

SK Open set and SK Closed set in Topological Spaces

K.Kaleeswari¹, K. Amutha²

M.Phil Scholar, Dept. of Mathematics, SCAS, Dgl, Tamil Nadu, India¹

Associate Professor, Dept. of Mathematics, SCAS, Dgl, Tamil Nadu, India²

ABSTRACT: In this paper, we introduce and investigate the notation of SK open set and SK closed set. We obtain fundamental properties of SK open set and SK closed set and discuss the relation between SK open set and SK closed set and other related sets.

KEYWORDS: SK open set, SK closed set.

I. INTRODUCTION

Levine [1] introduced generalized closed sets in topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Arya et al [2], Balachandran et al [3], Bhattacharya et al [4], Arockiarani et al [5], Gnanambal [6] Malghan [7], Nagaveni [8] and Palaniappan et al [9] have worked on generalized closed sets. In this paper, the notion of sg-interior is defined and some of its basic properties are investigated. Also we introduce the idea of sg-closure in topological spaces using the notions of sg-closed sets and obtain some related results. Throughout the paper, X and Y denote the topological spaces (X, τ) and (Y, σ) respectively and on which no separation axioms are assumed unless otherwise explicitly stated. In 1937, regular open sets were introduced and used to define the semi-regularization space of a topological space. Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X , the closure of A and the interior of A will be denoted by $cl(A)$ and $int(A)$, respectively. Stone [10] defined a subset A of a space X to be a regular open if $A = int(cl(A))$. Norman Levine [11] defined a subset A of a space X to be a semi-open if $A \subseteq cl(int(A))$, or equivalently, a set A of a space X will be termed semi-open if and only if there exists an open set U such that $U \subset A \subset cl(U)$. Mashhour *et al.* [12] defined a subset A of a space X to be a preopen if $A \subset int(cl(A))$. Njastad [13] defined a subset A of a space X to be an α -open if $A \subseteq int(cl(int(A)))$. The complement of a semi-open (resp., regular open) set is said to be semi-closed [14] (resp., regular closed). The intersection of all semi-closed sets of X containing A is called the semi-closure [15] of A . The union of semi-open sets of X contained in A is called the semi-interior of A . Joseph and Kwack [16] introduced the concept of θ -semi open sets using semi-open sets to improve the notion of S-closed spaces. Also Joseph and Kwack [16] introduced that a subset A of a space X is called θ -semi-open if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset cl(G \subset A)$. It is well-known that, a space X is called TI if to each pair of distinct points x, y of X , there exists a pair of open sets, one containing x but not y and the other containing y but not x , as well as is TI if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed. A space X is regular if for each $x \in X$ and each open set G containing x , there exists an open set H such that $x \in H \subset cl(H) \subset G$. Ahmed [17] defined a topological space (X, τ) to be s^{**} -normal if and only if for every semi-closed set F and every semi-open set G containing F , there exists an open set H such that $F \subset H \subset cl(H) \subset G$. In 1968, Velicko [18], defined the concepts of δ -open and θ -open as, a subset A of a space

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X is called δ -open (resp., θ -open) if for each $x \in A$, there exists an open set G such that $x \in G \subset \text{int}(cl(G)) \subset A$ (resp., $x \in G \subset cl(G) \subset A$). Di Maio and Noiri [19] introduced that a subset A of a space X is called semi- θ -open if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset scl(G) \subset A$. The family of all open (resp., semi-open, α -open, pre-open, θ -semi-open, semi- θ -open, θ -open, δ -open, regular open, semi-closed and regular closed) subsets of a topological space (X, τ) are denoted by τ (resp., $SO(X)$, $\alpha O(X)$, $PO(X)$, $\theta SO(X)$, $S\theta O(X)$, $\theta O(X)$, $\delta O(X)$, $RO(X)$, $SC(X)$ and $RC(X)$).

II. PRELIMINARIES

Definition :2.1 A subset A of a space X is called

- (i) A **preopen** set if $A \subseteq \text{int}(cl(A))$ and a **preclosed** if $cl(\text{int}(A)) \subseteq A$.
- (ii) A **regular open set** if $A = \text{int}(cl(A))$ and **regular closed set** if $A = cl(\text{int}(A))$.
- (iii) A **semi open set** if $A \subseteq cl(\text{int}(A))$ and **semi closed set** if $\text{int}(cl(A)) \subseteq A$.

Definition 2.2: A subset A of a space X is called

- (i) **g-closed set**[1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of g-closed set is g-open set.
- (ii) **semi generalized closed set** [4] (sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X . The complement of sg-closed set is sg-open set.
- (iii) **generalized preclosed set** [7] (gp-closed) if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X . The complement of gp-closed set is gp-open set.

Definition 2.3: Let X be a topological space and let $x \in X$. A subset N of X is said to be sg -neighbourhood of x if there exists a sg -open set G such that $x \in G \subset N$.

Definition 2.4: A subset A of a space X is called Bc-open if for each $x \in A \in BO(X)$, there exists a closed set F such that $x \in F \subset A$. The family of all Bc-open subsets of a topological space (X, τ) is denoted by $BcO(X, \tau)$ or (Briefly $BcO(X)$). The complement of Bc-open set is called Bc-closed set.

Definition 2.5:[20] A subset A of a space X is called b-open if $A \subseteq \text{int}(cl(A)) \cup cl(\text{int}(A))$. The family of all b-open subsets of a topological space (X, τ) is denoted by $BO(X, \tau)$ or (Briefly $BO(X)$). The complement of b-open set is called b-closed set.

Definition 2.6: Let A be a subset of X . A point $x \in A$ is said to be sg-interior point of A is a sg -neighbourhood of x . The set of all sg-interior points of A is called the sg-interior of A and is denoted by $sg\text{-int}(A)$.

Definition 2.7: Let A be a subset of a space X . We define the sg-closure of A to be the intersection of all sg-closed sets containing A . In symbols, $sg-cl(A) = \bigcap \{F : A \subset F \in sgc(X)\}$.

III. SK OPEN SET

Definition 3.1: A set A is said to be SK open set if

- (i) A is $BcO(X)$ [22] and
- (ii) A is Sg-open[21].

Example 3.1: Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau^c = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ and $U=X$. Then SK open set = $\{\emptyset, X, \{a, b\}, \{a, c\}\}$.

Theorem 3.1: Let (X, τ) be a topological space. Prove that if A is SK open set then A is regular open set in X . ($U=X$)

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Proof: Let A be SK open set in X.

Then A is BcO(X) and Sg-open in X.

$F \subset A$ where F is closed (By the definition of BcO(X)).

(ie) $Cl(F) = F \subset A$.

$\Rightarrow Cl(F) \subset A$.

$\Rightarrow \text{int}(Cl(F)) \subseteq \text{int}(A) = A$. since A is open

$\Rightarrow \text{int}(Cl(F)) \subseteq A$.

$\Rightarrow \text{int}(Cl(A)) \subseteq A$. since $F \subset A$(1)

$A \subseteq Cl(A)$.

$\text{int}(A) \subseteq \text{int}(Cl(A))$.

$A \subseteq \text{int}(Cl(A))$(2)

From (1) and (2)

$A = \text{int}(Cl(A))$

\therefore A is regular open.

Converse of the above theorem is also true.

Example 3.2: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and

$\tau^c = \{\emptyset, X, \{b\}, \{a\}, \{a, b\}, \{b, c\}\}$ and $U = X$. Then SK open set = $\{\emptyset, X, \{a\}, \{b, c\}\}$, regular open = $\{\emptyset, X, \{a\}, \{b, c\}\}$ and regular closed = $\{\emptyset, X, \{a\}, \{b, c\}\}$.

The elements {a}, {b,c} are in SK open set and in regular open.

Also the above example said regular open = regular closed.

Theorem 3.2: Let (X, τ) be a topological space. Prove that if A is SK open set then A is regular closed in X.

Proof: From the theorem3.1 and example3.1 obviously proved.

Theorem 3.3: Let (X, τ) be a topological space. Prove that if A is SK closed set then A is regular open and regular closed in X.

Example 3.3: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and

$\tau^c = \{\emptyset, X, \{b\}, \{a\}, \{a, b\}, \{b, c\}\}$ and $U = X$. Then SK closed set = $\{\emptyset, X, \{a\}, \{b, c\}\}$, regular open = $\{\emptyset, X, \{a\}, \{b, c\}\}$ and regular closed = $\{\emptyset, X, \{a\}, \{b, c\}\}$.

Hence regular open = regular closed.

Theorem 3.4: Let (X, τ) be a topological space. Prove that if A is SK open set then A is semiclosed in X. Converse is need not be true.(U=X)

Proof: Let A be SK open set in X.

Then A is BcO(X) and sg open in X.

$F \subset A$ Since A is Bc open

$cl(F) = F$ Since F is closed by the definition of BcO(X)

$cl(F) = F \subset A$

$\Rightarrow cl(F) \subset A$

$\text{int}(cl(F)) \subset \text{int}(A)$

$\Rightarrow \text{int}(cl(F)) \subset A$ Since A is Bc open

$\Rightarrow \text{int}(cl(A)) \subset A$

Hence A is semi closed.

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Converse,

Example:3.4 Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$,

$\tau^c = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $U = X$

(i) Semi closed set = $\{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$

(ii) SK open set = $\{\phi, X, \{a\}, \{a, b\}, \{b, c\}\}$

Here the element $\{a\}$ is in semi closed set but not in SK open set.

Theorem 3.5: Let (X, τ) be a topological space. Prove that every SK open set is semi open set in X . Converse is need not be true. ($U = X$)

Example 3.5: Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$

$\tau^c = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $U = X$

(i) SK open set = $\{\phi, X, \{b\}, \{a, c\}\}$ and

(ii) semi open set = $\{\phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$

Here the element $\{b\}$ is in both SK open set and semi open set and the element $\{c\}$ is in semi open set but not in SK open set.

Theorem 3.6: Let (X, τ) be a topological space. Prove that if A is SK open set then A is semi generalized closed set (sg-closed) in X . Converse is need not be true. ($U = X$)

Proof: Let A be SK open set in X .

Then A is $BcO(X)$ and sg open in X .

$F \subset A$

$cl(F) \subset A$ Since F is closed

$\Rightarrow scl(F) \subset A$ Since every closed set is semi closed

$\Rightarrow scl(A) \subset U$ Since $A \subset U$, U is semi open

Since A is sg open.

$\therefore A$ is sg closed.

Converse,

Example:3.6 Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$,

$\tau^c = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $U = X$

(i) SK open set = $\{\phi, X, \{c\}, \{a, b\}\}$

(ii) sg closed set = $\{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$

Here the element $\{a, c\}$ is in sg closed set but not in SK open set.

Theorem 3.7: Let (X, τ) be a topological space. Prove that every SK closed set is semi generalized open (sg-open) in X . Converse is need not be true. ($U = X$)

Example 3.7: Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$,

$\tau^c = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $U = X$

(i) SK closed set = $\{\phi, X, \{c\}, \{a, b\}\}$

(ii) sg open set = $\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$

Here the element $\{c\}$ is in both SK closed set and sg open set and $\{b, c\}$ is in sg open set but not in SK closed set.

Theorem 3.8: Let (X, τ) be a topological space. Prove that if A is SK open set then A is pre open set in X . Converse is need not be true. ($U = X$)

Proof: Let A be SK open set in X .

Then A is $BcO(X)$ and sg open in X .

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$$\begin{aligned}
 &F \subset A \\
 &cl(F) \subset cl(A) \\
 &F \subset cl(A) \\
 &\Rightarrow A \subset cl(A) \\
 &\Rightarrow int(A) \subset int(cl(A)) \quad \text{Since } int(A) = A \\
 &\Rightarrow A \subset int(cl(A))
 \end{aligned}$$

Hence the result.

Converse,

Example:3.8 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{c\}, \{a, b\}\}$, $\tau^c = \{\emptyset, X, \{c\}, \{a, b\}\}$ and $U = X$

- (i) SK open set = $\{\emptyset, X, \{c\}, \{a, b\}\}$
- (ii) Pre open set = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

Here the element $\{c\}$ is in both SK open set and pre open set. The element $\{a\}$ is in pre open but not in SK open set.

Theorem 3.9: Let (X, τ) be a topological space. Prove that if A is SK closed set then A is pre open set in X . Converse is need not be true. ($U = X$)

Example 3.9 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$, $\tau^c = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $U = X$

- (i) SK closed set = $\{\emptyset, X, \{a\}, \{b, c\}\}$
- (ii) Pre open set = $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$

Here the element $\{a\}$ is in both SK closed set and pre open set. The element $\{c\}$ is in pre open but not in SK closed set.

Theorem 3.10: Let (X, τ) be a topological space. Prove that if A is SK open set then A is generalized preclosed set (g-preclosed) in X . Converse is need not be true. ($U = X$)

Example 3.10 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$,

$\tau^c = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $U = X$

- (i) SK open set = $\{\emptyset, X, \{c\}, \{a, b\}\}$
- (ii) g-preclosed set = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

Here the element $\{c\}$ is in both SK open set and g-preclosed set. The element $\{a\}$ is in g-preclosed but not in SK open set.

Theorem 3.11: Let (X, τ) be a topological space. Prove that if A is SK closed set then A is generalized preclosed set (g-preclosed) in X . Converse is need not be true. ($U = X$)

Example:3.11 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$,

$\tau^c = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $U = X$

- (i) g-preclosed = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$
- (ii) SK closed set = $\{\emptyset, X, \{a\}, \{c\}, \{b, c\}\}$

Here the element $\{a\}$ is in both SK closed set and g-preclosed set. The element $\{b\}$ is in g-preclosed but not in SK closed set.

IV. PROPERTIES OF SK OPEN SETS

Let (X, τ) be a topological space. Let A and B be the SK open sets,

- (i) $A \cup B \supset A \cap B$
- (ii) $(A \cup B)^c = A^c \cup B^c$

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(iii) $(A \cap B)^c = A^c \cap B^c$

The above properties are true for SK closed sets.

V. CONCLUSION

1. SK open set is subset of the following sets
(i) Semi open, (ii) Pre open (iii) g-open (iv) g-preopen
2. SK closed set is subset of the following sets
(i) Semi closed (ii) Pre closed (iii) g-closed (iv) g-preclosed

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