

# Fuzzification of Some Snake Graphs

Mathew Varkey .T.K<sup>1</sup> , Sreena.T.D<sup>2</sup>

Department of Mathematics, Assistant Professor, T.K.M College of Engineering, Kollam, Kerala, India<sup>1</sup>

Department of Mathematics, Assistant Professor, Sree Narayana College, Nattika, Thrissur, Kerala, India<sup>2</sup>

**ABSTRACT:** Let  $G=(V,E)$  be a crisp graph. Then  $G$  has an evidence labeling if there exist two injections  $m$  and  $\rho$  such that  $m: V^* \rightarrow [0,1]$  defined by  $m(v_i) = \frac{i}{N}$ , where  $V^*=(v_1, v_2, \dots, v_N)$ ,  $v_i \in V$  and

$\rho: E \rightarrow [0,1]$  defined by  $\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N} m(v_k)$  if  $(v_j, v_k) \in E$  and  $m(v_k) = \bigwedge_{(v_j, v_p) \in E} m(v_p)$  and  $i < j < k$  and

$\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N^3}$  if  $(v_j, v_k) \notin E$  for any  $k$ , where  $1 \leq i \leq N - 1, 1 \leq j \leq N$ .

The graph  $G$  which admits evidence labeling is called an evidently labeled graph.

In this paper , we investigate evidence labeling of triangular snake, double triangular snake , quadrilateral snake and alternate quadrilateral snake etc.

**KEYWORDS:** Evidence Labeling, Triangular snake, Quadrilateral snake, Double triangular snake.

## I. INTRODUCTION

Graphs we discuss in this paper are simple and undirected. Graphs are used to explain relations between objects. To include fuzziness in such relations fuzzy graphs were introduced. The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. After that Rosenfield considered fuzzy relation on fuzzy sets and developed the structure of fuzzy graphs. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs.

In [5], A.Nagoor Gani and D.Rajalaxmi(a)Subahashini introduced a new concept of fuzzy labeling. A graph is said to be a fuzzy labeling graph if it has a fuzzy labeling. In [2], we introduced fuzzy digraph labeling. In [6] we introduced a special type of labeling called evidence labeling.

A triangular snake is a graph obtained from a path by joining each adjacent vertices  $v_i$  and  $v_{i+1}$  to a new vertex  $u_i$  for  $1 \leq i \leq n-1$ . A quadrilateral snake is the graph obtained from a path by joining each adjacent vertices to new vertices  $w_i, w_{i+1}$  respectively and then join  $w_i$  and  $w_{i+1}$ .

In this paper we present some snake graphs which admits evidence labeling .

### Definition

A fuzzy graph  $G = (V, \mu, \rho)$  is a non empty set  $V$  together with a pair of functions  $\mu: V \rightarrow [0,1]$  and  $\rho: V \times V \rightarrow [0,1]$  such that for all  $x, y$  in  $V$ ,  $\rho(x, y) \leq \mu(x) \wedge \mu(y)$ .

We call  $\mu$  the fuzzy vertex set of  $G$  and  $\rho$ , the fuzzy edge set of  $G$  respectively.

We denote the underlying graph of the fuzzy graph  $G = (\mu, \rho)$  by  $G^* = (\mu^*, \rho^*)$ , where  $\mu^* = \{x \in V : \mu(x) > 0\}$  and  $\rho^* = \{(x, y) \in V \times V : \rho(x, y) > 0\}$ .

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## II. MAIN RESULTS

### Theorem 2.1

The triangular snake graph  $T_n$  admits evidence labeling.

#### Proof

Consider the path graph  $T_n$ , where  $|V(G)| = 2n-1$ ,  $|E(G)| = 3n-3$ .

Let  $V^* = (v_1, v_2, \dots, v_{2n-1})$ .

Define  $m : V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{n}$ ,  $1 \leq i \leq 2n-1$  and

$\rho : E \rightarrow [0,1]$  as  $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n-1)^3}$ ,  $1 \leq i \leq 2n-3$ .

$\rho(v_i, v_{i+2}) = \frac{i(i+3)}{(2n-1)^3}$ ,  $1 \leq i \leq 2n-5$ .

$\rho(v_{2n-3}, v_{2n-1}) = \frac{2n-3}{(2n-1)^4}$ .

$\rho(v_{2n-2}, v_{2n-1}) = \frac{2n-2}{(2n-1)^4}$ .

If  $m(v_i) = m(v_j)$ , then  $i = j$ . So  $m$  is injective.

Suppose that  $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$  for some  $i \neq j$

$$\Rightarrow \frac{i(i+2)}{n^3} = \frac{j(j+2)}{n^3}$$

$$\Rightarrow i^2 - j^2 = 2(j-i)$$

$$\Rightarrow i + j = -2, \text{ contradiction.}$$

Suppose that  $\rho(v_i, v_{i+2}) = \rho(v_j, v_{j+2})$  for some  $i \neq j$

$$\Rightarrow \frac{i(i+3)}{n^3} = \frac{j(j+3)}{n^3}$$

$$\Rightarrow i^2 - j^2 = 3(j-i)$$

$$\Rightarrow i + j = -3, \text{ contradiction.}$$

Suppose that  $\rho(v_i, v_{i+1}) = \rho(v_j, v_{j+1})$  for some  $i \neq j$

$$\Rightarrow \frac{i(i+2)}{n^3} = \frac{j(j+3)}{n^3}$$

$$\Rightarrow i^2 - j^2 = 3j - 2i$$

$$\Rightarrow i^2 + 2i = 3j + j^2$$

This happens only if two quadratic equations in  $i$  and  $j$  have the same roots.

ie, when  $(i,j) = (0,0), (0,-3), (2,0)$  or  $(2,-3)$ , not possible.

Also,  $\frac{1}{n^4} < \frac{(2n-3)}{n^4} < \frac{(2n-3)}{n^4} < \frac{i(i+2)}{n^3} < \frac{i(i+3)}{n^3}$  for all  $i$ .

So  $\rho$  are injective.

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**Theorem 2.2**

The Quadrilateral Snake graph  $Q_n$  admits evidence labeling.

**Proof**

Consider the quadrilateral snake graph  $G=(V,E)$ , where  $|V(G)|=3n-2$ ,  $|E(G)|=4n-4$ .

Let  $V^*=(v_1,v_2,\dots,v_n,v_{n+1},v_{n+2},\dots,v_{3n-2})$ .

Define  $m : V^*\rightarrow[0,1]$  as  $m(v_i)=\frac{i}{3n-2}$ ,  $1 \leq i \leq 3n-2$

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+3}) = \frac{i(i+6)}{(3n-2)^3}, \quad 1 \leq i \leq 3n-8.$$

$$\rho(v_{3n-5}, v_{3n-2}) = \frac{(3n-5)}{(3n-2)^4}.$$

$$\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(3n-2)^3}, \quad 1 \leq i \leq 3n-4.$$

$$\rho(v_{3n-3}, v_{3n-2}) = \frac{3n-3}{(3n-2)^4}.$$

Clearly  $m$  and  $\rho$  are injective.

**Theorem 2.3**

The alternate triangular snake  $A(T_n)$  admits evidence labeling.

**Proof**

**Case I**

Consider the double triangular snake starting from first.

**Sub case (a) when  $n$  is odd**

Then  $|V| = \frac{3n-1}{2}$  and  $|E|=2(n-1)$ .

Let  $V(G)=\{v_1, v_2, \dots, v_{\frac{3n-1}{2}}\}$

Let  $V^*=(v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{\frac{3n-1}{2}})$ .

Define  $m : V^*\rightarrow[0,1]$  as  $m(v_i)=\frac{i}{\frac{3n-1}{2}}$ ,  $1 \leq i \leq \frac{3n-1}{2}$  and

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(\frac{3n-1}{2})^3}, \quad 1 \leq i \leq \frac{3n-5}{2}.$$

$$\rho(v_i, v_{i+2}) = \frac{i(i+3)}{(\frac{3n-1}{2})^3}, \quad i = 1, 4, \dots, \frac{3n-5}{2}$$

$$\rho(v_{\frac{3n-3}{2}}, v_{\frac{3n-1}{2}}) = \frac{\frac{3n-3}{2}}{(\frac{3n-1}{2})^4}.$$

Clearly  $m$  and  $\rho$  are injective.

**Sub case (b) when  $n$  is even.**

Then  $|V| = \frac{3n}{2}$  and  $|E|=2n-1$ .

Let  $V(G)=\{v_1, v_2, \dots, v_{\frac{3n}{2}}\}$

Let  $V^*=(v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{\frac{3n}{2}})$

Define  $m : V^*\rightarrow[0,1]$  as  $m(v_i)=\frac{2i}{3n}$ ,  $1 \leq i \leq \frac{3n}{2}$  and

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(\frac{3n}{2})^3}, \quad 1 \leq i \leq \frac{3n-4}{2}.$$

$$\rho(v_i, v_{i+2}) = \frac{i(i+3)}{(\frac{3n}{2})^3}, \quad i = 1, 4, \dots, \frac{3n-10}{2}.$$

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$$\rho\left(v_{\frac{3n-2}{2}}, v_{\frac{3n}{2}}\right) = \frac{\frac{3n-2}{2}}{\left(\frac{3n}{2}\right)^4}.$$

$$\rho\left(v_{\frac{3n-4}{2}}, v_{\frac{3n}{2}}\right) = \frac{\frac{3n-4}{2}}{\left(\frac{3n}{2}\right)^4}.$$

Clearly  $m$  and  $\rho$  are injective.

### Case II

Consider the double triangular snake starting from second.

#### Sub case (a) When $n$ is odd

Then  $|V| = \frac{3n-1}{2}$  and  $|E| = 2(n-1)$ .

Let  $V(G) = \{ v_1, v_2, \dots, v_{\frac{3n-1}{2}} \}$

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{\frac{3n-1}{2}})$ .

Define  $m : V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{\frac{3n-1}{2}}$ ,  $1 \leq i \leq \frac{3n-1}{2}$  and

$$\begin{aligned} \rho : E \rightarrow [0,1] \text{ as } \quad \rho(v_i, v_{i+1}) &= \frac{\frac{i(i+2)}{2}}{\left(\frac{3n-1}{2}\right)^3}, \quad 1 \leq i \leq \frac{3n-5}{2}. \\ \rho(v_i, v_{i+2}) &= \frac{\frac{i(i+3)}{2}}{\left(\frac{3n-1}{2}\right)^3}, \quad i = 2, 5, 8, \dots, \frac{3n-7}{2}. \\ \rho\left(v_{\frac{3n-3}{2}}, v_{\frac{3n-1}{2}}\right) &= \frac{\frac{3n-3}{2}}{\left(\frac{3n-1}{2}\right)^4}. \\ \rho\left(v_{\frac{3n-5}{2}}, v_{\frac{3n-1}{2}}\right) &= \frac{\frac{3n-5}{2}}{\left(\frac{3n-1}{2}\right)^4}. \end{aligned}$$

Clearly  $m$  and  $\rho$  are injective.

#### Sub case (b) when $n$ is even.

Then  $|V| = \frac{3n}{2}$  and  $|E| = 2n-1$ .

Let  $V(G) = \{ v_1, v_2, \dots, v_{\frac{3n}{2}} \}$

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{\frac{3n}{2}})$

Define  $m : V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{2i}{3n}$ ,  $1 \leq i \leq \frac{3n}{2}$  and

$$\begin{aligned} \rho : E \rightarrow [0,1] \text{ as } \quad \rho(v_i, v_{i+1}) &= \frac{\frac{i(i+2)}{2}}{\left(\frac{3n}{2}\right)^3}, \quad 1 \leq i \leq \frac{3n-4}{2}. \\ \rho(v_i, v_{i+2}) &= \frac{\frac{i(i+3)}{2}}{\left(\frac{3n}{2}\right)^3}, \quad i = 1, 4, \dots, \frac{3n-10}{2}. \\ \rho\left(v_{\frac{3n-2}{2}}, v_{\frac{3n}{2}}\right) &= \frac{\frac{3n-2}{2}}{\left(\frac{3n}{2}\right)^4}. \\ \rho\left(v_{\frac{3n-4}{2}}, v_{\frac{3n}{2}}\right) &= \frac{\frac{3n-4}{2}}{\left(\frac{3n}{2}\right)^4}. \end{aligned}$$

Clearly  $m$  and  $\rho$  are injective.

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## Theorem 2.4

The alternate quadrilateral snake  $A(Q_n)$  admits evidence labeling.

### Proof

#### Case I

Consider the alternate quadrilateral snake starting from first.

#### Sub case (a) When $n$ is odd

$$\text{Then } |V| = \frac{5n-1}{4} \quad \text{and } |E| = \frac{5n-5}{2}$$

$$\text{Let } V(G) = \{ v_1, v_2, \dots, v_{\frac{5n-5}{2}} \}$$

$$\text{Let } V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{\frac{5n-5}{2}}).$$

$$\text{Define } m : V^* \rightarrow [0,1] \text{ as } m(v_i) = \frac{i^2}{5n-1}, \quad 1 \leq i \leq \frac{5n-1}{4} \text{ and}$$

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{\left(\frac{5n-1}{4}\right)^3}, \quad 1 \leq i \leq \frac{5n-9}{4}.$$

$$\rho(v_i, v_{i+3}) = \frac{i(i+4)}{\left(\frac{5n-1}{4}\right)^3}, \quad i = 1, 5, \dots, \frac{5n-17}{4}$$

$$\rho\left(v_{\frac{5n-9}{4}}, v_{\frac{5n-1}{4}}\right) = \frac{\frac{5n-9}{4}}{\left(\frac{5n-1}{4}\right)^4}.$$

Clearly  $m$  and  $\rho$  are injective.

#### Sub case (b) When $n$ is even.

$$\text{Then } |V| = 2n \quad \text{and } |E| = \frac{5n-2}{2}.$$

$$\text{Let } V(G) = \{ v_1, v_2, \dots, v_{2n} \}$$

$$\text{Let } V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n})$$

$$\text{Define } m : V^* \rightarrow [0,1] \text{ as } m(v_i) = \frac{i}{2n}, \quad 1 \leq i \leq 2n \text{ and}$$

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}, \quad 1 \leq i \leq 2n-2.$$

$$\rho(v_i, v_{i+3}) = \frac{i(i+4)}{(2n)^3}, \quad i = 1, 5, 9, \dots, 2n-7.$$

$$\rho(v_{2n-1}, v_{2n}) = \frac{2n-1}{(2n)^4}.$$

$$\rho(v_{2n-3}, v_{2n}) = \frac{2n-3}{(2n)^4}.$$

Clearly  $m$  and  $\rho$  are injective.

### Case II

Consider the alternate quadrilateral snake starting from second.

#### Sub case (a) When $n$ is odd

$$\text{Then } |V| = 2n-1 \quad \text{and } |E| = \frac{3n-3}{2}$$

$$\text{Let } V(G) = \{ v_1, v_2, \dots, v_{2n-1} \}$$

$$\text{Let } V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n-1}).$$

$$\text{Define } m : V^* \rightarrow [0,1] \text{ as } m(v_i) = \frac{i}{2n-1}, \quad 1 \leq i \leq 2n-1 \text{ and}$$

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n-1)^3}, \quad 1 \leq i \leq 2n-3.$$

$$\rho(v_i, v_{i+3}) = \frac{i(i+4)}{(2n-1)^3}, \quad i = 2, 6, 10, \dots, 2n-7$$

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$$\rho(v_{2n-3}, v_{2n-1}) = \frac{2n-3}{(2n-1)^4}$$

$$\rho(v_{2n-4}, v_{2n-1}) = \frac{2n-4}{(2n-1)^4}$$

Clearly  $m$  and  $\rho$  are injective.

**Subcase (b) When  $n$  is even.**

Then  $|V| = 2n-2$  and  $|E| = \frac{5n-8}{2}$ .

Let  $V(G) = \{ v_1, v_2, \dots, v_{2n-2} \}$

Let  $V^* = (v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n-2})$

Define  $m : V^* \rightarrow [0,1]$  as  $m(v_i) = \frac{i}{2n-2}$ ,  $1 \leq i \leq 2n-2$  and

$$\rho : E \rightarrow [0,1] \text{ as } \rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n-2)^3}, \quad 1 \leq i \leq 2n-4.$$

$$\rho(v_i, v_{i+3}) = \frac{i(i+4)}{(2n-2)^3}, \quad i = 2, 6, \dots, 2n-6.$$

$$\rho(v_{2n-3}, v_{2n-2}) = \frac{2n-3}{(2n-2)^4}.$$

Clearly  $m$  and  $\rho$  are injective.

### III. CONCLUSION

In this paper we discuss about evidence labeling which is used to fuzzify an ordinary crisp graphs like snake graphs. We also prove some theorems which shows the admissibility of evidence labelling in such graphs.

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