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Family of Disjoint Sets and Its Applications

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ABSTRACT: In this paper we introduce the concept of family of disjoint sets as a concept in set theory and introduce some theorems related to this concept, we also provide examples and applications. The four colours problem is a problem in graph theory, in this paper we introduce four colours problem as an application of family of disjoint sets. Families of disjoint sets explain four colours problem as an example of m colours problem. Using families of disjoint sets we can explain any familiar map or unfamiliar map with any number of colours used to draw these maps.

KEYWORDS: family of disjoint sets, degree of intersection, standard of meeting, local maximum, local minimum.

I. INTRODUCTION

The purpose of this paper is to introduce the concept of family of disjoint sets as a topic of set theory. A family of nonempty sets is called family of disjoint sets if every two sets, belong to this family, are disjoint. Throughout this paper we assume any family of disjoint sets can contain finite or infinite sets. We also introduce definitions of degree of intersection, meeting and standard of meeting which are related to the concept of family of disjoint sets. Beside these definitions we introduce many examples.

In main results we establish theorems and corollaries explain properties of family of disjoint sets, degree of intersection and standard of meeting. We establish many theorems about local maximum and local minimum number of family of disjoint sets, these theorems have many applications.

In this paper we consider any map as families of disjoint sets, such that regions in the same colour as a family of disjoint sets. Regions in the same colour, are sets which are disjoint. Regions of map draw as sets intersect each other with degree of intersection three. We have seen from main theorems, minimum and maximum families of disjoint sets meet each other at W intersections depends on degree of intersection and standard of meeting. For any standard of meeting there is minimum and maximum families of disjoint sets, defined by local minimum of families of disjoint sets and local maximum of families of disjoint sets, these families of disjoint sets have properties which are explained by many theorems. Families of disjoint sets is generalization to colours problem, families of disjoint sets explain four colours problem as an example of m colours problem.

II. FAMILIES OF DISJOINT SETS

Here we introduce the concept of family of disjoint sets and concepts related to family of disjoint sets with some examples and propositions.

Definition 2.1. A family of nonempty sets is called family of disjoint sets if every two sets, belong to this family, are disjoint. One set or more belongs to the family of disjoint sets is called subfamily of disjoint sets. A family of disjoint sets may contain finite or infinite disjoint sets. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, we used the notation

$$\chi_i = X_{i1} \cup X_{i2} \cup X_{i3} \cup \dots \cup X_{ir}, \text{ or } \chi_i = \bigcup_{j=1}^r X_{ij}, 1 \leq i \leq m.$$

Example 2.2. In real line the family $\{(0,1), (1,2), (2,3), \dots\}$ is family of disjoint sets.

Definition 2.3. The family of disjoint sets χ_1 is called intersects the family of disjoint sets χ_2 if there exists a set belongs to χ_1 and a set belongs to χ_2 have nonempty intersection.

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Definition 2.4. Two families of disjoint sets χ_1 and χ_2 called two proper families of disjoint sets if $\chi_1 \cap \chi_2$ is nonempty, and n families of disjoint sets called n proper families of disjoint sets if any two have nonempty intersection.

Definition 2.5. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_n$ be n proper families of disjoint sets and X is nonempty set, $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_n$, $X = \{x : x \in X_{ij}, 1 \leq i \leq n, 1 \leq j, X_{ij} \in \chi_i\}$ then X is called intersection of degree n . X also can be written as

$$X = X_{1j} \cap X_{2j} \cap X_{3j} \cap \dots \cap X_{nj}, X = \{x : x \in X_{ij}, 1 \leq i \leq n, 1 \leq j, X_{ij} \in \chi_i\}$$

The index i for families of disjoint sets and j for sets belong to one family of disjoint sets.

Definition 2.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_{i1}, X_{i2}, X_{i3}, \dots, X_{in}$ be n sets such that any two sets belong to two different families of disjoint sets, and X is nonempty set, $X = X_{i1} \cap X_{i2} \cap X_{i3} \cap \dots \cap X_{in}$, $X = \{x : x \in X_{ij}, 1 \leq i \leq r, 1 \leq j \leq n, n \leq r, X_{ij} \in \chi_i\}$ then the positive integer n is called degree of intersection X .

Definition 2.7. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and X is nonempty set, $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, then the intersection X is called represent the families $\chi_1, \chi_2, \chi_3, \dots, \chi_r$, and we used the notation $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, or $X = (\chi_1, \chi_2, \chi_3, \dots, \chi_r)$.

Remark 2.8. As abbreviation instate of the notation $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \dots \cap \chi_r$, and $X = (\chi_1, \chi_2, \chi_3, \dots, \chi_r)$ we used the notation $X = (1, 2, 3, \dots, r)$.

Definition 2.9. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty different intersections each intersection of degree n , the family of disjoint sets χ_i is said to be appear t times at W intersection if there are t intersections each belong to χ_i , where $1 \leq i \leq r$, (χ_i has appearance equal t). If we say $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets at $X_1, X_2, X_3, \dots, X_w$, this means any one of $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ appear at $X_1, X_2, X_3, \dots, X_w$, (has appearance not equal zero).

Example 2.10. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_7$ be 7 families of disjoint sets and X, Y, Z are nonempty different intersections each of degree 4. $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \chi_5$, $Y = \chi_2 \cap \chi_3 \cap \chi_5 \cap \chi_7$ and $Z = \chi_1 \cap \chi_4 \cap \chi_6 \cap \chi_7$. We can use the notation $X = (1, 2, 3, 5)$, $Y = (2, 3, 5, 7)$, $Z = (1, 4, 6, 7)$ At intersections X, Y, Z , the family of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_5$ and χ_7 each appear twice at X, Y, Z , χ_4 and χ_6 each appear once. Also we can say at intersections X, Y, Z , the family of disjoint sets 1, 2, 3, 5, 7 each appear twice, and 4, 6 each appear once. As application to following proposition, each of intersections X, Y, Z the is of degree four we have $3(4) = 12 = 2 + 2 + 2 + 1 + 2 + 1 + 2$.

Proposition 2.11. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections each of degree n , and t_i number of times the family of disjoint sets χ_i appear at these W intersections, then we have $\sum_{i=1}^m t_i = wn$.

Proof: Since the appearance for m families of disjoint sets at W intersections $X_1, X_2, X_3, \dots, X_w$ each of degree n , then summation of all appearance for all m families of disjoint sets equal wn therefore $\sum_{i=1}^m t_i = wn$.

Definition 2.12. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty different intersections of these m families of disjoint sets, each intersection of degree n . The family of disjoint sets χ_i is said

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meets family of disjoint sets χ_j at these W intersections, if whatever we exchange families of disjoint sets at these W intersections, there exists at least one intersection the two families of disjoint sets χ_i and χ_j appear at this intersection.

Example 2.13. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_7$ be 7 families of disjoint sets and X, Y, Z are nonempty intersections each of degree 4. $X = \chi_1 \cap \chi_2 \cap \chi_3 \cap \chi_5$, $Y = \chi_2 \cap \chi_3 \cap \chi_5 \cap \chi_7$ and $Z = \chi_1 \cap \chi_4 \cap \chi_6 \cap \chi_7$. We can use the notation $X = (1,2,3,5)$, $Y = (2,3,5,7)$, $Z = (1,4,6,7)$. The following exchange is right $X = (1,2,4,5)$, $Y = (2,3,6,7)$, $Z = (1,3,5,7)$. The following exchange is not right $X = (1,2,3,7)$, $Y = (2,3,5,5)$, $Z = (1,4,6,7)$, because $Y = \chi_2 \cap \chi_3 \cap \chi_5 \cap \chi_5$ and the family of disjoint sets χ_5 appear twice at intersection Y , the family of disjoint sets χ_5 intersect χ_5 , a family of disjoint sets can't intersect itself. Any two of families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_5, \chi_7$ meet each other at X, Y, Z but χ_6 each χ_4 , not meet χ_6 at X, Y, Z .

Proposition 2.14. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections, the family of disjoint sets χ_i appear w_i times at W intersections and the family of disjoint sets χ_j appear w_j time at W intersections, then we say the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these W intersections if and only if $w_i + 1 \leq w_i + w_j$.

Proof: First suppose the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these W intersections, where $1 \leq i \leq m, 1 \leq j \leq m$, whatever we exchange families of disjoint sets represent these intersections, this means there exists intersection represented by the two families of disjoint sets χ_i and χ_j . Since from definitions of family of disjoint sets and degree of intersection there is no family of disjoint sets appear twice at one intersection, let us arrange each of $X_1, X_2, X_3, \dots, X_w$ to be represented either by χ_i or χ_j , but the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these W intersections, therefore exists intersection X_r represented by the two families of disjoint sets χ_i and χ_j , such that χ_i appear at $X_1, X_2, X_3, \dots, X_r$ and χ_j appear at $X_r, X_{r+1}, X_{r+2}, \dots, X_w$, then $w_i + 1 \leq w_i + w_j$.

Conversely suppose $w_i + 1 \leq w_i + w_j$, then if we exchange families of disjoint sets such that any intersection of $X_1, X_2, X_3, \dots, X_w$ either by χ_i or χ_j , there exists at least one intersection represented by the two families of disjoint sets χ_i and χ_j , since $w_i + 1 \leq w_i + w_j$, therefore the family of disjoint sets χ_i meet the family of disjoint sets χ_j at these W intersections.

We introduced the concept of meeting of two families of disjoint sets at number of intersections from what

happening in our life. Two persons A and B will meet each other at least once per week if A appear $\frac{7+1}{2} = 4$ days and

B appear 4 days. Where A meets B ? They meet each other at place allowed to A and allowed to B , place considered belongs (allowed) to A and place considered belongs (allowed) to B . Something belongs to A and belongs to B , it is intersection of A and B , let them be seven intersections, A and B will meet each other at this seven intersections if both A and B appear 4 time at these intersections or come to these intersections 4 time.

Proposition 2.15. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections, and any two families of disjoint sets meet each other at this W intersections then at least any family of disjoint sets appear $\frac{w+1}{2}$ time at W when W odd, and appear $\frac{w}{2} + 1$ time at W when W even.

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Proof: Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections, the family of disjoint sets χ_i appear w_i time at W intersections and the family of disjoint sets χ_j appear w_j time at W intersections, let $w_i = \frac{w+1}{2}$, $w_j = \frac{w+1}{2}$ where W is odd, from Proposition 2.14. we have $w+1 \leq w_i + w_j = w+1$. Let $w_i = \frac{w}{2} + 1$, $w_j = \frac{w}{2} + 1$ where W is even, from Proposition 2.14. we have $w+1 \leq w_i + w_j = w+2$.

Definition 2.16. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ each of degree n , and W be the minimum number of nonempty intersections, such that any two families of disjoint sets meet each other at these W intersections, then these W intersections called standard of meeting for these m families of disjoint sets.

Example 2.17. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be 5 nonempty intersections each of degree 3, where $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (3,4,5)$, $X_4 = (1,3,5)$, $X_5 = (2,4,5)$ and let $Y_1, Y_2, Y_3, \dots, Y_{10}$ be 10 nonempty intersections each of degree 3, where $Y_1 = (1,2,3)$, $Y_2 = (1,2,4)$, $Y_3 = (1,2,5)$, $Y_4 = (1,3,4)$, $Y_5 = (1,3,5)$, $Y_6 = (1,4,5)$, $Y_7 = (2,3,4)$, $Y_8 = (2,3,5)$, $Y_9 = (2,4,5)$, $Y_{10} = (3,4,5)$. At X_1, X_2, X_3, X_4, X_5 any two of the five families of disjoint sets meet each other, and at $Y_1, Y_2, Y_3, \dots, Y_{10}$ any two of the ten families of disjoint sets meet each other. From Definition 2.16. X_1, X_2, X_3, X_4, X_5 standard of meeting for these five families of disjoint sets, and $Y_1, Y_2, Y_3, \dots, Y_{10}$ is not standard of meeting for these five families of disjoint sets, but these five families of disjoint sets any two of them meet at intersections $Y_1, Y_2, Y_3, \dots, Y_{10}$.

Definition 2.18. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_r$ be r families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections each of degree n , any two families of disjoint sets meet each other at this W intersections. We say there are t families of disjoint sets can be added at W intersections if $\chi_1, \chi_2, \chi_3, \dots, \chi_r, \chi_{r+1}, \chi_{r+2}, \chi_{r+3}, \dots, \chi_{r+t}$ be $r+t$ families of disjoint sets any two of them meet each other at these W intersections and for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_r, \chi_{r+1}, \chi_{r+2}, \chi_{r+3}, \dots, \chi_{r+t}, \chi_{r+t+1}$, these $r+t+1$ families of disjoint sets there exist two families of disjoint sets not meet each other at W intersections.

Example 2.19. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,3,5)$, $X_3 = (4,5,1,2)$ any two families of disjoint sets meet each other at this 3 intersections. We can add a family of disjoint sets to these 5 families of disjoint sets. There is one family of disjoint sets can be added at the 3 intersections. We have $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$, 6 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,5,6)$, $X_3 = (3,4,5,6)$ any two of these six families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 . For seven families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6, \chi_7$ and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,5,6)$, $X_3 = (3,4,5,7)$. For any change in the intersections X_1, X_2, X_3 there is two family of disjoint sets not meet each other at the 3 intersections X_1, X_2, X_3 . Any two of families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ meet each other at X_1, X_2, X_3 , and χ_6, χ_7 not meet with any family of disjoint sets at the 3 intersections X_1, X_2, X_3 .

Definition 2.20. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m is called local maximum families of disjoint sets at W intersections, if any two families of disjoint sets meet each other at these W intersections, and any family of disjoint sets has appearance either equal $\frac{w+1}{2}$

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or $\frac{w+3}{2}$ for w odd such that not all families of disjoint sets has appearance equal $\frac{w+3}{2}$, and for w even we have three types, first all families of disjoint sets has appearance equal $\frac{w}{2}+1$, second appearance equal $\frac{w}{2}+1$ and $\frac{w}{2}+2$, not all $\frac{w}{2}+2$ but at least has one appearance equal $\frac{w}{2}+2$, third all families of disjoint sets has appearance equal $\frac{w}{2}+1$, except only one family of disjoint sets has appearance equal $\frac{w}{2}$. Maximum of local maximum families of disjoint sets called maximum families of disjoint sets.

Example 2.21. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets any two meet each other at w intersections, $w=8$, each intersection of degree 3, then $m=5$ be local maximum families of disjoint sets meet each other at intersections $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (2,4,5)$, $X_4 = (2,3,4)$, $X_5 = (1,3,5)$, $X_6 = (3,4,5)$, $X_7 = (1,3,4)$, $X_8 = (1,2,5)$. All families of disjoint sets has appearance equal $\frac{w}{2}+1=5$, except only one family of disjoint sets has appearance equal $\frac{w}{2}=4$.

Example 2.22 Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be five families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be five intersections each of degree three, $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (3,4,5)$, $X_4 = (1,3,5)$, $X_5 = (2,4,5)$ any two families of disjoint sets meet each other at these five intersections, if we add a family of disjoint sets χ_6 to $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ there exist two families of disjoint sets not meet each other at this five intersections. If $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (3,4,5)$, $X_4 = (1,3,5)$, $X_5 = (2,4,6)$ we have χ_2 not meet χ_5 and χ_1 not meet χ_6 . Then we called these five families of disjoint sets $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ local maximum families of disjoint sets meet each other at this five intersections.

Remark 2.23. For the relation between n, w, m till now we are using definitions and examples, if you prefer to see generalization of these examples, you can see Theorem 3.1.

Proposition 2.24. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, and $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , if $m(\frac{w+1}{2})+r=nw$ where $0 \leq r \leq \frac{w-1}{2}$ for w odd, and $m(\frac{w}{2}+1)+r=nw$ where $0 \leq r \leq \frac{w}{2}-1$ for w even. Then m be local maximum families of disjoint sets meet each other at w intersections.

Proof: Let w odd, from equation $m(\frac{w+1}{2})+r=nw$ any two families of disjoint sets meet each other if any family of disjoint sets has appearance not less than $\frac{w+1}{2}$, because $\frac{w+1}{2} + \frac{w+1}{2} = w+1 > w$. Suppose $m+1$ families of disjoint sets meet at w intersections, then from equation $m(\frac{w+1}{2})+r=nw$ there is family of disjoint sets has appearance equal r , where $0 \leq r \leq \frac{w-1}{2}$, this family of disjoint sets has appearance equal r , not meet any one of m families of disjoint sets at w intersections, since for local maximum families of disjoint sets meet each other at w intersections, value of r we have $r + \frac{w+1}{2} = \frac{w-1}{2} + \frac{w+1}{2} = w < w+1$. Therefore m be local maximum families of disjoint sets meet each other at w intersections. The proof will be the same for w even.

Definition 2.25. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be w nonempty intersections each of degree n , then m families of disjoint sets is called local minimum families of disjoint sets at w intersections, if $m(w-1) \leq nw$ such that $m \neq n$. When $m = n$, then m families of disjoint sets is called local minimum families of

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disjoint sets at W if $m(w-k) \leq nw < m(w-k+1)$ such that $m \neq n$, where $k = 2, 3, 4, \dots$. Minimum of local minimum families of disjoint sets called minimum families of disjoint sets.

Remark 2.26. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m local minimum families of disjoint sets at W intersections, each of degree n , if we say be m local minimum families of disjoint sets at W intersections, with t families of disjoint sets appear at each intersection, this means $m(w-1) + t = nw$ where $1 \leq t < w$.

Example 2.27. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1, 2, 3, 4)$, $X_2 = (1, 2, 3, 5)$, $X_3 = (4, 5, 1, 2)$ any two families of disjoint sets meet each other at this 3 intersections. We have 5 local minimum families of disjoint sets. The two families of disjoint sets χ_1, χ_2 appear at each intersection, this means $4(3) = nw = m(w-1) + t = 5(2) + 2 = 12$. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ be 6 local minimum families of disjoint sets and $X_1 = (1, 2, 3, 4)$, $X_2 = (1, 2, 5, 6)$, $X_3 = (3, 4, 5, 6)$ be 3 intersections each of degree four. There is no any family of disjoint sets appear at each intersection, this means $4(3) = nw = m(w-1) + t = 6(2) + 0 = 12$.

Proposition 2.28. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ be m_1 local minimum families of disjoint sets at W intersections, each of degree n , and let $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_2}$ be m_2 local minimum families of disjoint sets at W intersections, each of degree n . m_1 be local minimum families of disjoint sets with t_1 families of disjoint sets appear at each intersection and m_2 be local minimum families of disjoint sets with t_2 families of disjoint sets appear at each intersection, if $t_1 < t_2$ then $m_2 \leq m_1$.

Proof: Since m_1 be local minimum families of disjoint sets with t_1 families of disjoint sets appear at each intersection, then we can write $nw = m_1(w-1) + t_1$, and since m_2 be local minimum families of disjoint sets with t_2 families of disjoint sets appear at each intersection we can write $nw = m_2(w-1) + t_2$, we have $m_1(w-1) - m_2(w-1) = t_2 - t_1$, since $t_1 < t_2$ then we have $(m_1 - m_2)(w-1) = t_2 - t_1 < 0$, since $(w-1) > 0$ then $m_2 < m_1$, but both m_1, m_2 are integers then $m_2 \leq m_1$.

Definition 2.28. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections each of degree n , then m families of disjoint sets is called families of disjoint sets with minimum appearance $(w-i)$ at W intersections, if any family of disjoint sets has appearance not less than $w-i$, such that not all families of disjoint sets has appearance equal $w+1-i$, where $i = 1, 2, 3, \dots, \frac{w+1}{2}$

Remark 2.29. Local minimum families of disjoint sets at W intersections can be called families of disjoint sets with minimum appearance $(w-1)$ at W intersections. Also local maximum families of disjoint sets at W intersections can be called families of disjoint sets with minimum appearance $\frac{w+1}{2}$ at W intersections if W is odd, and with minimum appearance $\frac{w}{2} + 1$ at W intersections if W is even.

Example 2.30. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ be 6 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 5. $X_1 = (1, 2, 3, 4, 5)$, $X_2 = (1, 2, 3, 4, 6)$, $X_3 = (1, 2, 3, 5, 6)$. These 6 families of disjoint sets meet each other at X_1, X_2, X_3 are local minimum families of disjoint sets meet each other at X_1, X_2, X_3 . Families of disjoint sets represented (labeled) by 1, 2, 3 appear at each intersections, have appearance equal W . Also these 6 families of disjoint sets meet each other at X_1, X_2, X_3 are local maximum families of disjoint sets meet each other at X_1, X_2, X_3 , with families of disjoint sets represented (labeled) by 1, 2, 3 appear at each intersections. These 6 families of disjoint are local minimum and local maximum at X_1, X_2, X_3 , because $w-1 = \frac{w+1}{2}$, $3-1 = \frac{3+1}{2} = 2$.

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Example2.31. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6, \chi_7$ be 7 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 5. $X_1 = (1,2,3,4,5)$, $X_2 = (1,2,3,6,7)$, $X_3 = (1,4,5,6,7)$. These 7 families of disjoint sets meet each other at X_1, X_2, X_3 , are local minimum families of disjoint sets meet each other at X_1, X_2, X_3 because 6 families of disjoint sets each has appearance $w-1=2$ there is only one family of disjoint sets appear at all three intersections. Also these seven families of disjoint sets are local maximum meet each other at X_1, X_2, X_3 , because $w-1 = \frac{w+1}{2}$, $3-1 = \frac{3+1}{2} = 2$.

Example2.32. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be four families of disjoint sets and X_1, X_2, X_3 be three nonempty intersections each of degree three, $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (1,3,4)$. These four families of disjoint sets meet each other at X_1, X_2, X_3 are local minimum families of disjoint sets meet each other at X_1, X_2, X_3 with the same appearance (any family of disjoint sets appear twice at X_1, X_2, X_3). Also $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (1,3,4)$, these four families of disjoint sets meet each other at X_1, X_2, X_3 are local maximum families of disjoint sets meet each other at X_1, X_2, X_3 with different appearance (family of disjoint sets represented by 1 appear at each intersections and others appear twice).

Remark2.33. In case local minimum families of disjoint sets meet each other at w intersections itself is local maximum families of disjoint sets meet each other at w intersections, we have $w-1 = \frac{w+1}{2}$, $w = 3$ for w odd, and $w-1 = \frac{w}{2} + 1$, $w = 4$ for w even.

Remark 2.34. At $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , there are many local minimum families of disjoint sets meet each other at w intersections and there are many local maximum families of disjoint sets meet each other at w intersections, this for each w , depend on w and n . There is maximum families of disjoint sets meet each other at w intersections, this maximum is equal only one local maximum and greater than other local maximum, for $w \leq 2n - 1$. The maximum of all local maximum depend only on n , it is equal $2n - 1$. Table (1) explain this.

n	w	m local maximum at w	$m \leq 2n - 1$
3	3	4	$m < 2n - 1$
3	4	4	$m < 2n - 1$
3	5	5	$m = 2n - 1$
3	6	4	$m < 2n - 1$
3	7	5	$m = 2n - 1$
3	8	4	$m < 2n - 1$
3	10	5	$m = 2n - 1$
5	3	7	$m < 2n - 1$
5	5	8	$m < 2n - 1$
5	6	7	$m < 2n - 1$
5	7	8	$m < 2n - 1$
5	9	9	$m = 2n - 1$
5	10	8	$m < 2n - 1$
5	12	8	$m < 2n - 1$

Table (1)

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Example2.35. Let \mathcal{M} be local minimum families of disjoint sets meet each other at w intersections each of degree n , where $n = 6, w = 8$ we can't find \mathcal{M} such that $m(w-1) \leq nw, m = 6$, it is not true.

Example2.36. Let \mathcal{M} be local minimum families of disjoint sets meet each other at W intersections each of degree n , where $n = 5, w = 9$ we can't find \mathcal{M} such that $m(w-1) \leq nw, m = 5$, it is not true.

Definition2.37. Let $X_1, X_2, X_3, \dots, X_w$ be standard of meeting for \mathcal{M} families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ with degree of intersection n , and let $Y_1, Y_2, Y_3, \dots, Y_w$ be standard of meeting for the same \mathcal{M} families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$, with degree of intersection n , then we say standard of meeting $X_1, X_2, X_3, \dots, X_w$ is congruent to standard of meeting $Y_1, Y_2, Y_3, \dots, Y_w$. This means two standards of meeting are congruent if they satisfy three conditions: They are both intersections of the same \mathcal{M} families of disjoint sets, they have same number of intersections with same degree of intersection, but appearance of families of disjoint sets can be different at the two standards of meeting.

Example2.38. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_6$ be six families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be five intersections each with degree of intersection four, $X_1 = (1,2,3,4), X_2 = (1,2,3,5), X_3 = (1,3,4,6), X_4 = (1,4,5,6), X_5 = (1,2,5,6)$. These five intersections X_1, X_2, X_3, X_4, X_5 is standard of meeting for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_6$. Let Y_1, Y_2, Y_3, Y_4, Y_5 be five intersections each with degree of intersection four, $Y_1 = (1,2,3,4), Y_2 = (1,2,6,5), Y_3 = (1,3,4,6), Y_4 = (2,3,4,5), Y_5 = (1,2,5,6)$. These five intersections Y_1, Y_2, Y_3, Y_4, Y_5 is standard of meeting for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_6$. These five intersections X_1, X_2, X_3, X_4, X_5 is congruent to the five intersections Y_1, Y_2, Y_3, Y_4, Y_5 .

Remark2.39 Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets and $X_1, X_2, X_3, \dots, X_w$ be W nonempty intersections (each of degree n) standard of meeting for \mathcal{M} families of disjoint sets. The followings allowed:

- (1) Changes of standard of meeting such that standard of meeting before the change congruent to standard of meeting after the change.
- (2) Same intersection with two different symbols e.g. for $X_1 = (1,2,3,4) = (2,3,1,4)$. The followings not allowed:
 - (1) There is intersection of degree less than n .
 - (2) W intersections (standard of meeting for \mathcal{M} families of disjoint sets), where $w = 1, w = 2$ or $\frac{m}{n} < w$.
 - (3) For W intersections (standard of meeting for \mathcal{M} families of disjoint sets), two intersections have same representation (repeated), e.g. $X_1 = (1,2,3,5)$, and $X_2 = (1,2,3,5)$.
 - (4) Changes of standard of meeting such that standard of meeting before the change not congruent to standard of meeting after the change.

Example2.40. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be 5 nonempty intersections with maximum degree of intersection 3, $X_1 = (1,2,3), X_2 = (1,2,4), X_3 = (3,1,2), X_4 = (2,3,4), X_5 = (1,3,4)$. Intersections X_1 and X_3 have the same representation (repeated) intersection, (X_1 is intersection of families of disjoint sets χ_1, χ_2, χ_3 and X_3 is intersection of families of disjoint sets χ_1, χ_2, χ_3). Intersections have the same representation (repeated) are not allowed.

Example2.41. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be 5 nonempty intersections with maximum degree of intersection 3, $X_1 = (1,2,3), X_2 = (1,2,4), X_3 = (1,3,4), X_4 = (2,3,4), X_5 = (, ,)$ $X_5 = (, ,)$ is intersection of degree zero (empty intersection), and this is not allowed. Here $m = 4, n = 3, w = 5$ and $\frac{m}{n} = \frac{4}{3} < w$.

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From definition of combination, any intersections greater than four will be have the same representation (repeated) intersections or intersections of degree zero, and this is not allowed.

Remark2.42. Let $X_1, X_2, X_3, \dots, X_w$ be standard of meeting for m families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ with degree of intersection n , it is trivial case $w = 1$. When $w = 1$ we have $m = n$. Also standard of meeting consist of X_1 and X_2 where $X_1 = (1, 2, 3, \dots, n-1, n)$ and $X_2 = (1, 2, 3, \dots, n-2, n-1, n+1)$, X_1, X_2 is nottrivial. For $w = 2$, if standard of meeting for $n+1$ families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n, \chi_{n+1}$, because X_1 does not meet X_2 . If X_1 is standard of meeting for n families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n$, also X_1, X_2 is standard of meeting for n families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n$. Since $X_1 = X_2$, not allowed, two intersections have same representation (repeated), then standard of meeting X_1, X_2 means standard of meeting X_1 , and standard of meeting X_1, X_2 is trivial.

Trivial case when standard of meeting W equal one, we have two trivial cases, first degree of intersection n equal one and second $n = m$ where m is number of families of disjoint sets. When $n = 1$ this leads to $w = 1$, and when $n = m$ this also leads to $w = 1$. Let standard of meeting W equal two, X_1, X_2 each of degree n , for families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_n, \chi_{n+1}$. If $X_1 = (1, 2, 3, \dots, n-1, n)$ and $X_2 = (1, 2, 3, \dots, n-1, n+1)$, family of disjoint sets χ_n and family of disjoint sets χ_{n+1} not meet each other at X_1, X_2 , $w = 2$ is trivial because $X_1 = X_2$, and we can't find $w = 2$. (see Remark2.42).

Trivial case when standard of meeting W equal one, this means all intersections are same, we have one intersection. If standard of meeting consists of sets it can't be one set or two sets, if standard of meeting consists of time units it can't be one unit or two units, and so on for any type of units.

II. MAIN RESULTS

Here we establish theorems and corollaries explain properties of family of disjoint sets.

Theorem3.1. Let W intersections each with degree of intersection equal n , m number of local maximum families of disjoint sets, meet each other at W intersections, then we have $m(\frac{w+1}{2}) \leq wn$ for W odd, and $m(\frac{w}{2} + 1) \leq wn$ for W even.

Proof: Let t_i be number of times the family χ_i appear at these W intersections (W is odd), then from Proposition2.11.

we have $\sum_{i=1}^m t_i = wn$. Since every two families of disjoint sets meet each other at W intersections, for each i and odd

number W , we have $\frac{w+1}{2} \leq t_i$, then $m(\frac{w+1}{2}) \leq \sum_{i=1}^m t_i = wn$. For each i and W be even, we have $\frac{w}{2} + 1 \leq t_i$, then

$$m(\frac{w}{2} + 1) \leq \sum_{i=1}^m t_i = wn.$$

Corollary 3.2. Let W intersections with degree of intersection equal n , be standard of meeting for m families of disjoint sets. Then we have $m \leq 2n - 1$.

Proof: Using Theorem3.1 we have

$$m(\frac{w+1}{2}) \leq wn \text{ for } W \text{ odd, and } m(\frac{w}{2} + 1) \leq wn \text{ for } W \text{ even.}$$

Suppose $m > 2n + s$ where $s = 1, 2, 3, \dots$ then we have

$$2n + ws + s < 0 \text{ for } W \text{ odd, and } 4n + sw + 2s < 0 \text{ for } W \text{ even.}$$

This contradiction since w, n, s are positive integers. Therefore $m \leq 2n - 1$.

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Corollary3.3. In Theorem3.1 minimum value of W when $m = 2n - 1$ is equal $2n - 1$ when W is odd and equal $2(2n - 1)$ when W even.

Proof: First we want to prove it for W odd. Suppose $w = 2n - 1$ from Theorem3.1. it is true if we substitute in $m(\frac{w+1}{2}) \leq wn$. Suppose $w < 2n - 1$, let $w = 2n - 1 - k$, where $k = 2, 4, 6, \dots$, if we substitute in $m(\frac{w+1}{2}) \leq wn$, we have $(2n - 1)(\frac{2n - k}{2}) \leq n(2n - 1 - k)$ then we have $k \leq 0$ a contradiction. Therefore $w = 2n - 1$.

Second we want to prove it for W even. Suppose $w = 2(2n - 1)$ from Theorem3.1. it is true if we substitute in $m(\frac{w}{2} + 1) \leq wn$. Suppose $w < 2(2n - 1)$, let $w = 2(2n - 1) - k$, where $k = 2, 4, 6, \dots$, if we substitute in $m(\frac{w}{2} + 1) \leq wn$, we have $(2n - 1)(\frac{2(2n - 1) - k}{2} + 1) \leq n[2(2n - 1) - k]$ then we have $k \leq 0$ a contradiction. Therefore $w = 2(2n - 1)$.

Remark 3.4. From Corollary3.3. we have two value of W when $m = 2n - 1$, first $w = 2n - 1$ when W is odd and second $w = 2(2n - 1)$ when W even. From Definition2.16. standard of meeting W is minimum number of nonempty intersections, such that any two families of disjoint sets meet each other at these W intersections, therefore standard of meeting is $w = 2n - 1$ not $w = 2(2n - 1)$.

Example3.5. The following tables explain Corollary3.3. In Table(2) W odd and $n = 5$.

n	w	m
5	9	9
5	11	9
5	13	9
5	7	8
5	5	8
5	3	7

Table(2)

Corollary3.6. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets (local maximum) meet at $X_1, X_2, X_3, \dots, X_w$, nonempty intersections each of degree n . For any two intersections X_i, X_j such that $i \neq j, 1 \leq i \leq w, 1 \leq j \leq w$, then there is family of disjoint sets χ_i such that $X_i \in \chi_i$ and $X_j \in \chi_i$.

Proof: Suppose there is no any family of disjoint sets χ_i such that X_i and X_j both represented by χ_i , this is means X_i represented by $\chi_1, \chi_2, \chi_3, \dots, \chi_n$ and X_j represented by $\chi_{n+1}, \chi_{n+2}, \chi_{n+3}, \dots, \chi_{2n}$, then we have $m \geq 2n$, families which is a contradiction to Corollary3.2. Therefore there is family of disjoint sets χ_i such that X_i and X_j both represented by χ_i .

Example3.7. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ be 6 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1, 2, 3, 4)$, $X_2 = (1, 2, 5, 6)$, $X_3 = (3, 4, 5, 6)$ any two of these six families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , X_1 and X_2 both represented by 1 i.e. $X_1 \in \chi_1$ and $X_2 \in \chi_1$. Also X_1 and X_2 both represented by 2 i.e. $X_1 \in \chi_2$ and $X_2 \in \chi_2$. The two intersections X_2 and X_3 both represented by 5 i.e. $X_2 \in \chi_5$ and $X_3 \in \chi_5$.

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Theorem 3.8. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets any two meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , then m families of disjoint sets be local maximum families of disjoint sets at W intersections if and only if for $m+1$ families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_m, \chi_{m+1}$ at the W intersections there exist a family of disjoint sets not meet any one of other families of disjoint sets.

Proof: First suppose $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets any two meet each other at $X_1, X_2, X_3, \dots, X_w$, where W are nonempty intersections each of degree n , and m families of disjoint sets be local maximum families of disjoint sets at W intersections then from definition of local maximum families of disjoint sets at W intersections any family of disjoint sets has appearance either equal $\frac{w+1}{2}$ or $\frac{w+3}{2}$ for W odd and appearance either equal $\frac{w}{2}+1$ or $\frac{w}{2}+2$ for W even, such that not all families of disjoint sets has appearance equal $\frac{w+3}{2}$ for W odd and not all families of disjoint sets has appearance equal $\frac{w}{2}+2$ for W even. We can write $nw = m(\frac{w+1}{2}) + r$ where $\frac{w-1}{2} \leq r \leq 0$ for W odd, for $m+1$ families of disjoint sets there is m families of disjoint sets have appearance equal $\frac{w+1}{2}$ and one family of disjoint sets have appearance equal r , this family of disjoint sets have appearance equal r dose not meet any one has appearance equal $\frac{w+1}{2}$, since $\frac{w+1}{2} + \frac{w-1}{2} = w < w+1$. We can also write $nw = m(\frac{w}{2}+1) + r$ where $\frac{w}{2}-1 \leq r \leq 0$ for W even, for $m+1$ families of disjoint sets there is m families of disjoint sets have appearance equal $\frac{w}{2}+1$ and one family of disjoint sets have appearance equal r , since $\frac{w}{2}+1 + \frac{w}{2}-1 = w < w+1$. Therefore exists a family of disjoint sets not meet any one of other families of disjoint sets at this W intersections.

Conversely suppose $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets any two meet each other at $X_1, X_2, X_3, \dots, X_w$ and for $m+1$ families of disjoint sets there exists a family of disjoint sets not meet any one of other families of disjoint sets at this W intersections. Suppose W is odd, since m families of disjoint sets any two meet each other at W intersections and for $m+1$ families of disjoint sets exist there exists a family of disjoint sets not meet any one of other families of disjoint sets at this W intersections. We have m families of disjoint sets have appearance equal $\frac{w+1}{2}$ and one family of disjoint sets have appearance equal r , where $\frac{w-1}{2} \geq r \geq 0$, if there is a family of disjoint sets has appearance greater than $\frac{w+1}{2}$ using exchange between families of disjoint sets value of r can be equal or greater than $\frac{w+1}{2}$ and this contradict for $m+1$ families of disjoint sets there exists a family of disjoint sets not meet any one of other families of disjoint sets at these W intersections. We have m families of disjoint sets all have appearance equal $\frac{w+1}{2}$ if $r = 0$ or $r \neq 0$, therefore m families of disjoint sets is the local maximum families of disjoint sets at W intersections, for W odd. The proof is same for W even.

Corollary 3.9. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets any two meet each other at $X_1, X_2, X_3, \dots, X_w$ intersections each of degree n , and m families of disjoint sets be local maximum families of disjoint sets at W . Let

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$m = a + b$ such that $a \frac{w+3}{2} + b \frac{(w+1)}{2} = nw$, where a, b , are integers and $m \frac{w+1}{2} = nw$. Then

$$m = a + b \leq m' < m + 1.$$

Proof: From definitions of m and m' we have $m \leq m'$. From Theorem 3.8 for m families of disjoint sets be local maximum families of disjoint sets at W intersections and for $m+1$ families of disjoint set there exists a family of disjoint sets not meet any one of other families of disjoint sets at these W intersections, we have $m \frac{w+1}{2} \leq m' \frac{w+1}{2} = nw < (m+1) \frac{(w+1)}{2}$, then $a + b = m \leq m' < m + 1$.

Example 3.10. The following table, Table (3) explain Corollary 3.9.

n	W	$m = a + b$	$nw = a \left(\frac{w+3}{2} \right) + b \left(\frac{w+1}{2} \right)$	$m' = \frac{2nw}{w+1}$	$m+1$
3	3	4=1+3	9=3(3)=1(3)+3(2)	4.5	5
3	5	5=0+5	15=3(5)=0(4)+5(3)	5	6
5	3	7=1+6	15=1(3)+6(2)	7.5	8
5	5	8=1+7	25=5(5)=1(4)+7(3)	8.33	9
7	7	12=1+11	49=7(7)=1(5)+11(4)	12.25	13
7	9	12=3+9	63=7(9)=3(6)+9(5)	12.6	13
9	11	16=3+13	99=3(7)+13(6)	16.5	17
9	17	17=0+17	153=0(10)+17(9)	17	18

Table (3)

Theorem 3.11. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets at $X_1, X_2, X_3, \dots, X_w$, nonempty intersections each of degree n . If $m \left(\frac{w+1}{2} \right) > wn$ for W odd, and $m \left(\frac{w}{2} + 1 \right) > wn$ for W even, then there exists two families of disjoint sets not meet each other at this W intersections.

Proof: Suppose $m_1 + m_2 = m$, such that $m_1 \left(\frac{w+1}{2} \right) \leq wn$ for W odd, where m_1 local maximum families of disjoint sets meet each other at these W intersections and $m_2 \geq 1$. We have $m_1 \left(\frac{w+1}{2} \right) + m_2 \left(\frac{w+1}{2} \right) = m \left(\frac{w+1}{2} \right) > wn$ for W odd and

$m_1 \left(\frac{w}{2} + 1 \right) + m_2 \left(\frac{w}{2} + 1 \right) = m \left(\frac{w}{2} + 1 \right) > wn$ for W even. Using Theorem 3.8, m_1 local maximum families of disjoint sets meet each other at this W intersections and for m_2 families of disjoint sets there is at least one family of disjoint sets not meet any one of m_1 families of disjoint sets at this W intersections.

Lemma 3.12. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be local maximum m families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_w$, nonempty intersections with maximum degree of intersection n . If $m \left(\frac{w+1}{2} \right) \leq wn$ then for all W intersection of degree n , if

$wn - m \left(\frac{w+1}{2} \right) = \{1, 2, 3, \dots, n-1\}$ then there is intersection of degree less than n , and if

$wn - m \left(\frac{w+1}{2} \right) = \{n, n+1, n+2, \dots\}$ then there is an intersection of degree zero.

Proof: Proof direct by using Definition 2.9, Definition 2.12, and Proposition 2.11.

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Example3.13. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ be 6 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,5,6)$, $X_3 = (3,4,5,6)$ any two of these six families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , we have $n = 4$, $w = 3$, $m = 6$ and $m(\frac{w+1}{2}) = wn$ then all 3 intersections of degree 4.

Example3.14. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections with maximum degree 3. $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (3,4)$ any two of these four families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , we have $n = 3$, $w = 3$, $m = 4$ and $wn - m(\frac{w+1}{2}) = 1$ then there is intersection of degree less than n . This is the case $wn - m(\frac{w}{2} + 1) = \{1,2,3,\dots, n-1\}$ then there is an intersection of degree less than n .

Example3.15. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets and X_1, X_2, X_3, X_4, X_5 be 5 nonempty intersections with maximum degree of intersection 3, $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (3,4,1)$, $X_4 = (2,4,3)$, $X_5 = (.,.)$. any two of these four families of disjoint sets meet each other at the 5 intersections X_1, X_2, X_3, X_4, X_5 , we have $n = 3$, $w = 5$, $m = 4$ and $wn - m(\frac{w+1}{2}) = 3$ then there is intersection of degree zero. This is the case $wn - m(\frac{w}{2} + 1) = \{n, n+1, n+2, \dots\}$ then there is an intersection of degree zero.

Lemma3.16. If $b-a = 0,1,2,3,\dots, n-1$ and $c-a = n, n+1, n+2, \dots$ then $a \leq b, b < c$.

Proof: If $b-a = 0$, $b = a$, and if $b-a$ equal any one of the set $\{1,2,3,\dots, n-1\}$, then $b < a$, therefore $b \leq a$. Since $c-b = (c-a) - (b-a)$ and $(c-a) - (b-a)$ equal any one of the set $\{1,2,3,\dots, n, n+1, n+2, \dots\}$, therefore $c < b$.

Corollary 3.17. Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be any m families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_w$, nonempty intersections each of degree n . Let $k = 2,4,6, \dots$ then $m(\frac{w+1}{2}) \leq wn < m(\frac{w+1+k}{2})$ for W odd, and $m(\frac{w}{2} + 1) \leq wn < m(\frac{w+k}{2} + 1)$ for W even.

Proof: Let $m\frac{w+1}{2} = a$, $wn = b$ and $m(\frac{w+1+k}{2}) = c$. From Lemma3.4. we have $b-a = 0,1,2,3,\dots, n-1$, if $b-a = \{n, n+1, n+2, \dots\}$ then there is an intersection of degree zero, so we will take $b-a = \{0,1,2,3,\dots, n-1\}$. Since $c-a = \{n, n+1, n+2, \dots\}$, using Lemma3.9. we have $a \leq b, b < c$, therefore $m(\frac{w+1}{2}) \leq wn < m(\frac{w+1+k}{2})$ for W odd. The

proof will be the same for $m(\frac{w}{2} + 1) \leq wn < m(\frac{w+k}{2} + 1)$ where W even.

Example3.18. Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ be 6 families of disjoint sets and X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,5,6)$, $X_3 = (3,4,5,6)$ any two of these six families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , $n = 4$, $w = 3$, $m = 6$ and let $k = 2$ (minimum value of k). This satisfies $m(\frac{w+1}{2}) \leq wn < m(\frac{w+1+k}{2})$ for W odd. We have $6(\frac{3+1}{2}) \leq 3(4) < 6(\frac{3+1+2}{2})$.

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Let $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ be 5 families of disjoint sets and X_1, X_2, X_3, X_4 be 4 nonempty intersections each of degree 4. $X_1 = (1,2,3,4)$, $X_2 = (1,2,3,5)$, $X_3 = (1,3,4,5)$, $X_4 = (2,4,5)$ any two of these five families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3, X_4 , $n = 4$, $w = 4$, $m = 5$ and let $k = 2$ (minimum value of k). This satisfies $m(\frac{w}{2} + 1) \leq wn < m(\frac{w+k}{2} + 1)$ for W even. We have $5(\frac{4}{2} + 1) \leq 4(4) < 5(\frac{4+2}{2} + 1)$.

Remark3.19. From division of a by m we have $a = mb + r$, $0 \leq r \leq m - 1$. From division by 5 we can write:

$$15 = 3(5) + 0 = 3 + 3 + 3 + 3 + 3$$

$$16 = 3(5) + 1 = 3 + 3 + 3 + 3 + 4$$

$$17 = 3(5) + 2 = 3 + 3 + 3 + 4 + 4$$

$$18 = 3(5) + 3 = 3 + 3 + 4 + 4 + 4$$

$$19 = 3(5) + 4 = 3 + 4 + 4 + 4 + 4$$

Generally we can write

$$a - r = (b + 0) + (b + 0) + (b + 0) + \dots + (b + 0) \quad b \text{ repeted } m \text{ time}$$

$$a - r + 1 = (b + 1) + (b + 0) + (b + 0) + \dots + (b + 0)$$

$$a - r + 2 = (b + 1) + (b + 1) + (b + 0) + \dots + (b + 0)$$

$$a - r + 3 = (b + 1) + (b + 1) + (b + 1) + (b + 0) + \dots + (b + 0)$$

$$\begin{matrix} \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \end{matrix}$$

$$a = (b + 1) + \dots + (b + 1) + (b + 0) + (b + 0) + (b + 0) + \dots + (b + 0). \quad 1 \text{ repeted } r \text{ time}$$

Remark3.20. Since families of disjoint sets meet at W intersections each of degree n , and number of families of disjoint will be local maximum if we divide nW by smallest number of appearance, such that any two families of disjoint sets meet each other at W intersections, which equal $\frac{w+1}{2}$ when W is odd or equal $\frac{w}{2} + 1$ when W is even.

Let W is the number of nonempty intersections each of degree n , and m is number of local maximum families of disjoint sets meet at intersections $X_1, X_2, X_3, \dots, X_w$, from Definition2.20. the appearance of any family of disjoint sets equal $\frac{w+1}{2}$ or equal $\frac{w+1}{2} + 1$ when W is odd and for W even we have three types, first all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, second appearance equal $\frac{w}{2} + 1$ and $\frac{w}{2} + 2$, not all $\frac{w}{2} + 2$ but at least has appearance equal $\frac{w}{2} + 2$, third all families of disjoint sets has appearance equal $\frac{w}{2} + 1$, except only one family of disjoint sets has appearance equal $\frac{w}{2}$. Table (4) explain Definition2.20 where m is local maximum families of disjoint sets meet each other at w intersections and W is odd.

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n	w	$m = a + b$	$nw = a\left(\frac{w+3}{2}\right) + b\left(\frac{w+1}{2}\right)$
3	3	4=1+3	9=3(3)=1(3)+3(2)
3	5	5=0+5	15=3(5)=0(4)+5(3)
5	3	7=1+6	15=1(3)+6(2)
5	5	8=1+7	25=5(5)=1(4)+7(3)
7	7	12=1+11	49=7(7)=5(5)+6(4)
7	9	12=3+9	63=7(9)=3(6)+9(5)
9	11	16=3+13	99=9(7)+6(6)
9	17	17=0+17	153=0(10)+17(9)

Table (4)

Table (5) explain Definition 2.20 where m is local maximum families of disjoint sets meet each other at w intersections and W is even.

n	w	$m = a_1 + a_2 + a_3$	$nw = a_1\left(\frac{w}{2} + 2\right) + a_2\left(\frac{w}{2} + 1\right) + a_3\left(\frac{w}{2}\right)$
3	4	4=0+4+0	3(4)=12=0(4)+4(3)+0(2)
4	4	5=1+4+0	4(4)=16=1(4)+4(3)+0(2)
5	6	7=2+5+0	5(6)=30=2(5)+5(4)+0(3)
3	8	5=0+4+1	3(8)=24=0(6)+4(5)+1(4)
7	8	11=1+10+0	7(8)=56=1(6)+10(5)+0(4)
7	10	11=4+7+0	7(10)=70=4(7)+7(6)+0(5)
9	10	15=0+15+0	9(10)=90=0(7)+15(6)+0(5)
7	12	12=0+12+0	7(12)=84=0(8)+12(7)+0(6)

Table (5)

Table (6) explain Definition 2.26 where m is local minimum families of disjoint sets meet each other at W intersections and W is odd.

n	w	$m = a_1 + a_2$	$nw = a_1w + a_2(w-1)$
3	3	4=1+3	9=3(3)=1(3)+3(2)
3	5	m not exists	15=3(5)=0(4)+5(3)
5	3	7=1+6	15=5(3)=1(3)+6(2)
5	5	6=1+5	25=5(5)=1(5)+5(4)
7	7	8=1+7	49=7(7)=1(7)+7(6)
7	9	m not exists	
9	11	m not exists	
9	17	m not exists	

Table (6)

Table (7) explain Definition 2.26 where m is local minimum families of disjoint sets meet each other at W intersections and W is even.

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n	W	$m = a_1 + a_2$	$nw = a_1w + a_2(w-1)$
3	4	$4 = 0 + 4$	$12 = 3(4) = 0(4) + 4(3)$
4	4	$5 = 1 + 4$	$16 = 4(4) = 1(4) + 4(3)$
5	6	$6 = 0 + 6$	$30 = 5(6) = 0(6) + 6(5)$
3	8	m not exists	
7	8	$8 = 0 + 8$	$56 = 7(8) = 0(8) + 8(7)$
7	10	m not exists	
9	10	$10 = 0 + 10$	$90 = 9(10) = 0(10) + 10(9)$
7	12	m not exists	

Table (7)

Theorem 3.21. Let m families of disjoint sets meet each other at $X_1, X_2, X_3, \dots, X_w$, w intersections each of degree n , and let $m = n + k$ is where $k = 1, 2, 3, \dots, n - 1$, then m is families of disjoint sets meet each other at w intersections then m families of disjoint sets be local minimum families of disjoint sets at W intersections if and only if $k(w - 1) \leq nw$.

Proof: First suppose m families of disjoint sets be local minimum families of disjoint sets at w intersections. From Theorem 3.1. and definition of local minimum families of disjoint sets meet each other at w intersections we have $m(w - 1) = (n + k)(w - 1) \leq nw$,

then $nw + kw - n - k \leq nw$, and we have $k(w - 1) \leq n$.

Conversely suppose m families of disjoint sets meet each other at w intersections. Since we have $k(w - 1) \leq n$, we can write $nw + kw - k \leq nw + n$, and $(n + k)(w - 1) \leq nw$, we can write this as $m(w - 1) = (n + k)(w - 1) \leq nw$, therefore m families of disjoint sets be local minimum families of disjoint sets at w intersections.

Corollary 3.22. Let W be number of nonempty intersections each of degree n , and m is number of local maximum families of disjoint sets meet at intersections $X_1, X_2, X_3, \dots, X_w$, if a is the number of families of disjoint sets each its appearance equal

$\frac{w+1}{2} + 1$ where W is odd then we have $a(\frac{w+3}{2}) + (m-a)(\frac{w+1}{2}) = nw$. If C is the number of families of disjoint sets each its

appearance equal $(\frac{w}{2} + 2)$ where W is even then we have $c(\frac{w}{2} + 2) + (m-c)(\frac{w}{2} + 1) = nw$ where W is even integer.

Proof: Let $\chi_1, \chi_2, \chi_3, \dots, \chi_m$ be m families of disjoint sets, $X_1, X_2, X_3, \dots, X_w$ are nonempty intersections each of degree n . Using

Remark 3.20. we have appearance of families of disjoint equal $\frac{w+1}{2}$ or equal $\frac{w+1}{2} + 1$ when W is odd and the appearance of

families of disjoint equal $\frac{w}{2} + 1$ or equal $\frac{w}{2} + 2$ when W is even. Let t_i be number of time the family of disjoint sets χ_i appear at

these W intersections, using Proposition 2.11. we have $\sum_{i=1}^m t_i = wn$. Therefore if a is the number of families of disjoint sets each its

appearance equal $\frac{w+1}{2} + 1$ where W is odd then we have $a(\frac{w+3}{2}) + (m-a)(\frac{w+1}{2}) = nw$. If b is the number of families of

disjoint sets each its appearance equal $(\frac{w}{2} + 1) + 1$ where W is even then we have $b(\frac{w+4}{2}) + (m-b)(\frac{w}{2} + 1) = nw$ where W is

even integer.

Theorem 3.23. Let W be odd integer, m_i is local maximum families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_w$, (nonempty intersections each of degree n), such that

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$$m_1\left(\frac{w+1}{2}\right) \leq nw \tag{3.1} \text{ and } m_2 \text{ is local}$$

maximum families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_w, X_{w+1}$ (nonempty intersections each of degree n), such that

$$m_2\left(\frac{w+1}{2} + 1\right) \leq n(w+1) \tag{3.2}$$

then $m_2 \leq m_1$.

Proof: Suppose $m_1 < m_2$ let $m_2 = m_1 + k$ where $k = 1, 2, 3, \dots$. We can write (3.2) as

$$(m_1 + k)\left(\frac{w+1}{2} + 1\right) \leq n(w+1) \tag{3.3}$$

We can write (3.3) as

$$m_1\left(\frac{w+1}{2}\right) + m_1 + k\left(\frac{w+1}{2}\right) + k \leq nw + n \tag{3.4}$$

Using Corollary 3.17. we can write (3.1) as

$$m_1\left(\frac{w+1}{2}\right) \leq nw < m_1\left(\frac{w+1}{2}\right) + \frac{w+1}{2} \tag{3.5}$$

From (3.4) and (3.5) we can write

$$m_1\left(\frac{w+1}{2}\right) + m_1 + k\left(\frac{w+1}{2}\right) + k \leq nw + n < m_1\left(\frac{w+1}{2}\right) + \frac{w+1}{2} + n \tag{3.6}$$

From (3.6) we have

$$m_1 + k\left(\frac{w+1}{2}\right) + k < \frac{w+1}{2} + n \tag{3.7}$$

In (3.7) since $n < m_1$, and $\frac{w+1}{2} \leq k\left(\frac{w+1}{2}\right)$ where $k = 1, 2, 3, \dots$ therefore $m_1 < m_2$ is a contradiction. Hence $m_2 \leq m_1$.

Example 3.24. The below table, Table (8) explain Theorem 3.23.

n	w	$m_1\left(\frac{w+1}{2}\right) \leq nw$	$m_2\left(\frac{w+1}{2} + 1\right) \leq n(w+1)$
3	3	$m_1 = 4$	$m_2 = 4$
3	5	$m_1 = 5$	$m_2 = 4$
4	3	$m_1 = 6$	$m_2 = 5$
4	5	$m_1 = 6$	$m_2 = 6$
4	7	$m_1 = 7$	$m_2 = 6$
5	3	$m_1 = 7$	$m_2 = 6$
5	5	$m_1 = 8$	$m_2 = 7$
5	7	$m_1 = 8$	$m_2 = 8$
7	3	$m_1 = 10$	$m_2 = 9$
7	5	$m_1 = 11$	$m_2 = 10$

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7	7	$m_1 = 12$	$m_2 = 11$
7	9	$m_1 = 12$	$m_2 = 11$
7	11	$m_1 = 12$	$m_2 = 12$
7	13	$m_1 = 13$	$m_2 = 12$

Table (8)

Theorem 3.25. Let $w_1 < w_2$, m_1 is local minimum families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_{w_1}$, (nonempty intersections each of degree n), such that m_2 is local minimum families of disjoint sets meet at $X_1, X_2, X_3, \dots, X_{w_2}$, (nonempty intersections each of degree n), $m_1 = \lceil \frac{nw_1}{w_1 - 1} \rceil$ and $m_2 = \lceil \frac{nw_2}{w_2 - 1} \rceil$, where $\lceil x \rceil$ is integral part of x , such that

$$m_1(w_1 - 1) \leq w_1 n \tag{3.8}$$

and

$$m_2(w_2 - 1) \leq n w_2 \tag{3.9}$$

then $m_2 \leq m_1$.

Proof: Let $m_1 = a_1 + b_1$ such that $a_1 w_1 + b_1(w_1 - 1) = n w_1$, and $m_2 = a_2 + b_2$ such that $a_2 w_2 + b_2(w_2 - 1) = n w_2$

where a_1, b_1, a_2, b_2 are integers. Let $m_1 = a_1 + b_1 \leq m_1^* < m_1 + 1$ where $m_1^*(w_1 - 1) = n w_1$ and let $m_2 = a_2 + b_2 \leq m_2^* < m_2 + 1$ where $m_2^*(w_2 - 1) = n w_2$. From (3.8) we have

$$m_1^* = n \left(\frac{w_1}{w_1 - 1} \right) = n \left(1 + \frac{1}{w_1 - 1} \right) \tag{3.10}$$

and from (3.10) we have

$$m_2^* = n \left(\frac{w_2}{w_2 - 1} \right) = n \left(1 + \frac{1}{w_2 - 1} \right) \tag{3.11}$$

Since $w_1 < w_2$, from (3.10) and (3.11) we have $m_2^* < m_1^*$ and hence $m_1 \leq m_2$.

Example 3.26. The table below explain Theorem 3.25. In Table (9) n is constant $n = 7$.

w	$m^*(w - 1) = n w$	$m(w - 1) \leq n w$ where m is integer
4	$m^* = \frac{28}{3} = 9.33$	$m = 9$
5	$m^* = \frac{35}{4} = 8.75$	$m = 8$
6	$m^* = \frac{42}{5} = 8.40$	$m = 8$
7	$m^* = \frac{49}{6} = 8.17$	$m = 8$
8	$m^* = \frac{56}{7} = 8.00$	$m = 8$

Table (9)

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The table below Table (10) explain Theorem 3.25. In Table (10) n is constant $n = 8$. Table (10) is the same as Table (9), difference between them the value of n .

w	$m \setminus (w - 1) = nw$	$m(w - 1) \leq nw$ where m is integer
5	$m \setminus = \frac{40}{4} = 10$	$m = 10$
6	$m \setminus = \frac{48}{5} = 9.60$	$m = 9$
7	$m \setminus = \frac{56}{6} = 9.33$	$m = 9$
8	$m \setminus = \frac{64}{7} = 9.14$	$m = 9$
9	$m \setminus = \frac{72}{8} = 9.00$	$m = 9$

Table (10)

Remark 3.27. In equation $a(\frac{w+3}{2}) + (m-a)(\frac{w+1}{2}) = nw$ from Corollary 3.18, if we put $m - a = b$ the equation can be written as

$$a(\frac{w+3}{2}) + b(\frac{w+1}{2}) = nw \tag{3.12}$$

Since $m = a + b$, it is true if $a = 0$ then we have $b = m$ and $b(\frac{w+1}{2}) = nw$, it is not true if $b = 0$, when $b = 0$, then we have $a = m$ and $a(\frac{w+3}{2}) = nw$.

Let W be number of nonempty intersections each of degree n , and m_i is number of local maximum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_i}$ meet at these W intersections. If we add two intersections to these W intersections local maximum number of families of disjoint sets meet at $w + 2$ intersections become m_2 . Let $m_1 = a_1 + b_1$ and $m_2 = a_2 + b_2$ then equation (12) can be written as

$$a_1(\frac{w+3}{2}) + b_1(\frac{w+1}{2}) = nw \tag{3.13}$$

$$a_2(\frac{w+5}{2}) + b_2(\frac{w+3}{2}) = nw + 2n \tag{3.14}$$

From (3.13) and (3.14) we have

$$[a_2 + b_2 - a_1 - b_1](\frac{w+1}{2}) + 2a_2 + b_2 - a_1 = 2n \tag{3.15}$$

In case $m_1 = m_2$ we have $a_1 + b_1 = a_2 + b_2$ and (3.15) can be written as:

$$2a_2 + b_2 - a_1 = 2n \tag{3.16}$$

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Remark3.28. Next we will prove corollary which states: Let W be number of nonempty intersections each of degree n , and m_1 is number of local maximum families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ meet at these W intersections. If we add two intersections to these W intersections, local maximum families of disjoint sets meet at $w + 2$ intersections become m_2 , then $m_1 \leq m_2$. Here we want to explain this by the following table. In table (11) we calculate m_1 as real number $m_1 = \lceil \frac{2nw}{w+1} \rceil$.

n degree of intersections	W standard of meeting	$\frac{2nw}{w+1}$	$\lceil \frac{2nw}{w+1} \rceil$ is local maximum families of disjoint sets
5	3	7.5	7
5	5	8.33	8
5	7	8.75	8
5	9	9	9
7	3	10.5	10
7	5	11.67	11
7	7	12.25	12
7	9	12.6	12
7	11	12.83	12
7	13	13	13

Table (11)

Theorem3.29. Let W be number of nonempty intersections each of degree n , and m_1 is local maximum ‘families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ meet at these W intersections, where W is odd. m_2 is local maximum number of families of disjoint sets at $w + 2$. Let $m_1 = a_1 + b_1$ such that $a_1 \frac{w+3}{2} + b_1 \frac{w+1}{2} = nw$, and $m_2 = a_2 + b_2$ such that

$a_2 \frac{w+5}{2} + b_2 \frac{w+3}{2} = n(w+2)$, where a_1, b_1, a_2, b_2 are integers. If $2n + a_1 < m_1 + \frac{w+3}{2}$, then $m_1 = m_2$, and if $m_1 + \frac{w+3}{2} \leq 2n + a_1$ then $m_1 + 1 = m_2$.

Proof: Since

$$m_1 \left(\frac{w+1}{2} \right) = (a_1 + b_1) \left(\frac{w+1}{2} \right) = nw - a_1, \tag{3.17}$$

we can write

$$(m_1 + 1) \left(\frac{w+1}{2} \right) = nw - a_1 + \frac{w+1}{2} \tag{3.18}$$

since we have

$$n(w+2) = (nw - a_1) + (2n + a_1) \tag{3.19}$$

$$\text{If } m_1 + \frac{w+3}{2} \leq 2n + a_1 \tag{3.20}$$

From (3.17), (3.18), (3.19) and (3.20) we can write

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$$m_2 \left(\frac{w+3}{2} \right) = (m_1 + 1) \left(\frac{w+3}{2} \right) \leq n(w+2) \tag{3.21}$$

$$\text{If } 2n + a_1 \leq m_1 + \frac{w+1}{2} \tag{3.22}$$

From (3.17), (3.18), (3.19) and (3.22) we can write

$$m_1 \left(\frac{w+1}{2} \right) = m_2 \left(\frac{w+1}{2} \right) \leq n(w+2) \tag{3.23}$$

Hence from (3.21) and (3.23) the theorem holds.

Example 3.30. The below table, Table (12) explain Theorem 3.29 for W odd

n	w	$m_1 = a_1 + b_1$ $a_1 \frac{w+3}{2} + b_1 \frac{w+1}{2} = nw$	$m_2 = a_2 + b_2$ $a_2 \frac{w+5}{2} + b_2 \frac{w+3}{2} = n(w+2)$	$m_1 + \frac{w+3}{2}$	$2n + a_1$
3	3	$4 = 1 + 3$ $1(3) + 3(2) = 3(3)$	$5 = 0 + 5$ $0(4) + 5(3) = 3(5)$	$4 + 3$	$6 + 1$
4	3	$6 = 0 + 6$ $0(3) + 6(2) = 4(3)$	$6 = 2 + 4$ $2(4) + 4(3) = 4(5)$	$6 + 3$	$8 + 0$
4	5	$6 = 2 + 4$ $2(4) + 4(3) = 4(5)$	$7 = 0 + 7$ $0(5) + 7(4) = 4(7)$	$6 + 4$	$8 + 2$
5	3	$7 = 1 + 6$ $1(3) + 6(2) = 5(3)$	$8 = 1 + 7$ $1(4) + 7(3) = 5(5)$	$7 + 3$	$10 + 1$
5	7	$8 = 3 + 5$ $3(5) + 5(4) = 5(7)$	$9 = 0 + 9$ $0(6) + 9(5) = 5(9)$	$8 + 5$	$10 + 3$
7	7	$12 = 1 + 11$ $1(5) + 11(4) = 7(7)$	$12 = 3 + 9$ $3(6) + 9(5) = 7(9)$	$12 + 5$	$14 + 1$
7	9	$12 = 3 + 9$ $3(6) + 9(5) = 7(9)$	$12 = 5 + 7$ $5(7) + 7(6) = 7(11)$	$12 + 6$	$14 + 3$
7	11	$12 = 5 + 7$ $5(7) + 7(6) = 7(11)$	$13 = 0 + 13$ $0(8) + 13(7) = 7(13)$	$12 + 7$	$14 + 5$
9	7	$15 = 3 + 12$ $3(5) + 12(4) = 9(7)$	$16 = 1 + 15$ $1(6) + 15(5) = 9(9)$	$15 + 5$	$18 + 3$
9	9	$16 = 1 + 15$ $1(6) + 15(5) = 9(9)$	$16 = 3 + 13$ $3(7) + 13(6) = 9(11)$	$16 + 6$	$18 + 1$

Table (12)

Theorem 3.31. Let W be number of nonempty intersections each of degree n , and m_1 is local maximum 'families of disjoint sets $\chi_1, \chi_2, \chi_3, \dots, \chi_{m_1}$ at these W intersections, where W is even. m_2 is local maximum number of families of disjoint sets at $w + 2$. Let $m_1 = a_1 + b_1$ such that $a_1 \left(\frac{w}{2} + 2 \right) + b_1 \left(\frac{w}{2} + 1 \right) = nw$, and $m_2 = a_2 + b_2$ such that

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$a_2(\frac{w}{2} + 3) + b_2(\frac{w}{2} + 2) = n(w + 2)$, where a_1, b_1, a_2, b_2 are integers. If $2n + a_1 < m_1 + (\frac{w}{2} + 2)$, then $m_1 = m_2$, and if

$m_1 + (\frac{w}{2} + 2) \leq 2n + a_1$ then $m_1 + 1 = m_2$.

Proof: Since

$$m_1(\frac{w}{2} + 1) = (a_1 + b_1)(\frac{w}{2} + 1) = nw - a_1, \tag{3.24}$$

we can write

$$(m_1 + 1)(\frac{w}{2} + 1) = nw - a_1 + (\frac{w}{2} + 1) \tag{3.25}$$

since we have

$$n(w + 2) = (nw - a_1) + (2n + a_1) \tag{3.26}$$

If we have

$$m_1 + (\frac{w}{2} + 2) \leq 2n + a_1 \tag{3.27}$$

From (3.24), (3.25) and (3.26) we can write

$$m_2(\frac{w}{2} + 1) = (m_1 + 1)(\frac{w}{2} + 2) \leq n(w + 2) \tag{3.28}$$

If we have

$$2n + a_1 \leq m_1 + (\frac{w}{2} + 2) \tag{3.29}$$

From (3.24), (3.25), (3.26) and (3.29) we can write

$$m_1(\frac{w}{2} + 1) = m_2(\frac{w}{2} + 1) \leq n(w + 2) \tag{3.30}$$

Hence from (3.28) and (3.30) the theorem holds.

Example 3.32. The below table, Table (13) explain Theorem 3.31 for w even

n	w	$m_1 = a_1 + b_1$ $a_1(\frac{w}{2} + 2) + b_1(\frac{w}{2} + 1) = n$	$m_2 = a_2 + b_2$ $a_2(\frac{w}{2} + 3) + b_2(\frac{w}{2} + 2) = n(w + 2)$	$m_1 + (\frac{w}{2} + 3)$	$2n + a_1$
3	4	$4 = 0 + 4$ $0(4) + 4(3) = 3(4)$	$4 = 2 + 2$ $2(5) + 2(4) = 3(6)$	$4 + 4$	$6 + 0$
4	4	$5 = 1 + 4$ $1(4) + 4(3) = 4(4)$	$6 = 0 + 6$ $0(5) + 6(4) = 4(6)$	$4 + 4$	$8 + 1$
4	6	$6 = 0 + 6$ $0(5) + 6(4) = 4(6)$	$6 = 2 + 4$ $2(6) + 4(5) = 4(6)$	$6 + 5$	$8 + 0$
5	4	$6 = 2 + 4$ $2(4) + 4(3) = 5(4)$	$7 = 2 + 5$ $2(5) + 5(4) = 5(6)$	$6 + 4$	$10 + 2$
5	6	$7 = 2 + 5$ $2(5) + 5(4) = 5(6)$	$8 = 0 + 8$ $0(6) + 8(5) = 5(8)$	$7 + 5$	$10 + 2$
5	8	$8 = 0 + 8$	$8 = 2 + 6$	$8 + 6$	$10 + 0$

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		$0(6) + 8(5) = 5(8)$	$2(7) + 6(6) = 5(10)$		
6	4	$8 = 0 + 8$ $0(4) + 8(3) = 6(4)$	$9 = 0 + 9$ $0(5) + 9(4) = 6(6)$	$8 + 4$	$12 + 0$
6	6	$9 = 0 + 9$ $0(5) + 9(4) = 6(6)$	$9 = 3 + 6$ $3(6) + 6(5) = 6(8)$	$9 + 5$	$12 + 0$
6	8	$9 = 3 + 6$ $3(6) + 6(5) = 6(8)$	$10 = 0 + 10$ $0(7) + 10(6) = 6(10)$	$9 + 6$	$12 + 3$
6	10	$10 = 0 + 10$ $0(7) + 10(6) = 6(10)$	$10 = 2 + 8$ $2(8) + 8(7) = 6(12)$	$10 + 7$	$12 + 0$

Table (13)

Remark3.33. From Theorem3.29 and Theorem3.31 if m_1 is number of local maximum families of disjoint sets at W intersections withof degree of intersection n , and m_2 is number of local maximum families of disjoint sets at $w + 2$ intersections with degree of intersection n , then $m_1 \leq m_2$ for all W odd or even.

Remark3.34. If m is number of local minimum families of disjoint sets at W intersections withof degree of intersection n , since $m \neq n$ is ($m = n$ is trivial case) then we have $(n+k)(w-1) \leq nw$ where $k=1,2,3,\dots$ and $k(w-1) \leq n$. Number oflocal minimum families of disjoint sets at W not always equal $n+1$. If $n=8, w=5$, then $m = n + 2 = 10$. If $n=15, w=6$, then $m = n + 3 = 18$.

Remark3.35. From Corollary3.27 minimum families of disjoint sets will be smallest local minimum families of disjoint sets meet each other at smallest standard of meeting for W odd, $w = 3$ and for W even, $w = 4$. By using Theorem3.1 at $w = 3, w - 1 = 2$ we have $2m_1 \leq 3n$, at $w = 4, w - 1 = 3$ we have $3m_2 \leq 4n$, then we have $m_2 \leq m_1$.

Corollary3.36. For smallest odd standard of meeting and smallest even standard of meeting local minimum families of disjoint sets m equal $m \leq \frac{2n}{3}, m \leq \frac{3n}{4}$ respectively and local maximum families of disjoint sets meet each other at

smallest standard of meeting equal $m \leq \frac{2n}{3}, m \leq \frac{3n}{4}$ respectively.

Proof: Smallest odd standard of meeting $w = 3$ and smallest even standard of meeting $w = 4$. From definition of local minimum families of disjoint sets for $w = 3$ we have $m(w-1) = m(3-1) \leq 3n$ and for $w = 4$ we have

$m(w-1) = m(4-1) \leq 4n$. From definition of local maximum families of disjoint sets for $w = 3$ we have

$m(\frac{w+1}{2}) = 2m \leq 3n$ and for $w = 4$ we have $m(\frac{w}{2} + 1) = 3m \leq 4n$.

Remark3.37. For each degree of intersection n we have $2n - 3$ cases of local maximum and have $2n - 3$ cases of local minimum. The values of standards of meeting W , for each degree of intersection n is $w = 3, 4, 5, \dots, 2n - 1$.

Remark3.38. For w intersection each degree of intersection equal n , if we determine the appearance of the family of disjoint sets, we can find the number m of family of disjoint sets which satisfies appearance which we want. If we select any appearance $w - k$, where $1 \leq k \leq \frac{w-1}{2}$ for w odd and $1 \leq k \leq \frac{w}{2} - 1$ for w even, it is easy to find the number m of family of disjoint sets. The following table explain this.

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$n = 10$ even

w	Selected appearance of families	Number of families
15	14	11
15	13	11
15	12	12
15	11	13
15	10	15
15	9	16
15	8	18
16	15	11
16	14	11
16	13	12
16	12	13
16	11	14
16	10	16
16	9	17

Table (14)

III. APPLICATIONS

In this section we introduce examples satisfying main results in this paper, in details we introduce Four Colour Problem as an example.

The Four Colour Problem dates back to 1852 when Francis Guthrie, while trying to colour the map of the counties of England, noticed that four colours succeed. He asked his brother Frederick if it was true that any map can be coloured using four colours in such a way that adjacent regions (i.e., those sharing a common boundary segment, not just a point) receive different colours. Frederick Guthrie then communicated the conjecture to De Morgan. The first printed reference is by Cayley in 1878 [14].

A year later the first proof by Kempe appeared; its incorrectness was pointed out by Heawood eleven years later. Another failed proof was published by Tait in 1880; a gap in the argument was pointed out by Petersen in 1891. Both failed proofs did have some value, though. Kempe proved the five-colour theorem as: (Every plane graph has a 5-coloring), and discovered what became known as Kempe chains, and Tait found an equivalent formulation of the Four Colour Theorem in terms of edge 3-coloring, stated here as: (Every cubic plane graph with no cut-edge has an edge 3-coloring)[14].

The next major contribution came from G. D. Birkhoff in 1913, whose work allowed Franklin to prove in 1922 that the four-colour conjecture is true for maps with at most 25 regions. The same method was used by other mathematicians to make progress on the four colour problem. Important here is the work by Heesch who developed the two main ingredients needed for the ultimate proof (reducibility) and (discharging). While the concept of reducibility was studied by other researchers as well, the idea of discharging, crucial for the unavoidability part of the proof, is due to Heesch, and he also conjectured that a suitable development of this method would solve the Four Colour Problem. This was confirmed by Appel and Haken when they published their proof of the Four Colour Theorem in two 1977 papers [14].

The four colour problem is a problem in graph theory, in this paper we introduce four colour problem as a problem in set theory. Here we consider regions have same colour in the map as sets belong to one family of disjoint sets. Families of disjoint sets is a generalization to the colours problems. Four colours problems is an example of

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Theorem3.1. From Definition 4.1. and Definition 4.2. we can consider regions have one colour as a family of disjoint sets with degree of intersection three.

Definition 4.1. A point is a corner of a map if and only if it belongs to the closures of at least three regions [17].

Definition 4.2. Two regions of a map are adjacent if their respective closures have a common point that is not a corner of the map [17].

In this paper we consider any map as families of disjoint sets, such that regions in the same colour as a family of disjoint sets. Regions in the same colour, are sets which are disjoint. Regions of map draw as sets intersect each other with degree of intersection three. We have seen from main theorems, local minimum and local maximum families of disjoint sets meet each other at W intersections depends on degree of intersection and standard of meeting. Minimum and maximum families of disjoint sets are general concepts, minimum and maximum colours used in maps are examples of these concepts.

Example4.3. Let degree of intersection n equal three, regions of a map are sets with degree of intersection three. Let for m be number families of disjoint sets, (number of colours used to draw map), from Theorem3.2 $m \leq 2n - 1$, substitute $n = 3$, we have $m \leq 5$, this means maximum number of colour to draw a map is 5. If you used six or more than six colours, this means by small effort you can reduce the number of colours to five.

Example4.4. Let degree of intersection n equal three, and W standard of meeting equal five, from Theorem3.1 $m(\frac{w+1}{2}) \leq wn$, for substitute $n = 3$, $w = 5$ we have $m(\frac{5+1}{2}) \leq 5(3)$, m is local maximum families of disjoint sets

(maximum colours used in maps) at $w = 5$, $m = 5$. Suppose $m = 6$, and X_1, X_2, X_3, X_4, X_5 be 5 nonempty intersections with maximum degree of intersection 3, $X_1 = (1,2,3)$, $X_2 = (1,2,5)$, $X_3 = (3,4,6)$, $X_4 = (2,4,3)$, $X_5 = (4,5,6)$. The families of disjoint sets (colours used in maps) labeled by 1,5,6 appear twice at X_1, X_2, X_3, X_4, X_5 , this means families of disjoint sets labeled by 1,5,6 not meet each other at these intersections. We change these intersections as

$X_1 = (1,2,3)$, $X_2 = (1,2,5)$, $X_3 = (3,4,5)$, $X_4 = (2,4,3)$, $X_5 = (4,5,1)$. Therefore any two of these families of disjoint sets (colours used in maps) labeled by 1,2,3,4,5 meet each other at X_1, X_2, X_3, X_4, X_5 , because any one appear three time.

Example4.5. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets (colours used to draw map), and X_1, X_2, X_3 be 3, nonempty intersections each of degree 3, $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (1,3,4)$. These 4, families of disjoint sets meet each other at X_1, X_2, X_3 are local maximum families of disjoint sets meet each other at minimum standard of meeting X_1, X_2, X_3 . If $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (1,3,4)$. These 4, families of disjoint sets meet each other at X_1, X_2, X_3 , are local minimum families of disjoint sets meet each other at minimum standard of meeting X_1, X_2, X_3 .

Example4.6. Let $\chi_1, \chi_2, \chi_3, \chi_4$ be 4 families of disjoint sets (colours used to draw map), and X_1, X_2, X_3, X_4 be 4 nonempty intersections each of degree 3. $X_1 = (1,2,3)$, $X_2 = (1,2,4)$, $X_3 = (1,3,4)$, $X_4 = (2,3,4)$. These 4 families of disjoint sets meet each other at X_1, X_2, X_3, X_4 are local minimum families of disjoint sets meet each other at minimum standard of meeting X_1, X_2, X_3, X_4 . Also local maximum families of disjoint sets meet each other at this standard of meeting are 4.

Remark4.7. In both Example4.5 and Example4.6 families of disjoint sets (colours used to draw map) are four, and nonempty intersections of degree 3. In Example4.5 standard of meeting $w = 3$, and in Example4.6 standard of meeting $w = 4$. In Example4.5 if we take any 3 intersections each of degree 3, such that they form standard of meeting, we find that these three intersections with the property there is one family of disjoint sets (one colour) appear at all intersections. If we have B for blue, G for green, O for orange, R for red, and Y for yellow. Take any three intersections X_1, X_2, X_3 , such that they formed standard of meeting for the colours B, G, R, Y . Let $X_1 = (B, G, R)$, $X_2 = (B, G, Y)$, $X_3 = (B, R, Y)$, the four colours B, G, R, Y are local minimum colours at intersections X_1, X_2, X_3 , with

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property one colour B appear at each intersections. In Example 4.6 if we take any 4 intersections each of degree 3, such that they form standard of meeting, we find that these four intersections with the property all families of disjoint sets (all colours) have the same appearance at all intersections. Take any four intersections X_1, X_2, X_3, X_4 such that they formed standard of meeting for the colours B, G, R, Y . Let $X_1 = (B, G, R)$, $X_2 = (B, G, Y)$, $X_3 = (B, R, Y)$, $X_4 = (G, R, Y)$, the four colours B, G, R, Y are local minimum colours at intersections X_1, X_2, X_3, X_4 with property four colours have the same appearance at all intersections.

Example 4.8. Let X_1, X_2, X_3 be 3 nonempty intersections each of degree 4. $X_1 = (1, 2, 3, 4)$, $X_2 = (1, 2, 5, 6)$, $X_3 = (3, 4, 5, 6)$ any two of these six families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , $n = 4$, $w = 3$, These 6 families of disjoint sets meet each other at X_1, X_2, X_3 , are local maximum families of disjoint sets meet each other at minimum standard of meeting X_1, X_2, X_3 , and are local minimum families of disjoint sets meet each other at minimum standard of meeting X_1, X_2, X_3 . Let Y_1, Y_2, Y_3 be 3 nonempty intersections each of degree 4. $Y_1 = (1, 2, 3, 4)$, $Y_2 = (1, 2, 3, 5)$, $Y_3 = (1, 2, 4, 5)$ any two of these five families of disjoint sets meet each other at the 3 intersections Y_1, Y_2, Y_3 , $n = 4$, $w = 3$, $m = 5$. These 5 families of disjoint sets meet each other at Y_1, Y_2, Y_3 , are minimum families of disjoint sets meet each other at minimum standard of meeting Y_1, Y_2, Y_3 . Families of disjoint sets (colours) represent (labeled) by 1 and 2 appear at all three intersections Y_1, Y_2, Y_3 .

Example 4.9 Let X_1, X_2, X_3 be 3 nonempty intersections each of degree 6. $X_1 = (1, 2, 3, 4, 5, 6)$, $X_2 = (1, 2, 3, 4, 5, 7)$, $X_3 = (1, 2, 3, 4, 6, 7)$ any two of these seven families of disjoint sets meet each other at the 3 intersections X_1, X_2, X_3 , $n = 6$, $w = 3$, $m = 7$. These 7 families of disjoint sets meet each other at X_1, X_2, X_3 , are local minimum families of disjoint sets, with four families of disjoint sets appear at each intersection, meet each other at minimum standard of meeting X_1, X_2, X_3 . Let Y_1, Y_2, Y_3 be 3 nonempty intersections each of degree 6. $Y_1 = (1, 2, 3, 4, 5, 6)$, $Y_2 = (1, 2, 3, 4, 7, 8)$, $Y_3 = (1, 2, 5, 6, 7, 8)$ any two of these eight families of disjoint sets meet each other at the three intersections Y_1, Y_2, Y_3 , these eight families of disjoint sets with two families of disjoint sets appear at each intersection, $n = 6$, $w = 3$, $m = 8$. Let Z_1, Z_2, Z_3 be 3 nonempty intersections each of degree 6. $Z_1 = (1, 2, 3, 4, 5, 6)$, $Z_2 = (1, 2, 3, 7, 8, 9)$, $Z_3 = (4, 5, 6, 7, 8, 9)$ any two of these nine families of disjoint sets meet each other at the 3 intersections Z_1, Z_2, Z_3 , $n = 6$, $w = 3$, $m = 9$. These 9 families of disjoint sets meet each other at Z_1, Z_2, Z_3 , are local minimum all families of disjoint sets have the same appearance and local maximum families of disjoint sets meet each other at minimum standard of meeting Z_1, Z_2, Z_3 .

Remark 4.10. Four colours problem is an application of families of disjoint sets with degree of intersection equal three at all standards of meeting three, four and five local minimum families of disjoint sets (colours) equal four. At standards of meeting three and four local maximum families of disjoint sets (colours) equal four. In this case local maximum families of disjoint sets (colours) equal local minimum families of disjoint sets (colours). At standard of meeting equal five local maximum families of disjoint sets (colours) equal five and local minimum families of disjoint sets equal four.

For families of disjoint sets with degree of intersection equal four at standard of meeting equal three, in this case local minimum families of disjoint sets (colours) equal six and local maximum families (colours) equal six. For families of disjoint sets with degree of intersection equal four at standard of meeting equal four, in this case local minimum families of disjoint sets (colours) equal five and local maximum families of disjoint sets (colours) equal five. For families of disjoint sets with degree of intersection equal six at standard of meeting equal three, in this case local minimum families of disjoint sets (colours) equal nine and local maximum families of disjoint sets (colours) also equal nine. For families of disjoint sets with degree of intersection equal six at standard of meeting equal four, in this case

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local minimum families of disjoint sets (colours) equal eight and local maximum families of disjoint sets (colours) also equal eight.

Example 4.11. Suppose we have any number of rectangles such that no point belongs to more than two rectangles. If we consider these rectangles families of disjoint sets with degree of intersection two, when we need to colour these rectangles minimum number of colour used three and maximum number of colour used three. Map of three colours is a map with degree of intersection two.

Example 4.12. Suppose we have any number of bodies (cubes) arrange such that maximum number of bodies touch each other at one point is eight point. If we coloured these bodies such that any two bodies touch each other even at one point have different colours. We want to calculate maximum and minimum number of colours used to any number of bodies. We consider bodies here as families of disjoint sets, and one family of disjoint sets, is a set of bodies such that every two not touch each other even at one point. This bodies are families of disjoint sets with degree of intersection equal eight. Using Corollary 3.3 we have maximum families of disjoint sets (maximum number of colours used to colour these bodies) equal $2n - 1 = 15$. We have many local minimum families of disjoint sets with degree of intersection eight, minimum families of disjoint sets (minimum number of colours used to colour these bodies) equal nine.

Example 4.13. Suppose we have ten papers connected together by spiral wire and each paper from other side connected with nine other papers by spiral wire, so that each paper from two different sides connected to nine other papers by spiral wire. If each ten papers connected together by spiral wire having ten different colours. We want to calculate maximum and minimum number of colours used to coloured any number of papers. We consider papers here as families of disjoint sets, and one family of disjoint sets, is a set of papers such that every two not connected by one spiral wire. These papers are families of disjoint sets with degree of intersection equal ten. Using Corollary 3.3 we have maximum families of disjoint sets (maximum number of colours used to colour these papers) equal $2n - 1 = 19$.

We have minimum families of disjoint sets (minimum number of colours used to colour these papers), from definition of local minimum families of disjoint sets we have minimum families of disjoint $m = 11$.

Example 4.14. Suppose we have huge balloon, and we draw all surface of this balloon by congruent equilateral triangles. Every point of the surface either inside a triangle, located on common rib belongs to two triangles or located on common head of six triangles. To colour these triangles, such that any six triangles have common head coloured with six different colours, at $w = 3$, local minimum families of disjoint sets is nine families of disjoint sets (colours) minimized to seven families of disjoint sets (colours) such that four colours appear at each intersection. The maximum number of colours used equal eleven $m = 2n - 1 = 2(6) - 1 = 11$. If number of triangles on surface of this balloon is large we can't use more than eleven colours, if you used more than eleven colours any number of intersections of degree six can't be standard of meeting. If you used more than eleven colours it become like two persons every one come one day per week, both met at Monday, it can't be consider meet each other once per week.

Remark 4.15. For all degree of intersections equal n we care about $w = 3$ and $w = 2n - 1$ because minimum number of families of disjoint sets when $w = 3$ and maximum number of families of disjoint sets when $w = 2n - 1$.

Example 4.16. In a city let m be number of bus routes (families of disjoint sets), W be number of bus stops (standard of meeting), and n be maximum number of bus routes pass through any bus stop (degree of intersection). This net of bus routes and bus stops built with conditions if you want to move between any two bus stops it is enough to used two route only, This net with the following properties:

- (1) To move between any two bus stops you can used one bus (bus route). See corollary 3.6.
- (2) Any two routes meet each other at least at one bus stop. See Definition 2.13.
- (3) $n < m$, if $n = m$ this means any routes stop at all bus stops.

Remark 4.17. For purpose of high efficient, airport administration may consider airline companies as families of disjoint sets, airplanes belong to one airline company form a family of disjoint sets. According to number of airplanes arrive to airport per week they have select suitable unit time. The number of airline companies (families of disjoint sets) be m , and suitable unit time can be a day, hours, one hour, have an hour or minutes. Number of airplanes arrive to airport per unit time n , is degree of intersection n . We can select suitable standard of meeting w ,

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number of airplanes arrive to airport (families of disjoint sets) m , depend on degree of intersection n , and standard of meeting w . We can select suitable number of airplanes arrive to airport m , using results of families of disjoint sets to determine local maximum, local minimum and properties of families of disjoint sets. By properties of families of disjoint sets we mean the appearance of airline companies at standard of meeting w .

V. COLOURING OF MAPS

In this paper we introduced the concept of families of disjoint sets as a topic in sets theory. If we look map of countries, any point of map either belongs to one set (one country), two sets (two countries) or three sets (three countries), for this reason we consider countries families of disjoint sets with degree of intersection three. Results which we proved here can be applied in space of two dimensions, three dimensions and n dimensions.

In this paper the word map means sets with degree of intersection equal n , where n can take any value except one. Here colouring of maps depends on degree of intersection equal n and standard of meeting equal w . Standard of meeting also depend on n , minimum value for standard of meeting equal 3, and maximum value for standard of meeting equal $2n - 1$. In map of countries degree of intersection is three, if the map coloured by four colours, and we select standard of meeting equal three will be like this $X_1 = (B, G, R)$, $X_2 = (B, G, Y)$, $X_3 = (B, R, Y)$, every colour appear twice and one colour appear three times. If standard of meeting equal four will be like this $X_1 = (B, G, R)$, $X_2 = (B, G, Y)$, $X_3 = (B, R, Y)$, $X_4 = (G, R, Y)$, the four colours B, G, R, Y each colour appear twice. If the map coloured by five colours B, G, O, R, Y each colour appear three times, and standard of meeting equal five will be like this $X_1 = (B, G, O)$, $X_2 = (B, R, Y)$, $X_3 = (B, O, Y)$, $X_4 = (G, R, Y)$, $X_5 = (G, O, R)$. If the map coloured by six colours, we know five colours (five families of disjoint sets) is maximum number of families of disjoint sets for degree of intersection is three. We can exchange the six colours to be reduced to five colours. Also we can exchange the five colours to be reduced to four colours.

If we have a map with degree of intersection equal five, to colour this map, we can use six colours, seven colours, eight colours or nine colours. If we have a map with degree of intersection equal six, to colour this map, we can use seven colours, eight colours, nine colours, ten colours or eleven colours.

In this paper we introduced the concept of families of disjoint sets as a topic in sets theory. If we look map of countries, any point of map either belongs to one set (one country), two sets (two countries) or three sets (three countries), so the degree of intersection in countries maps is three. Table (16) explain maps of countries, number of minimum colours used is four and number of maximum colours used is five. In Table (16) $n = 3$.

w	Local minimum	Local maximum
3	4	4
4	4	4
5	4	5

Table (16)

Table (17) explain maps of countries, number of minimum colours used is five and number of maximum colours used is seven. In Table (17) $n = 4$, in maps of countries usually there is no common point belong to four countries.

w	Local minimum	Local maximum
3	6	6
4	5	5
5	5	6
6	5	6
7	5	7

Table (17)

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Table (18) explain maps of countries, number of minimum colours used is six and number of maximum colours used is nine. In Table (18) $n = 5$, in maps of countries usually there is no common point belong to five countries.

w	Local minimum	Local maximum
3	7	7
4	6	6
5	6	8
6	6	7
7	6	8
8	6	8
9	6	9

Table (18)

Table (19) explain maps of countries, number of minimum colours used is seven and number of maximum colours used is eleven. In Table (19) $n = 6$, in maps of countries usually there is no common point belong to six countries.

w	Local minimum	Local maximum
3	9	9
4	8	8
5	7	10
6	7	9
7	7	10
8	7	9
9	7	10
10	7	10
11	7	11

Table (19)

Table (20) explain maps of countries, number of minimum colours used is eight and number of maximum colours used is thirteen. In Table (20) $n = 7$, in maps of countries usually there is no common point belong to seven countries.

w	Local minimum	Local maximum
3	10	10
4	9	9
5	8	11
6	8	10
7	8	12
8	8	11
9	8	12
10	8	11
11	8	12
12	8	12
13	8	13

Table (20)

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Table (20) explain if we have map of eight colours and degree of intersection seven, here is nine different types of eight colours, this different types due tonine different standards of meeting w . Same thing in maps of four colours, at $w = 3$ we have $m = 4$, at $w = 4$ we have $m = 4$, andat $w = 5$ we have $m = 4$. For this you can return to Table (16).

In these tables above for any degree of intersection n at W intersections, we observe that local minimum at $w = 3$ and at $w = 4$ is greater than anylocal minimum in the same table, this because local minimum equallocal maximum at $w = 3$ and at $w = 4$.

In Table (16)degree of intersection $n = 3$ can explain maps of countries. The number of colour four and five, there is local maximum and local minimumat eachstandard of meeting w . For differentstandards of meeting if we have the same number of colour, but may have different properties. e.g. fourcolours whenstandard of meeting consist of threeintersections, there is a colour appear at eachintersection, fourcolours whenstandard of meeting consist of four intersections, each colour appear three times at these four intersections, maximumnumber of colour is five and minimumnumber of colour is four.

In Table (20)degree of intersection $n = 7$, here in this map there are many points each of them belong to seven sets, and these seven sets coloured by seven different colours,maximumnumber of colour is $m = 13$, and minimumnumber of colour $m = 8$. There are fourstandards of meeting at each onelocal maximum $m = 12$, is $m = 13$, and they have different properties (appearance of a colour at each element of intersection).

The following table, Table (21) explain map of eight colours with degree of intersection $n = 7$. We can take different standard of meeting W for this map and have different properties as shown in the table.

w	Local minimum	Properties of local minimum
3	10	Local minimum equal local maximum
4	9	Local minimum equal local maximum
5	8	All families appear four times and three families appear at each intersection.
6	8	All families appear five times and two families appear at each intersection.
7	8	All families appear six times and one family appears at each intersection.
8	8	All families appear seven times at these intersections.
A	8	All families appear eight times except one family appear seven times. .
10	8	All families appear nine times except two families appear eight times.
11	8	All families appear ten times except three families appear nine times.
12	8	Four families appear eleven times andfour families appear ten times.
13	8	Three families appear twelve times andfive families appear eleven times.

Table (21)

In Table (21) we explain if we have map of eight colours and degree of intersection seven, here is nine different types of eight colours, this different types due tonine different standards of meeting w . Same thing in maps of four colours, at $w = 3$ we have $m = 4$, at $w = 4$ we have $m = 4$, andat $w = 5$ we have $m = 4$. For this you can return to Table (16). For eachdegree of intersection n we have $2n - 3$ cases oflocal maximum andhave $2n - 3$ cases oflocal minimum, so famous problem (Four colour problem), has simple cases, three cases oflocal maximum andthree cases oflocal minimum.

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For constant m families of disjoint sets and constant degree of intersection n , we can find different standards of meeting w , $3 \leq w \leq 2n - 1$ and different appearances for these m families of disjoint sets. This can be explained in Table (22), where m families of disjoint sets means four colours and degree of intersection n means degree of intersection three. Table (22) explains all standards of meeting for map of four colours.

m	n	w	nw	$\frac{nw}{m}$	$a + b = m = 11$	$a(\lfloor \frac{nw}{m} \rfloor \pm 1) + b(\lfloor \frac{nw}{m} \rfloor) = nw$
4	3	3	9	2.25	$1 + 3 = 4$	$1(3) + 3(2) = 9$
4	3	4	12	3	$0 + 4 = 4$	$0(4) + 4(3) = 12$

Table (22)

In Table (23), where m families of disjoint sets means eleven colours and degree of intersection n means degree of intersection eight. Table (23) explains all standards of meeting for map of eleven colours, with degree of intersection eight. Also Table (23) explains a drawing coloured by eleven colours with degree of intersection eight, such that regions with one colour formed one family of disjoint sets. About Table (23) we can say: (For constant m families of disjoint sets and constant degree of intersection n , we can find different standards W and different appearances for these m families of disjoint sets).

m	n	w	nw	$\frac{nw}{m}$	$a + b = m = 11$	$a(\lfloor \frac{nw}{m} \rfloor \pm 1) + b(\lfloor \frac{nw}{m} \rfloor) = nw$
11	8	3	24	2.18	$2 + 9 = 11$	$2(3) + 9(2) = 24$
11	8	4	32	2.9	$1 + 10 = 11$	$1(2) + 10(3) = 32$
11	8	5	40	3.6	$7 + 4 = 11$	$7(4) + 4(3) = 40$
11	8	6	48	4.36	$4 + 7 = 11$	$4(5) + 7(4) = 48$
11	8	7	56	5.09	$1 + 10 = 11$	$1(6) + 10(5) = 56$
11	8	8	64	5.81	$9 + 2 = 11$	$9(6) + 2(5) = 64$
11	8	9	72	6.55	$6 + 5 = 11$	$6(7) + 5(6) = 72$
11	8	10	80	7.27	$3 + 8 = 11$	$3(8) + 8(7) = 80$
11	8	11	88	8	$0 + 11 = 11$	$0(9) + 11(8) = 88$
11	8	12	96	8.73	$8 + 3 = 11$	$8(9) + 3(8) = 96$
11	8	13	104	9.81	$5 + 6 = 11$	$5(10) + 6(9) = 104$
11	8	14	112	10.18	$2 + 9 = 11$	$2(11) + 9(10) = 112$
11	8	15	120	10.90	$10 + 1 = 11$	$10(11) + 1(10) = 120$

Table (23)

Here when we mention families of disjoint sets, we have to mention with families of disjoint sets the degree of intersection. Same thing for number of colours used, we have to say m colours with degree of intersection n . Four colours here related only to degree of intersection equal three, and three colours related only to degree of intersection equal two. Five colours related to degree of intersection equal three and four. We have two maps of five colours, five colours with degree of intersection equal three and five colours with degree of intersection equal four. Six colours related to degree of intersection equal four and five. We have two maps of six colours, six colours with degree of intersection equal four and six colours with degree of intersection equal five. Eleven colours related to degree of intersection equal 6,7,8,9,10. There are different maps of eleven colours depend on different degrees of intersection. Table (22) explain

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map of even colours with degree of intersection equal eight. There is only map of four colours, four colours with degrees of intersection equal three. Table (21) explain map four colours with degrees of intersection equal three.

Here we explain m colours problem as an application to theory of families of disjoint sets. m colours problem have many maps. We have different maps of m colours with degree of intersection n , where m is constant and n has different values, n equal $\frac{m+1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, \dots, m-1$, where m is odd, and equal $\frac{m}{2}+1, \frac{m}{2}+2, \frac{m}{2}+3, \dots, m-1$, where m is even, this is relation from $n+1 \leq m \leq 2n-1$. For each map we have different values standard of meeting w , w equal $3, 4, 5, \dots, 2n-1$.

VI. CONCLUSION

Families of disjoint sets with concepts which we have introduced, have many applications because sets in several problems in mathematics similar to sets in families of disjoint sets, such as map, regions in the same colour as a family of disjoint sets. We apply theorems of local minimum and local maximum families of disjoint sets to find minimum and maximum colours used in maps. Number of bus routes in a city and number of plane routes between cities can be consider families of disjoint sets, also number of bus stops number of airports can be consider standard of meeting. As we have seen in examples in this papers problems can be changed, without essential changes, families of disjoint sets where studying of minimum and maximum and some properties become easy.

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