

Large-Scale Mixed Integer Programming model for Open Periodic Vehicle Routing Problem with Hard Time Windows

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ABSTRACT: Vehicle routing problem (VRP) model is a very useful tool in managing efficiently the distribution of commercial products. As the objective of the model is to find the best route and schedule of a fleet of vehicles such that to satisfy customers demand. This paper considers a variant of VRP which is called open periodic vehicle routing problem (OVPRP), in which the vehicles used are not required to come back to the distribution centre after delivering periodically customers demand. Due to the number of customers is large it is then necessary to impose a restricted time windows for the vehicles whenever on duty to serve. The problem which is modelled as a mixed integer program is solved using a combined of heuristic and exact method.

KEYWORDS: Vehicle routing problem, Hard time windows, Modelling, Combined method.

I. INTRODUCTION

In the logistic system which involves delivery to meet customers' demand taking into account fleet of vehicle and their routing is called Vehicle Routing Problem (VRP). In terms of operations research the objective of this problem is to minimize the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used) such that customers' demand are satisfied within scheduled time. Due to its important role in the logistic system, the research domain of VRP has been widely explored (see for example, the survey in [1], [2], [3]). The classical vehicle routing problem (VRP) is defined as follows: vehicles with a fixed capacity Q must deliver order quantities q_i ($i = 1, \dots, n$) of goods to n customers from a single depot ($i = 0$). It is assumed that the distance d_{ij} between customers i and j ($i, j = 1, \dots, n$) is known, the objective of the problem is then to minimize the total distance travelled by the vehicles with condition that only one vehicle carries out the deliveries for a given customer.

In literature there are some variants of VRP which are grouped according to specific conditions imposed. Some of the well known variants are: Capacitated VRP (CVRP), the vehicles are restricted to carry limited capacity; VRP with time windows (VRPTW), each customer is served within a defined time frame; multiple depots VRP (MDVRP), in this variant goods can be delivered to a customer from a set of depots; VRP with pick-up and delivery (VRPPD), goods not only need to brought from the depot to the customers, but also must be picked-up at a number of customers and brought back to the depot. Taking into account several days of planning for routing problems is another variant of the VRP, known as the periodic VRP (PVRP).

In PVRP, within an extending planning horizon to several days in which customers generally do not require to have a delivery on every day in this period, but rather within one of a limited number of possible combinations of delivery days. There would be a visiting schedules associated with each customer. It is assumed that a fleet of vehicles is available and each vehicle leaves the depot, serves a set of customers, when its work shift or capacity is over, returns to the depot. As this is a VRP problem the objective is then to minimize the total length of the routes travelled by the

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vehicles on the time horizon. There are many commercial distribution problems turn up to be suitable with this model, such as, distribution for bakery companies [4], blood product distribution [5], or pick-up of raw materials for a manufacture of automobile parts [6]. Other miscellaneous applications can be found in [7].

A survey on PVRP and its extensions can be found in [8]. Due to the complexity of the problem most of the works present heuristic approaches, nevertheless [9] proposed an exact method. [10] addressed a combined of heuristic and exact method for solving PVRP. [11] present another algorithm: The solution algorithm is adopted from travelling salesman heuristic concept which, differently from the other heuristics, may allow infeasible solutions during the search process. [12] used particle swarm optimization to tackle the problem.

The Open vehicle routing problem (OVRP) has received in the literature relatively less attention than the VRP. The problem is first described in a paper by [13] without any solution attempt. [14] address the express airmail distribution in the USA and solve two separate routing problems, one for the delivery and another one for the pick up using a modified Clarke-Wright saving algorithm. The open vehicle routing problem (OVRP) describes efficient routes with minimum total distance and cost for a fleet of vehicles that serve some commodity to a given number of customers. Each customer is visited exactly once by one vehicle, while vehicle activity is bounded by capacity constraints, duration constraints and time constraints. Either each route is a sequence of customers who starts at the depot and finishes at one of the customers to whom the goods are delivered, or each route is a sequence of customers who begins at a certain customer and ends at the distribution depot, where goods are gathered. The OVRP differs from the well-known vehicle routing problem (VRP) in that the vehicles do not necessarily return to their original locations after serving to the customers; if they do, they must follow the same path in the reverse order. Therefore the routes in the OVRP consist of Hamiltonian paths originating at the depot and ending at one of the customer side. In other words, the best Hamiltonian path is NP-hard, since the Hamiltonian path problem is equivalent to the travelling salesman problem, which is known to be NP-hard. The best Hamiltonian path problem with a fixed source node must be solved for each vehicle in the OVRP, and OVRP solutions involve finding the best Hamiltonian path for each set of customers assigned to a vehicle. Consequently, the OVRP is also an NP-hard problem. It is not surprising that most of the approach for solving OVRP belong to heuristic, such as tabu search [15], simulated annealing [16], adaptive large neighbourhood search [17] and combined approach of heuristic and exact method [18].

The other variant of VRP which is the focus of this paper is open periodic vehicle routing problem (OPVRP). This problem has a new characteristic rather than those in PVRP: a vehicle is not obliged to return to depot. The objective of the problem is to find the minimum paths for each vehicle needed to service the customers. Like OVRP, OPVRP can be easily applied to any couriers to reduce costs. Many of these companies work with contractors which do the service (whether pickup or delivery) and are not obliged to come back to the depot [19].

This paper considers an open periodic vehicle routing problem with hard time windows. For abbreviation we will use OPVRPHTW.

II. DESCRIPTION OF THE PROBLEM

Firstly we consider the OVRP, Using graph, the problem can be defined as follows. Let a graph $G = (V, E)$, where $V = \{v_0, v_1, \dots, v_n\}$ is the vertex set and $E = \{(v_i, v_j): v_i, v_j \in V, i \neq j, j \neq 0\}$ is the arc set. Vertex v_0 represents the central depot where a fleet of vehicles is located, each of them with maximum carrying load equal to Q . The remaining n vertices of $V \setminus \{v_0\}$ represent the customer set. With each customer vertex is associated a non-negative known demand q_i , whereas with each arc $(v_i, v_j) \in E$ is associated a cost c_{ij} which corresponds to the cost (travel time, distance) for transiting from v_i to v_j . As with most previous OVRP approaches, we consider that the cost matrix is obtained by calculating the Euclidean distances between vertex pairs, so that $c_{ij} = c_{ji}$ ($0 < i, j \leq n, i \neq j$). The objective of the problem is to design the set of Hamiltonian paths to serve all customers such that: the number of vehicles is minimized, as well as to minimize the total cost of the generated paths. There are some restrictions which must be satisfied, such as, every path originates from the central depot v_0 , each customer vertex is assigned to a single path, and the total demand of the customer set assigned to a single path does not exceed the maximum carrying load Q of the vehicles (capacity constraint). Now we need to define for the periodic case.

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In the PVRP it is not allowed to assign directly customers to directly to vehicles, as it is normally done in VRP model. We must firstly assign a customer to an allowable delivery combination. Therefore it is necessary to define the periodic time (hours or days or whatsoever) for the customer and then assign him/her to an available delivery vehicle on each of the chosen time period.

To formulate the model, firstly we denote T as the horizon of time period. Let K denote the set of vehicles. For each vehicle $k \in K$, let Q_k denote the capacity of vehicle k . As this is a periodic problem, it is then necessary to define the number of distinct delivery combinations, which is denoted with ND . Along with that, let S_i be the set of allowable delivery combinations for customer i .

Denote the set of n customers (/nodes) by $N = \{1, 2, \dots, n\}$. Denote the depot by $\{0, n+1\}$. Each vehicle starts from $\{0\}$ and terminates at $\{n+1\}$. Each customer $i \in N$ specifies a set of days to be visited, denoted by $T_i \subseteq T$. On each day $t \in T_i$, customer $i \in N$ requests service with demand of q_i^t in weight and p_i^t in volume, service duration d_i^t and time window $[a_i, b_i]$. Note that, for the depot $i \in \{0, n+1\}$ on day t , we set $q_i^t = p_i^t = d_i^t = 0$. The travel time between customer i and j is given by c_{ij} .

In the service of delivery, a time window for vehicles is to be imposed and is given by $[a_i, b_i]$. The arrival time of a vehicle at customer i is denoted by a_i and its departure time is b_i . Vehicle routes are restricted to a maximum duration of H_k , $k = 1, \dots, K$. Costs occur for travelling through route $(i, j) \in A$, defined as $\alpha_{ij}^m = d_{ij} g_{ij}$, where d_{ij} is the distance travelled between route i to route j and g_{ij} a factor cost for travelling, for m type of vehicle. Each route originates from the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before a_i and after b_i however, the vehicle can arrive before a_i and wait for service. Due to the logistic company considered in this research has a lot of customers, the time scheduled for deliveries is so tight. Mean while the operation time daily is within busy traffic condition, therefore we should impose a strict travel time.

Notations used are defined as follows.

Set:

T	The set of workdays in the time period horizon,
K	The set of vehicles,
N	The set of customers,
N_0	The set of customers and depot $N_0 = \{0, n+1\} \cup N$,
T_i	The set of days on which customer $i \in N$ orders,
ND	The number of distinct delivery combinations
S	The set of allowable delivery combinations, in this case $S \subseteq [1, \dots, ND]$

Parameter:

Q_k	The capacity of vehicle $k \in K$,
c_{ij}	The travel time (or cost) from node $i \in N_0$ to node $j \in N_0$,
z_{st}	Equals to 1 if day $t \in T$ is in combination $s \in S$
$[a_i, b_i]$	The earliest and the latest visit time at node $i \in N_0$,
d_i^t	The service time of node $i \in N_0$ on day $t \in T_i$,
q_i^t	The demand of node $i \in N_0$ on day $t \in T_i$,

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β_{ik}^t Delivery quantity for customer $i \in N_0$ on day $t \in T_i$ using vehicle $k \in K$

Variables:

x_{ilk}^t Binary variable indicating whether vehicle $k \in K$ travels from node $i \in N_0$ to $j \in N_0$ on day $t \in T$,

w_i^s Binary variable indicating whether customer $i \in N_0$ is served with s delivery combination,

y_i^t Binary variable stating whether customer i is visited on day t

l_i^m Arrival time for vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)

u_i^m Duration of service of vehicle type $m \in K_m$ at customer $i \in V_c$ (non-negative continuous variable)

III. THE MATHEMATICAL MODEL

The OPVRPHTW problem is to design route to deliver customers demand periodically such that to minimize the total cost which consists of travelling cost of all vehicle used and the cost for the availability of vehicle in the horizon time period of a day.

$$\text{Minimize } \sum_{t \in T} \sum_{i \in N_0} \sum_{j \in N_0} \sum_{k \in K} c_{ij} x_{ijk}^t \quad (1)$$

Subject to

$$\sum_{s \in S} w_i^s = 1, \quad \forall i \in N_0 \quad (2)$$

Constraint (2) is to make sure that one and only one combination is chosen for each customer.

$$y_i^t = \sum_{s \in S} w_i^s z_{st}, \quad \forall t \in T, \forall i \in N_0 \quad (3)$$

In Constraint (3), we schedule service days for each customer which related to the chosen service combination.

$$\sum_{k \in K} x_{0j}^k = 1, \quad \forall j \in N \quad (4)$$

Constraint (4) is to guarantee that only one vehicle will leave the depot (distribution center) to serve a customer.

$$\sum_{k \in K} x_{ijk}^t \leq \frac{y_i^t + y_j^t}{2}, \quad \forall t \in T, \forall i, j \in N, (i \neq j) \quad (5)$$

In Constraint (5), we define that the route chosen by a vehicle is related to visited-day of a customer.

$$\sum_{j \in N} x_{ijk}^t = \sum_{j \in N_0} x_{ijk}^t, \quad \forall k \in K, \forall i \in N_0, \forall t \in T \quad (6)$$

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In order to ensure that vehicles will visit and leave a node of customer on the same day it is necessary to have Constraint (6).

$$\sum_{k \in K} \sum_{i \in N_0} x_{ijk}^t = y_j^t, \quad \forall j \in N, \forall t \in T \quad (7)$$

Constraint (6) states that to serve a customer should be on the selected day.

$$\sum_{i \in N_0} x_{ij+1,k}^t \leq 1, \quad \forall j \in N, \forall k \in K, \forall t \in T \quad (8)$$

Constraint (8) is to guarantee that the route of each vehicle should end at the artificial node.

$$\sum_{k \in K} \sum_{i \in N_0} x_{(j+1)ik}^t = 0, \quad \forall j \in N, \forall t \in T \quad (9)$$

However, it is necessary to impose restriction that a vehicle is not allowed to start service from artificial node, as stated in Constraint (9).

$$\sum_{j \in N} x_{0jk}^t \leq 1, \quad \forall k \in K, \forall t \in T \quad (10)$$

Constraint (10) states that only one vehicle is to be used once a day.

$$\sum_{k \in K} \sum_{i \in N} x_{i0k}^t = 0, \quad \forall t \in T \quad (11)$$

As this is an open VRP, Constraint (11) is to ensure that a vehicle is not allowed to return to the depot.

$$\sum_{(i,j) \in N} x_{ijk}^t \leq ND - 1, \quad \forall k \in K, \forall t \in T \quad (12)$$

Constraint (12) is about the sub-tour elimination process.

$$\sum_{i \in N_0} \beta_{ik}^t \sum_{j \in N} x_{ijk}^t \leq Q_k, \quad \forall k \in K, \forall t \in T \quad (13)$$

Constraint (13) is imposed for the allowable capacity of each vehicle.

$$\sum_{i \in N_0} \sum_{j \in N} c_{ij} x_{ijk}^t \leq H_k, \quad \forall k \in K, \forall t \in T \quad (14)$$

Constraint (14) is to confirm the travel distance of each vehicle.

$$\sum_{k \in K} \sum_{i \in N_0} x_{iik}^t = 0, \quad \forall t \in T \quad (15)$$

Constraint (15) is to avoid loops.

$$\sum_{i \in N_0} y_i^t = 1, \quad \forall t \in T \quad (16)$$

Constraint (16) is to ensure that only one customer is to be visited on each scheduled day.

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$$l_{ik}^t \leq a_i \sum_{j \in N} x_{ijk}^t, \quad \forall i \in N_o, \forall k \in K, \forall t \in T \quad (17)$$

$$a_i \sum_{j \in N} x_{ijk}^t \leq l_{ik}^t + u_{ik}^t \leq b_i \sum_{j \in N} x_{ijk}^t, \quad \forall i \in N_o, \forall k \in K, \forall t \in T \quad (18)$$

Constraints (17) and (18) present the hard time window imposed for each customer.

$$x_{ijk}^t \in \{0,1\}, \quad \forall i, j \in N_o, \forall k \in K, \forall t \in T \quad (19)$$

$$w_i^s \in \{0,1\}, \quad \forall i \in N_o, \forall s \in S \quad (20)$$

$$y_i^t \in \{0,1\}, \quad \forall t \in T \quad (21)$$

$$l_{ik}^t, u_{ik}^t \geq 0, \quad \forall i \in N_o, \forall k \in K, \forall t \in T \quad (22)$$

Constraints (19) to (22) state the nature of each variable.

It is easy to observe that the model as presented in expression (1) to (22) is a large scale Mixed-Integer Linear Programs.

IV. THE ALGORITHM

We use the following Step for solving the problem expressed in Eq. (1) to Eq. (22).

1. Obtain solution of the continuous relaxation (Exact method)
2. CYCLE1: remove integer variables from the basis by moving a suitable nonbasic away from its bound. The hope is to drive an infeasible integer basic variable to an integer value, and then to pivot it into the superbasic set; the previous nonbasic replacing it in the basis. (Heuristic and Exact method)
3. CYCLE2, pass1: adjust integer-infeasible superbasics by fractional steps to reach complete integer-feasibility. (Heuristic method)
4. CYCLE2, pass2: adjust integer feasible superbasics. This phase aims to conduct a highly-localized neighbourhood search. (Exact method).

V. CONCLUSION

The OPVRPHTW can be modelled as a large scale mixed integer programming (MIP) problem. Generally OPVRP is already a large scale problem, it is then reasonably logic that the problem has become more larger after imposing hard time windows for services. We propose a combined of heuristic and exact method for solving the MIP problem. The exact method is used in order to obtain the lower bound. The heuristic approach is used to find an integer feasible solution such that the deterioration of the optimal continuous solution is minimal.

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