s$ . Without loss of generality, let  $y \in V(B_2) \setminus \{v\}$ . If  $d(x, v) \neq d(y, v)$ , then  $r(x | W) \neq r(x | W)$ . So we may assume that  $d(x, v) = d(y, v)$ . Clearly,  $d(x, v_{22}) = d(y, v_{22}) + 2$ . It follows that  $r(x | W) \neq r(x | W)$ . Let  $x \in V(B_i) \setminus \{v\}$  for some  $2 \leq i \leq s$ . Without loss of generality, let  $x \in V(B_2) \setminus \{v\}$  and  $y$

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$\in V(B_3) \setminus \{v\}$ . If  $d(x, v) \neq d(y, v)$ , then  $r(x | W) \neq r(y | W)$ . So we may assume that  $d(x, v) = d(y, v)$ . Clearly,  $d(y, v_{22}) = d(x, v_{22}) + 2$ . It follows that  $r(x | W) \neq r(y | W)$ . Hence  $W$  is a resolving set of  $G$ . Since  $\langle W \rangle$  has no isolates,  $\text{tr}(G) \leq d(v)$ . Hence in this case  $\text{tr}(G) = d(v)$ .

### Case 2: $\delta(G) \geq 2$

Then all the  $r$  branches are cycles. By Lemma 2.3,  $\text{tr}(G) \geq 2e_c = d(v)$ . Next, we claim that  $\text{tr}(G) \leq d(v)$ . Let  $W = \{v_{\lfloor \frac{k_i}{2} \rfloor}, v_{\lfloor \frac{k_i}{2} \rfloor + 1} \mid 1 \leq i \leq r\}$ . If  $x, y \in V(B_i)$  for some  $1 \leq i \leq r$ , then  $r(x | (W \cap V(B_i))) \neq r(y | (W \cap V(B_i)))$ . It follows that  $r(x | W) \neq r(y | W)$ . If  $x \in V(B_i)$  and  $y \in V(B_j)$  for some  $1 \leq i \neq j \leq r$ , then  $r(x | (W \cap V(B_i))) \neq r(y | (W \cap V(B_j)))$ . It follows that  $r(x | W) \neq r(y | W)$ . Hence  $W$  is a resolving set of  $G$ . Since  $\langle W \rangle$  has no isolates,  $\text{tr}(G) \leq d(v)$ . Hence in this case  $\text{tr}(G) = d(v)$ .

**Theorem 3.4.** Let  $G$  be a cactus with no end cycle and no vertex of  $G$  be simple path support. Then  $\text{tr}(G) = \sigma(G) - \text{ex}(G) + \Theta(G)$  if and only if the following three conditions are satisfied.

- (i)  $G$  has no even cycle  $C_r$  with exactly two vertices in  $C_r$  at a distance  $\frac{r}{2}$  having degree at least 3
- (ii) there is no two consecutive cycles  $C_s$  and  $C_t$  such that  $u_1 \in V(C_s) \cap V(C_t)$  and  $u_1u_2, u_1u_3 \in E(G)$  with  $d(u_2) \geq 3$  and  $d(u_3) \geq 3$  where  $u_2 \in V(C_s), u_3 \in V(C_t)$  and  $d(x) = 2$  for all  $x \in (V(C_s) \cup V(C_t)) \setminus \{u_1, u_2, u_3\}$  and
- (iii) there is no cycle  $C_k: v_1v_2 \dots v_kv_1$  with  $d(v_1) = d(v_2) = \dots = d(v_{\lfloor \frac{k}{2} \rfloor}) = 2$ , and there exist at least two major vertices in  $C_k$  but there is no vertex of  $C_k$  is an exterior major vertex of  $G$ .

**Proof.** Let  $\text{tr}(G) = \sigma(G) - \text{ex}(G) + \Theta(G)$ . Let  $W$  be a minimum total resolving set of  $G$ . Let  $v_1, v_2, \dots, v_m$  be the exterior major vertices of  $G$  with terminal degree at least 2 and  $x_{i1}, x_{i2}, \dots, x_{ik_i}$  be the neighbours of  $v_i, 1 \leq i \leq m$ . Let  $k_i = \text{ter}(v_i)$  and  $P_{ij}$  be the  $v_i$ - $u_{ij}$  path. Then proof of Lemma 2.6,  $W \cap (V(G) \setminus V(P_{ij}))$  is empty.

Suppose  $G$  does not satisfy the condition (i). Let  $C_r: v_1v_2 \dots v_rv_1$  and  $d(v_1) \geq 3$  and  $d(v_{\frac{r}{2}+1}) \geq 3$ . Then  $d(v_2, w) = d(v_r, w)$  for all  $w \in W$ . It follows that  $r(v_2 | W) = r(v_r | W)$ , which is a contradiction. Thus  $G$  satisfies (i).

Suppose  $G$  does not satisfy the condition (ii). Let  $a, b \in N(u_1) \cap (V(C_s) \cup V(C_t))$ . Then  $d(a, w) = d(b, w)$  for all  $w \in W$ . It follows that  $r(a | W) = r(b | W)$ , which is a contradiction. Thus  $G$  satisfies (ii).

Suppose  $G$  does not satisfy the condition (iii). Let  $C_k: v_1v_2 \dots v_kv_1$  with  $d(v_1) = d(v_2) = \dots = d(v_{\lfloor \frac{k}{2} \rfloor}) = 2$ , and there exist at least two major vertices  $a, b$  in  $C_k$  such that  $a$  or  $b$  is an exterior major vertex. Without loss of generality, let  $a = v_1 = v_k$ . Let  $v \in V(P_{1k_1}) \cap N(v_1)$ . Then  $d(v_1, w) = d(v, w)$  for all  $w \in W$ . It follows that  $r(v_1 | W) = r(v | W)$ , which is a contradiction. Thus  $G$  satisfies (iii).

Conversely, suppose that  $G$  satisfies (i), (ii) and (iii). Let  $W = \{x_{i1}, x_{i2}, \dots, x_{ik_{i-1}} \mid 1 \leq i \leq m\} \cup \{v_1, v_2, \dots, v_m\}$ . Let  $x$  and  $y$  be two distinct vertices of  $V(G) \setminus W$ . Then we consider the following two cases.

**Case 1:**  $x$  lies on  $P_{ij}$  path for some  $1 \leq i \leq m, 1 \leq j \leq k_i$ .

Without loss of generality, let  $x$  lie on  $P_{ij}$  path for some  $1 \leq j \leq k_1$ . If  $d(x, w) \neq d(y, w)$  for some  $w \in W \cap V(P_{ij})$ , then  $r(x | W) \neq r(y | W)$ . So we may assume that  $d(x, w) = d(y, w)$  for all  $w \in W \cap V(P_{ij})$ . If  $y$  lies on the path between  $v_1$  and a vertex  $w$  of  $W \setminus V(P_{ij})$ , then  $d(x, w) > d(y, w)$ . It follows that  $r(x | W) \neq r(y | W)$ . So we may assume that  $y$  does not lie on the path between  $v_1$  and a vertex of  $W \setminus V(P_{ij})$ . Clearly,  $y$  lies on odd cycle  $u_1u_2 \dots u_ku_1$  such that  $d(u_1) \geq 3, d(u_{\lfloor \frac{k}{2} \rfloor + 1}) \geq 3$  and  $d(u_i) \geq 2, i \neq 1, i \neq \lfloor \frac{k}{2} \rfloor + 1$  or  $y$  lies on  $P_{ij}$  path for some  $2 \leq i \leq m, 1 \leq i \leq$

$k_i$ . If  $y$  lies on odd cycle  $u_1u_2 \dots u_ku_1$  such that  $d(u_1) \geq 3, d(u_{\lfloor \frac{k}{2} \rfloor + 1}) \geq 3$  and  $d(u_i) \geq 2, i \neq 1, i \neq \lfloor \frac{k}{2} \rfloor + 1$ , then  $x$  lies on branch at  $u_1$  or  $u_{\lfloor \frac{k}{2} \rfloor + 1}$ . Without loss of generality, let  $x$  lies on the branch at  $u_1$ . Then  $d(x, w) > d(y, w)$  for all  $w$  lies

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on the branch at  $u_{\lfloor \frac{k_i}{2} \rfloor + 1}$ . It follows that  $r(x | W) \neq r(y | W)$ . If  $y$  lies on some  $P_{ij}$  path for some  $2 \leq i \leq m$ ,  $1 \leq i \leq k_i$ , then  $d(x, w) > d(y, w)$  for all  $w \in W \cap V(P_{ij})$  for some  $2 \leq i \leq m$ ,  $1 \leq i \leq k_i$ . It follows that  $r(x | W) \neq r(y | W)$ .

**Case 2:**  $x$  does not lie on any path  $P_{ij}$ ,  $1 \leq i \leq m$ ,  $1 \leq i \leq k_i$ .

If  $y$  lies on some  $P_{ij}$  path, then by Case 1,  $r(x | W) \neq r(y | W)$ . So we may assume that  $y$  does not lie on any path  $P_{ij}$ ,  $1 \leq i \leq m$ ,  $1 \leq i \leq k_i$ . If  $x$  lies on the path between  $y$  and a vertex  $w \in W$ , then  $d(x, w) < d(y, w)$ . If  $y$  lies on the path between  $x$  and a vertex  $w \in W$ , then  $d(x, w) > d(y, w)$ . It follows that  $r(x | W) \neq r(y | W)$ . Clearly,  $x$  or  $y$ , say  $x$  lies on the path between two exterior major vertices, say  $w_1$  and  $w_2$  of  $G$ . If  $d(x, w_1) \neq d(y, w_1)$ , then  $r(x | W) \neq r(y | W)$ . So we may assume that  $d(x, w_1) = d(y, w_1)$ . Clearly,  $d(y, w_2) > d(x, w_2)$ . It follows that  $r(x | W) \neq r(y | W)$ .

Thus  $W$  is a resolving set of  $G$ . Since  $\langle W \rangle$  has no isolates  $\text{tr}(G) \leq \sigma(G) - \text{ex}(G) + \Theta(G)$ . By Lemma 2.6,  $\text{tr}(G) \geq \sigma(G) - \text{ex}(G) + \Theta(G)$  and hence  $\text{tr}(G) = \sigma(G) - \text{ex}(G) + \Theta(G)$ .

#### IV. CONCLUSION

We have conclude that if any cactus graph  $G$  has an exterior major vertex with terminal degree at least 2 and  $G$  has an end cycle, then we have by Theorems 3.2 and 3.4,  $\text{tr}(G) \geq \sigma(G) - \text{ex}(G) + \Theta(G) + 2e_c$ . Then characterize cactus graphs  $G$  for which  $\text{tr}(G) = \sigma(G) - \text{ex}(G) + \Theta(G) + 2e_c$  and  $\text{tr}(G) > \sigma(G) - \text{ex}(G) + \Theta(G) + 2e_c$  as the open problem.

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