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Infinite Series with Common Difference Zero Will Have At Least One Regularized Finite Value—Zero

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ABSTRACT: In this paper, we will find a plausible regularized finite value of infinite series with common difference zero, i.e., series which look like—$a + a + a + a + \ldots$

KEYWORDS: Infinite Series(divergent), Riemann zeta function, Ramanujan Summation.

I. INTRODUCTION

In mathematics, a series is, roughly speaking, a description of the operation of adding infinitely many quantities, one after the other, to a given starting quantity. The study of series is a major part of calculus and its generalization, mathematical analysis. Series are used in most areas of mathematics, even for studying finite structures (such as in combinatorics), through generating functions. In addition to their ubiquity in mathematics, infinite series are also widely used in other quantitative disciplines such as physics, computer science, statistics and finance. [1]
Regularized finite values of infinite series, thus, have wide range applications, such as, in explaining the Casimir effect. In this paper, we will manipulate infinite series to assign a finite value to them—whose applications may be wide range.

II. RELATED WORK

The regularized sum of any divergent infinite series such as, 1+2+3… was considered to be infinity—because terms keep on adding and enlarging the sum—until ground-breaking results by great mathematicians likeLeonhard Euler, Bernhard Riemann, and Srinivasa Ramanujan were discovered, for example, 1+2+3…= -1/12 and 1+1+1…= -1/2. These values seem to be counter-intuitive, but they have real-life applications in various fields.
[2]It was in the systematic theory of infinite series (beyond explicit evaluations) that Euler made one of his greatest contributions, namely his creation of a theory of divergent series, or at least the first steps towards such a theory. He knew that by attempting to associate numbers as values of diverging series he was going beyond the confines of what people were used to thinking about, and yet he insisted that one has to build a theory of divergent series in order to free analytical methods from artificial limitations. There is no better place to hear his viewpoint than from his great 1760 paper (communicated in 1755) *De seriebusdivergentibus*[3]
Notable enough, however, are the controversies over the series 1− 1+1−1+1−1+ ... whose sum was given by Leibniz as 1/2, although others disagree. No one has yet assigned another value to that sum, and so the controversy turns on the question whether the series of this type have a certain sum. Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived—namely the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken—has relevance only for convergent series, and we should in general give up this idea of sum for divergent series. Wherefore, those who thus define a sum cannot be blamed if they claim they are unable to assign a sum to a series. On the other hand, as series in analysis arise from the expansion of fractions or irrational quantities or even transcendentals, it will in turn be permissible in calculation to substitute in place of such a series that quantity out of whose development it is produced. For this reason, if we employ this definition of sum, that is to say, the sum of a series is that quantity which generates the series, all doubts with respect to divergent series vanishes and no further controversy remains on this score, in as much as this definition is applicable equally to

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convergent or divergent series. Accordingly, Leibniz, without any hesitation, accepted for the series 1 − 1+1 − 1+1 − 1 + ... , the sum 1/2, which arises out of the expansion of the fraction 1/1 + 1, and for the series 1 − 2+3 − 4+5 − 6 + ..., the sum 1/4, which arises out of the expansion of the formula 1/(1+1)2. In a similar way a decision for all divergent series will be reached, where always a closed formula from whose expansion the series arises should be investigated. However, it can happen very often that this formula itself is difficult to find, as here where the author treats an exceptional example, that divergent series par excellence 1−1+2−6+24−120+720−5040+..., which is Wallis' hypergeometric series, set out with alternating signs; this series, in whatever formula it finds its origin and however much this formula is valid, is seen to be determinable by only the deepest study of higher Analysis. Finally, after various attempts, the author by a wholly singular method using continued fractions found that the sum of this series is about 0.596347362123, and in this decimal fraction the error does not affect even the last digit. Then he proceeds to other similar series of wider application and he explains how to assign them a sum in the same way, where the word "sum" has that meaning which he has here established and by which all controversies are cut off. (From the translation of E. J. Barbeau and P. J. Leah [4]).

The paper shows clearly the fact that he understood what was involved, that the number we associate to an infinite series is a matter of convention, and that a systematic treatment of this question would be very beneficial to the development of Analysis.

In 1745 he had discussed these matters in letters to Goldbach and Nicholas Bernoulli, especially the summing of the factorial series

$$\sum_{n\geq 0}(-1)^n n!,$$

which he called the divergent series par excellence. In it Euler has this to say (free translation from German):

.. . I believe that every series should be assigned a certain value. However, to account for all the difficulties that have been pointed out in this connection, this value should not be denoted by the name sum, because usually this word is connected with the notion that a sum has been obtained by a real summation: this idea however is not applicable to "seriebusdivergentibus". .. .

In the nineteenth century, with Abel, Cauchy, Dedekind, and Weierstrass, rigorous foundations were laid for Analysis, and divergent series were banned as "the work of the devil". With the rise of these formal principles Euler's reputation also suffered, and people began to misunderstand what he was trying to do and started thinking of him as a loose mathematician. It should have been clear that no one who calculated the values of many convergent series to tremendous accuracy would have loose ideas about when series diverge and what their values are. His true greatness in these matters was not appreciated till a century afterwards when a genuine theory of divergent series was created and it became clear how far he was ahead of his time [5], [6].

It is true that Euler had some misconceptions regarding summation of divergent series. He appeared to believe that all series could be summed by some procedure or other and also that in general all summation procedures would lead to the same value. The actual theory of divergent series shows that the situation is much more subtle. However, in my opinion these are differences in detail that do not alter the fact that he took the first steps in creating a true theory of divergent series.

Euler had several different methods of summing divergent series, but most of all he worked with what we now call Abel summation.

But, contributions are just not limited to the aforementioned. Contributions by great Indian mathematicians, like Ramanujan, cannot be neglected. Ramanujan, for example, provided us with the Ramanujan Summation.Ramanujan summation is a technique invented by the mathematician SrinivasaRamanujan for assigning a value to divergent infinite series. Although the Ramanujan summation of a divergent series is not a sum in the traditional sense, it has properties which make it mathematically useful in the study of divergent infinite series, for which conventional summation is undefined. It's not to be confused with Ramanujan's sum.[7]

Starting with simpler methods to assign sums to such series, eventually, led to the development of more rigorous and complex methods such as, the Riemann zeta function.[8]

III. METHODS

Let there be an infinite series,S, with a common difference equal to zero, i.e.,

$S = a + a + a + a + \ldots$, here common difference $= a - a = 0$ (1)

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Now, the series can also be written as:

$S = (a + a) + (a + a) + \ldots = 2a + 2a + 2a + 2a + \ldots = 2(a + a + a + a + \ldots) = 2S\ (from\ (1))$ (2)

$\Rightarrow S = 2S$

$\Rightarrow S = 0$

The series can also be written by clubbing three, four, five and so on a's, but each method would, indeed, result the same finite value— zero.

Using the same methodology, as in (2), grouping na's (generalizing the method) would give:

$S = (a + a + \ldots n\ times) + (a + a + \ldots n\ times) + (a + a + \ldots n\ times) + \ldots ; n > 1$ (3)

$= na + na + na + \ldots = n(a + a + a + a + \ldots) = nS$

$\Rightarrow S = nS$

$\Rightarrow (n - 1)S = 0$

$\Rightarrow S = 0\ [\because n - 1 \neq 0\ (from\ (3))]$

IV. CONCLUSION

Thus, every infinite series will at least have zero as a plausible regularized sum (finite numerical value), provided the common difference is zero. For example, the infinite series—1+1+1+… , using the above methodology, would result zero as a possible sum, but it can also take up a different value (= -1/2), according to the Riemann zeta function, i.e. ,

$1 + 1 + 1 + \ldots = 0 \text{and} 1 + 1 + 1 + \ldots = -\frac{1}{2}$ [9],[10]

Although the zeta function may be a more rigorous method of solving infinite series, but this doesn't mean that the zeta function is the only way—there could be many more ways of assigning regularized finite values to infinite series. We get such astounding results because of there being infinite number of terms; the multiple values of infinite series can be used according to our needs and the required conditions, in various fields such as, mathematics, physics (string theory), and so on.

Although, infinite series seem at first sight not to have any meaningful value at all, but they can be manipulated to yield a number of mathematically interesting results, some of which have applications in other fields such as complex analysis, quantum field theory, and string theory.

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