

# Modular Product on Intuitionistic Fuzzy Graphs

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**ABSTRACT:** Intuitionistic fuzzy sets as a generalization of fuzzy set. Intuitionistic fuzzy graph is an extension of fuzzy graph which gives more flexibility and compatibility to the model when compared with ordinary and fuzzy situations. In this paper, we define modular product on intuitionistic fuzzy graph. Some properties of modular product on intuitionistic fuzzy graph are studied.

**KEYWORDS:** Intuitionistic fuzzy graph, modular product, regular IFG.

**2010 AMS Classification:** 16Y60, 08A72, 03E72

## I. INTRODUCTION

In the year 1983, Atanassov[1] initiated the notion of intuitionistic fuzzy set as a generalisation of fuzzy sets. Atanassov[1] added new components that determine the degree of non-membership in the definition of fuzzy set. He introduced a new component degree of non-membership in addition to the degree of membership in the case of fuzzy sets with the condition that their sum be less than or equal to one. Intuitionistic fuzzy sets have been applied in a wide variety of areas, including engineering, computer science, medicine, mathematics, chemistry, and economics.

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths, cycles and connectedness are introduced by Rosenfeld [6]. In the year 2015 S.Dogra [4] defined some new operations on fuzzy graphs.

In 1994, Shannon and Atanassov[7] introduced the concept of intuitionistic fuzzy graphs. Parvathi and Karunambigai [5] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Shannon and Atanassov[7].

In this paper, the modular product on Intuitionistic fuzzy graphs and their properties are studied. Theorems related to the above concepts are stated and proved with examples.

## II. PRELIMINARIES

### Definition 2.1

An intuitionistic fuzzy graph is of the form  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  on  $G^* = (V, E)$ , where

1.  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\sigma_1: V \rightarrow [0,1]$  and  $\sigma_2: V \rightarrow [0,1]$  denote the degree of membership and non-membership of the element  $v_i \in V$  respectively, such that  $\sigma_1(v_i) + \sigma_2(v_i) \leq 1$  for all  $v_i \in V$ .
2.  $E \subseteq V \times V$  where  $\mu_1: E \rightarrow [0,1]$  and  $\mu_2: E \rightarrow [0,1]$  are defined by  $\mu_1(v_i, v_j) \leq \sigma_1(v_i) \wedge \sigma_1(v_j)$  and  $\mu_2(v_i, v_j) \leq \sigma_2(v_i) \vee \sigma_2(v_j)$  such that  $\mu_1(v_i, v_j) + \mu_2(v_i, v_j) \leq 1, \forall (v_i, v_j) \in E$ .

### Definition 2.2

An intuitionistic fuzzy graph  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  is called *strong* intuitionistic fuzzy graph if  $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$  and  $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j), \forall (v_i, v_j) \in E$ .

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**Definition 2.3**

An intuitionistic fuzzy graph  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  is called *complete* intuitionistic fuzzy graph if  $\mu_1(v_i, v_j) = \sigma_1(v_i) \wedge \sigma_1(v_j)$  and  $\mu_2(v_i, v_j) = \sigma_2(v_i) \vee \sigma_2(v_j), \forall v_i, v_j \in V, i \neq j$ .

**Definition 2.4**

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph, then the scalar cardinality of  $V$  defined by  $|V| = \sum_{v_i \in V} \frac{1 + \sigma_1(v_i) - \sigma_2(v_i)}{2} = \sum_{v_i \in V} w(v_i)$  is called *order* of  $G$  and is denoted by  $O(G)$  or  $p$ .

**Definition 2.5**

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph, then the scalar cardinality of  $E$  defined by  $|E| = \sum_{v_i, v_j \in E} \frac{1 + \mu_1(v_i, v_j) - \mu_2(v_i, v_j)}{2} = \sum_{v_i, v_j \in E} w(v_i, v_j)$  is called *size* of  $G$  and is denoted by  $S(G)$  or  $q$ .

**Definition 2.6**

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph. The degree of a vertex  $u$  in  $G$  is denoted by  $d_G(u) = (d_1^G(u), d_2^G(u))$  and defined by  $d_1^G(u) = \sum_{u \neq v} \mu_1^G(u, v) = \sum_{u, v \in E} \mu_1^G(u, v)$  and  $d_2^G(u) = \sum_{u \neq v} \mu_2^G(u, v) = \sum_{(u, v) \in E} \mu_2^G(u, v)$ .

### III. MODULAR PRODUCT ON INTUITIONISTIC FUZZY GRAPHS

**Definition 3.1**

Let  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two fuzzy graphs with underlying vertex sets  $V_1$  and  $V_2$  and edge sets  $E_1$  and  $E_2$  respectively.

Then modular product of  $G_1$  and  $G_2$  is a pair of functions  $((\sigma_1^{G_1} \odot \sigma_1^{G_2})(\sigma_2^{G_1} \odot \sigma_2^{G_2}), (\mu_1^{G_1} \odot \mu_1^{G_2})(\mu_2^{G_1} \odot \mu_2^{G_2}))$  with underlying vertex set  $V_1 \odot V_2 = \{(x_1, y_1) / x_1 \in V_1, y_1 \in V_2\}$  and underlying edge set  $E_1 \odot E_2 = \{(x_1, y_1)(x_2, y_2) / x_1 x_2 \in E_1, y_1 y_2 \in E_2 \text{ or } x_1 x_2 \notin E_1, y_1 y_2 \notin E_2\}$  with

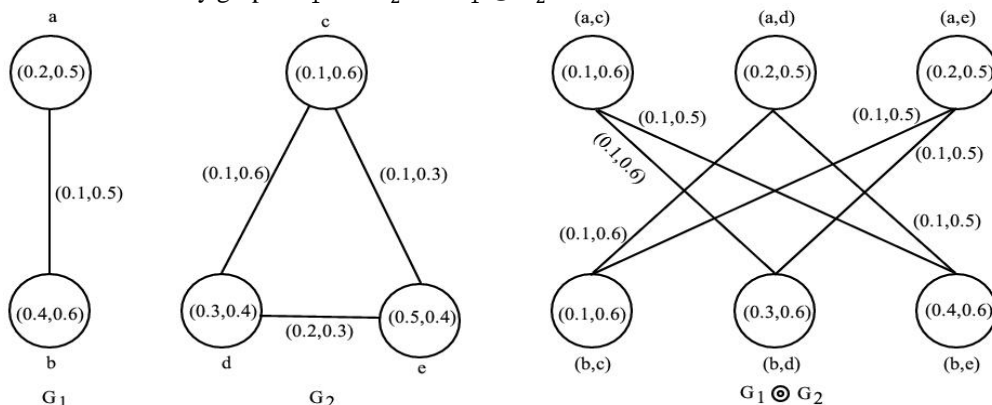
$(\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_1, y_1) = \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_2}(y_1), (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_1, y_1) = \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_2}(y_1),$  where  $x_1 \in V_1$  and  $y_1 \in V_2$ .

$$(\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) = \begin{cases} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2), & \text{if } x_1 x_2 \in E_1, y_1 y_2 \in E_2 \\ \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) & \text{if } x_1 x_2 \notin E_1, y_1 y_2 \notin E_2. \end{cases}$$

$$(\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) = \begin{cases} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), & \text{if } x_1 x_2 \in E_1, y_1 y_2 \in E_2 \\ \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) & \text{if } x_1 x_2 \notin E_1, y_1 y_2 \notin E_2 \end{cases}$$

**Example 3.1**

Let  $G_1^* = (V_1, E_1)$  and  $G_2^* = (V_2, E_2)$  be graphs such that  $V_1 = \{a, b\}, V_2 = \{c, d, e\}, E_1 = \{ab\}, E_2 = \{cd, ce, de\}$ . Consider two intuitionistic fuzzy graphs  $G_1$  and  $G_2$  and  $G_1 \odot G_2$  as follows:



Modular Product

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### Theorem 3.1

Let  $G_1$  and  $G_2$  be the intuitionistic fuzzy graphs of  $G_1^*$  and  $G_2^*$  respectively. Then modular product  $G_1 \odot G_2$  is an intuitionistic fuzzy graph of  $G_1^* \odot G_2^*$ .

#### Proof:

Let  $G_1 = (\sigma_1, \mu_1)$  and  $G_2 = (\sigma_2, \mu_2)$  be intuitionistic fuzzy graphs on  $G_1^*$  and  $G_2^*$  respectively.

We have to prove that  $G_1 \odot G_2$  is an intuitionistic fuzzy graph.

By the definition,  $\sigma^{G_1 \odot G_2}$  is an intuitionistic fuzzy set of  $V_1 \odot V_2$  and  $\mu^{G_1 \odot G_2}$  is an intuitionistic fuzzy set of  $E_1 \odot E_2$ .

$\forall (x_1, y_1)(x_2, y_2) \in E_1 \odot E_2$ ,

$$\begin{aligned} (\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) &= \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) \text{ if } x_1 x_2 \in E_1 \text{ and } y_1 y_2 \in E_2 \\ &\leq \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2), \end{aligned}$$

Since  $G_1$  and  $G_2$  are intuitionistic fuzzy graphs

$$\begin{aligned} (\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) &= \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), \text{ if } x_1 x_2 \in E_1 \text{ and } y_1 y_2 \in E_2 \\ &\leq \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) \end{aligned}$$

Since  $G_1$  and  $G_2$  are intuitionistic fuzzy graphs

$$\begin{aligned} (\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) &= \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2), \\ &\text{if } x_1 x_2 \notin E_1 \text{ and } y_1 y_2 \notin E_2 \\ &= (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_1, y_1) \wedge (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_2, y_2) \end{aligned}$$

$$\begin{aligned} (\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) &= \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2), \\ &\text{if } x_1 x_2 \notin E_1 \text{ and } y_1 y_2 \notin E_2 \\ &= (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_1, y_1) \vee (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_2, y_2) \end{aligned}$$

Hence,  $(\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) \leq (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_1, y_1) \wedge (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_2, y_2)$

$$(\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) \leq (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_1, y_1) \vee (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_2, y_2)$$

Thus  $\mu^{G_1 \odot G_2}$  is an intuitionistic fuzzy relation on  $\sigma^{G_1 \odot G_2}$ .

Hence  $G_1 \odot G_2$  is an intuitionistic fuzzy graph.

### Theorem 3.2

Let  $G_1$  and  $G_2$  be strong intuitionistic fuzzy graphs of the graphs  $G_1^*$  and  $G_2^*$ . Then Modular product  $G_1 \odot G_2$  is also an strong intuitionistic fuzzy graph of  $G_1^* \odot G_2^*$ .

#### Proof:

Let  $G_1 = (\sigma_1, \mu_1)$  and  $G_2 = (\sigma_2, \mu_2)$  be strong intuitionistic fuzzy graphs on  $G_1^*$  and  $G_2^*$  respectively.

We have to prove that  $G_1 \odot G_2$  is a strong intuitionistic fuzzy graph.

By the definition,  $\sigma^{G_1 \odot G_2}$  is an intuitionistic fuzzy set of  $V_1 \odot V_2$  and  $\mu^{G_1 \odot G_2}$  is an intuitionistic fuzzy set of  $E_1 \odot E_2$ .

$\forall (x_1, y_1)(x_2, y_2) \in E_1 \odot E_2$ .

$$\begin{aligned} (\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) &= \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) \text{ if } x_1 x_2 \in E_1 \text{ and } y_1 y_2 \in E_2 \\ &= \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) \end{aligned}$$

Since  $G_1$  and  $G_2$  are strong intuitionistic fuzzy graphs

$$\begin{aligned} (\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) &= \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), \text{ if } x_1 x_2 \in E_1 \text{ and } y_1 y_2 \in E_2 \\ &= \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) \end{aligned}$$

Since  $G_1$  and  $G_2$  are strong intuitionistic fuzzy graphs

$$\begin{aligned} (\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) &= \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2), \\ &\text{if } x_1 x_2 \notin E_1 \text{ and } y_1 y_2 \notin E_2 \\ &= (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_1, y_1) \wedge (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_2, y_2) \end{aligned}$$

$$\begin{aligned} (\mu_2^{G_1} \odot \mu_2^{G_2})(x_1, y_1), (x_2, y_2) &= \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2), \\ &\text{if } x_1 x_2 \notin E_1 \text{ and } y_1 y_2 \notin E_2 \\ &= (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_1, y_1) \vee (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_2, y_2) \end{aligned}$$

Hence,  $(\mu_1^{G_1} \odot \mu_1^{G_2})(x_1, y_1), (x_2, y_2) = (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_1, y_1) \wedge (\sigma_1^{G_1} \odot \sigma_1^{G_2})(x_2, y_2)$

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$$(\mu_2^{G_1} \odot \mu_2^{G_2})((x_1, y_1), (x_2, y_2)) = (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_1, y_1) \vee (\sigma_2^{G_1} \odot \sigma_2^{G_2})(x_2, y_2).$$

Thus  $\mu^{G_1 \odot G_2}$  is an strong intuitionistic fuzzy relation on  $\sigma^{G_1 \odot G_2}$ .

Hence  $G_1 \odot G_2$  is an strong intuitionistic fuzzy graph.

### Theorem 3.3

Let  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two complete intuitionistic fuzzy graphs.

1. If  $\mu_1^{G_1} \leq \mu_1^{G_2}, \mu_2^{G_1} \geq \mu_2^{G_2}$ , then  $d^{G_1 \odot G_2}(x, y) = d^{G_1}(x)$ ,  
where  $d^{G_1 \odot G_2}(x, y) = (d_1^{G_1 \odot G_2}(x, y), d_2^{G_1 \odot G_2}(x, y)), d^{G_1}(x) = (d_1^{G_1}(x), d_2^{G_1}(x))$

2. If  $\mu_1^{G_1} \geq \mu_1^{G_2}, \mu_2^{G_1} \leq \mu_2^{G_2}$ , then  $d^{G_1 \odot G_2}(x, y) = d^{G_2}(x)$ ,  
where  $d^{G_1 \odot G_2}(x, y) = (d_1^{G_1 \odot G_2}(x, y), d_2^{G_1 \odot G_2}(x, y)), d^{G_2}(x) = (d_1^{G_2}(x), d_2^{G_2}(x))$

### Proof:

Let  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two complete intuitionistic fuzzy graphs.

The degree of a vertex  $(x, y)$  in the modular product =

$$\begin{aligned} d^{G_1 \odot G_2}(x_1, y_1) &= \left( d_1^{G_1 \odot G_2}(x_1, y_1), d_2^{G_1 \odot G_2}(x_1, y_1) \right) \\ d_1^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_1^{G_2}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) \\ &\quad \text{Since both intuitionistic fuzzy graphs are complete.} \\ &= \sum_{x_1 x_2 \in E_1} \mu_1^{G_1}(x_1 x_2), \text{ Since } \mu_1^{G_1} \leq \mu_1^{G_2} \\ &= d_1^{G_1}(x_1) \end{aligned}$$

Similarly,

$$\begin{aligned} d_2^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2) \\ &\quad + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), \\ &\quad \text{Since both intuitionistic fuzzy graphs are complete} \\ &= \sum_{x_1 x_2 \in E_1} \mu_2^{G_1}(x_1 x_2) \text{ Since } \mu_2^{G_1} \geq \mu_2^{G_2} \\ &= d_2^{G_1}(x_1) \end{aligned}$$

$$\begin{aligned} d_1^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2), \\ &\quad \text{Since both intuitionistic fuzzy graph are complete.} \\ &= \sum_{x_1 x_2 \in E_1} \mu_1^{G_2}(x_1 x_2) . \text{ Since } \mu_1^{G_1} \geq \mu_1^{G_2} = d_1^{G_2}(x_1) . \end{aligned}$$

Similarly,

$$\begin{aligned} d_2^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), \\ &\quad \text{Since both intuitionistic fuzzy graph are complete.} \\ &= \sum_{x_1 x_2 \in E_1} \mu_2^{G_2}(x_1 x_2), \text{ Since } \mu_2^{G_1} \leq \mu_2^{G_2} \\ &= d_2^{G_2}(x_1) . \text{ Hence the proof.} \end{aligned}$$

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### Theorem 3.4

Let  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  is complete and  $G_2^*$  is regular. If  $\mu^{G_2} \subseteq \sigma^{G_1}$ ,  $\mu^{G_1} \subseteq \sigma^{G_2}$  and  $\mu^{G_1} = \mu^{G_2}$ , then  $G_1 \odot G_2$  is a regular if and only if  $G_1$  is a regular intuitionistic fuzzy graph.

#### Proof:

Let  $G_2^*$  be a regular graph of degree  $d$  and  $G_1^*$  is complete graphs. Let  $\mu^{G_1} = \mu^{G_2} = k$  for all  $E_1$  and  $E_2$ , where  $k = (k_1, k_2)$  is constant. Let  $\sigma^{G_1} \subseteq \mu^{G_2}$ ;  $\mu^{G_1} \subseteq \sigma^{G_2}$ . Suppose that  $G_1$  is a regular fuzzy graph of degree  $d^{G_1}$ .  
By definition for any  $(u, v) \in \sigma^{G_1} \odot \sigma^{G_2}$

$$\begin{aligned} d_1^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2), \text{ Since } G_1 \text{ is complete.} \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2), \text{ Since } \mu^{G_1} = \mu^{G_2}, \\ &= d_1^{G_1}(x_1) \cdot d_1^{G_2}(x_2) \\ d_2^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_2^{G_1}(x_1) \vee \sigma_2^{G_1}(x_2) \vee \sigma_2^{G_2}(y_1) \vee \sigma_2^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \vee \mu_2^{G_2}(y_1 y_2), \text{ Since } G_1 \text{ is complete.} \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \quad \text{Since } \mu^{G_1} = \mu^{G_2} \\ &= d_2^{G_1}(x_1) \cdot d_2^{G_2}(x_2) \\ &\therefore G_1 \odot G_2 \text{ is a regular intuitionistic fuzzy graph.} \end{aligned}$$

Conversely, Let us assume that  $G_1 \odot G_2$  be a regular intuitionistic fuzzy graph,  $G_2^*$  be a regular graph of degree  $d^{G_2^*}$  and  $G_1^*$  be a complete graph.

For any two points  $(x_1, x_2), (y_1, y_2)$  in  $\sigma^{G_1 \odot G_2}$

$$\begin{aligned} d_1^{G_1 \odot G_2}(x_1, x_2) &= d_1^{G_1 \odot G_2}(y_1, y_2) \\ d_1^{G_1}(x_1) d_2^{G_2}(x_2) &= d_1^{G_1}(y_1) d_2^{G_2}(y_2) \end{aligned}$$

$d_1^{G_1}(x_1) = d_1^{G_1}(y_1)$ , Since  $G_2^*$  is regular.

$$\begin{aligned} d_2^{G_1 \odot G_2}(x_1, x_2) &= d_2^{G_1 \odot G_2}(y_1, y_2) \\ d_2^{G_1}(x_1) d_2^{G_2}(x_2) &= d_2^{G_1}(y_1) d_2^{G_2}(y_2) \end{aligned}$$

$d_2^{G_1}(x_1) = d_2^{G_1}(y_1)$  Since  $G_2^*$  is regular.

Hence,  $G_1$  is regular intuitionistic fuzzy graph.

### Theorem 3.5

Let  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  is regular and  $G_2^*$  is complete. If  $\mu^{G_2} \subseteq \sigma^{G_1}$ ,  $\mu^{G_1} \subseteq \sigma^{G_2}$  and  $\mu^{G_1} = \mu^{G_2}$ , then  $G_1 \odot G_2$  is a regular if and only if  $G_2$  is a regular intuitionistic fuzzy graph.

#### Proof:

Let  $G_1^*$  be a regular graph of degree  $d^{G_1^*}$  and  $G_2^*$  is complete graphs. Let  $\mu^{G_1} = \mu^{G_2} = k$  for all  $E_1$  and  $E_2$ , where  $k = (k_1, k_2)$  is constant. Let  $\sigma^{G_1} \subseteq \mu^{G_2}$ ;  $\mu^{G_1} \subseteq \sigma^{G_2}$ . Suppose that  $G_2$  is a regular intuitionistic fuzzy graph of degree  $d^{G_2}$ .

By definition for any  $(u, v) \in \sigma^{G_1} \odot \sigma^{G_2}$ .

$$\begin{aligned} d_1^{(G_1 \odot G_2)(x_1, y_1)} &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2) + \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_1^{G_1}(x_1) \wedge \sigma_1^{G_1}(x_2) \wedge \sigma_1^{G_2}(y_1) \wedge \sigma_1^{G_2}(y_2) \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_1}(x_1 x_2) \wedge \mu_1^{G_2}(y_1 y_2), \text{ Since } G_2^* \text{ is complete.} \\ &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_1^{G_2}(y_1 y_2), \text{ Since } \mu^{G_1} = \mu^{G_2} \qquad \qquad \qquad = d_1^{G_2}(y_1) \cdot d_1^{G_1}(y_2) \end{aligned}$$

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$$\begin{aligned}
 d_2^{G_1 \odot G_2}(x_1, y_1) &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \wedge \mu_2^{G_2}(y_1 y_2) \\
 &+ \sum_{x_1 x_2 \notin E_1, y_1 y_2 \notin E_2} \sigma_2^{G_1}(x_1) \wedge \sigma_2^{G_1}(x_2) \wedge \sigma_2^{G_2}(y_1) \wedge \sigma_2^{G_2}(y_2) \\
 &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_1}(x_1 x_2) \wedge \mu_2^{G_2}(y_1 y_2), \text{ Since } G_2^* \text{ is complete.} \\
 &= \sum_{x_1 x_2 \in E_1, y_1 y_2 \in E_2} \mu_2^{G_2}(y_1 y_2) \text{ Since } \mu^{G_1} = \mu^{G_2} \\
 &= d_2^{G_2}(y_1) \cdot d_1^{G_1^*}(y_2) \\
 &\therefore G_1 \odot G_2 \text{ is a regular intuitionistic fuzzy graph}
 \end{aligned}$$

Conversely, Let us assume that  $G_1 \odot G_2$  be a regular intuitionistic fuzzy graph,  $G_1^*$  be a regular graph of degree  $d^{G_1^*}$  and  $G_2^*$  be a complete graph

For any two points  $(x_1, x_2), (y_1, y_2)$  in  $\sigma^{G_1} \odot \sigma^{G_2}$ ,

$$\begin{aligned}
 d_1^{G_1 \odot G_2}(x_1, x_2) &= d_1^{G_1 \odot G_2}(y_1, y_2) \\
 d_1^{G_2}(x_1) d_1^{G_1^*}(x_2) &= d_1^{G_2}(y_1) d_1^{G_1^*}(y_2) \\
 d_1^{G_2}(x_1) &= d_1^{G_2}(y_1), \text{ Since } G_1^* \text{ is regular.} \\
 d_2^{G_1 \odot G_2}(x_1, x_2) &= d_2^{G_1 \odot G_2}(y_1, y_2) \\
 d_2^{G_2}(x_1) d_2^{G_1^*}(x_2) &= d_2^{G_2}(y_1) d_2^{G_1^*}(y_2) \\
 d_2^{G_2}(x_1) &= d_2^{G_2}(y_1), \text{ Since } G_1^* \text{ is regular.} \\
 \text{Hence, } G_2 &\text{ is regular intuitionistic fuzzy graph.}
 \end{aligned}$$

## IV. CONCLUSION

Here, we discussed the modular product on Intuitionistic fuzzy graphs and their properties with examples. Theorems related to modular product on intuitionistic fuzzy graphs are stated and proved. Also examples related to the theorems are explained. Further, we propose to discuss some other operations on IFG with examples.

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