

# **Comparative Study of Attitude Control for Rigid Body of Launch Vehicle Using Sliding Mode Control for Different Constant Rate Reaching Laws**

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**ABSTRACT:** Attitude control is controlling the orientation of the object with respect to an inertial frame of reference or another entity. The major hurdle here is the instability of the launch vehicle. The general features of the attitude control problem, especially as it relates to trajectory deviations. The design of the attitude control system is thus conducted in several phases. One is concerned primarily with developing an autopilot configuration that achieves adequate short-period stability and performance. This makes use of the so-called time slice approach, in which time-varying mass and inertial properties are frozen over a short period of time. The total launch vehicle dynamics consists of rigid body and flexible body dynamics. The rigid body is made stable by stability augmentation. A PID controller is designed and applied to the rigid body. The simulation results show satisfactory responses.

**KEYWORDS:** Attitude Control, Stability Augmentation, PID control

## **I. INTRODUCTION**

Launch vehicles are used to transport and put satellites or spacecraft into space. When vehicle flies through the atmosphere it experiences aerodynamic forces and moments. Vehicle is aerodynamically stable if it can maintain its orientation under wind disturbances. Vehicle is aerodynamically unstable if its orientation diverges due to angle of attack by wind disturbances. In general, launch vehicles are aerodynamically unstable.

The aim of an attitude control problem is to achieve adequate stability, rapid response to input command and moderate insensitivity to external disturbance. A conventional PID controller is easy to design and implemented to the system. The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces or eliminates the steady state error. The derivative controller reduces the rate of change of error. Thus, a suitable combination of the three basic modes can be improve all aspects of the system performance. [4].

## **II. RELATED WORK**

Attitude control system stabilizes the vehicle and track guidance commands by providing appropriate signals to the actuators. The design of the launch vehicle attitude control system represents a challenging nonlinear time varying flexible body problem. A typical method applied to overcome this is the linear perturbation, which initializes the system at nominal operation condition on the ascent trajectory. The launch vehicle guidance system provides the inputs necessary to fly in a predefined direction. These control inputs are generally in the form of attitude commands. The aim of an attitude control problem is to good tracking performance, achieve adequate stability, rapid (well damped) response to input command and moderate insensitivity to external disturbances. Major design problems arise because a

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(A High Impact Factor, Monthly, Peer Reviewed Journal)

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launch vehicle is aerodynamically unstable. The attitude control problem of the launch vehicles requires an effective controller to track the recommended trajectory. The prime significance lies in building up an autopilot arrangement that accomplishes sufficient short period stability and performance. This makes utilization of the alleged time slice approach, in which the time varying mass and inertial properties are considered as constant over a short period of time. [1].

In [10] PD and PID controller is designed based on LQR technique and it cannot offer a good time domain response due to the higher aerodynamic coupling. Then a PID plus phase lag is proposed to overcome this problem. Attitude control of launch vehicle is done in this paper by using a ground experimental model and by using some stabilization strategies. In this it was shown that the adaptive filter stabilizes an uncertain system successfully in conjunction with thruster vector control actuation using an air thruster. PID controller design can be done by root locus technique. The integral gain value is designed from the specified error constant. [4]

The dynamical systems are usually affected by external matched disturbances, parametric variations, modeling uncertainties and nonlinearities (hysteresis, friction, etc.). The sliding mode control (SMC) is a well-known solution used to solve these problems. Indeed, SMC is able to overcome these barriers in regulation and tracking. Such control technique has amply demonstrated its effectiveness through theoretical and practical studies in main areas of applications including robotics, chemical reactors, mobile manipulators and electrical machinery. The robustness property of SMC is achieved by using a high frequency switching to steer the states of a system into the sliding surface. The high-frequency switching leads, generally, to the appearing of an undesirable chattering phenomenon at outputs of the system and on the control input. This leads to the dissipation of a large quantity of energy in electric actuators and a rapid wear of mechanical actuators. This limits seriously the implementation of SMC in real time. To overcome this difficulty, several solutions have been proposed in the literature. Among them, the “sign” function is replaced by a smooth similar function such as the saturation function or the sigmoid function. [9]

### III. MATHEMATICAL MODELLING

Under this study, the launch vehicle is assumed as completely rigid. The assumption that the bodies are rigid, which means that they do not deform under the action of applied forces, simplifies the analysis by reducing the parameters that describe the configuration of the system to the translation and rotation of reference frames attached to each body. [1][2]

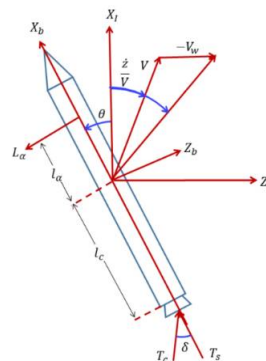


Fig 1: Geometrical diagram for rigid body dynamic model

The linearized equations of motions of the launch vehicle can be derived.

The force equation,

$$\ddot{x} = \frac{(T_s - D)}{m} \theta - \frac{L_x}{m} \alpha + \frac{T_s}{m} \delta \quad (1)$$

The moment equations,

$$\ddot{\theta} = \mu_\alpha \alpha + \mu_c \delta \quad (2)$$

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$$\alpha = \theta + \frac{\delta}{V} + \alpha_W \tag{3}$$

Where,

$D$  = drag which is the force acting opposite to relative motion of the launch vehicle

$I_{yy}$  = moment of inertia of vehicle about pitch axis which is a quantity that expresses the body's tendency to resist angular acceleration

$m$  = mass of vehicle

$T_c$  = control thrust

$T_s$  = sustainer (nonswivelled) thrust

$T_T = T_c + T_s$  is the total thrust acting which is the sum of control thrust and sustainer thrust

$V$  = forward velocity of vehicle

$V_W$  = wind velocity

$z$  = normal displacement of vehicle along the  $X_T$  axis.

$\theta_0$  = nominal attitude angle relative to the earth-fixed reference frame

$L_\alpha$  = aerodynamic load per unit angle of attack

$l_c$  = distance from mass center of vehicle ( $c_g$ ) to engine swivel point

$l_\alpha$  = distance from mass center of vehicle to center of pressure (cp)

$\delta$  = thrust deflection angle which is the quantity need to be controlled

$\alpha$  = angle of attack-angle between the wing velocity and the reference line on the body.

$\theta$  = attitude angle error relative to trajectory inertial frame.

$\alpha_W$  = gust angle of attack which is the angle between wind velocity and forward velocity of the vehicle =  $-\frac{V_W}{V}$

During the flight, all the vehicle parameters and flight conditions change as the time evolves. The variations of these coefficients are primarily due to changes in dynamic pressure.

Taking Laplace transform of equation (2)

$$s^2 \theta(s) = \mu_\alpha \alpha(s) + \mu_c \delta(s) \tag{4}$$

where thrust and aerodynamic moment coefficients are given as follows

$$\mu_c = \frac{T_c l_c}{I_{yy}} \text{ and } \mu_\alpha = \frac{L_\alpha l_\alpha}{I_{yy}} \tag{5}$$

Taking an assumption that  $\alpha(s) = \theta(s)$  that is gust angle of attack is neglected and normal velocity is assumed to be same as forward velocity of the launch vehicle (4) becomes

$$(s^2 - \mu_\alpha) \theta(s) = \mu_c \delta(s) \tag{6}$$

$$\frac{\theta(s)}{\delta(s)} = \frac{\mu_c}{s^2 - \mu_\alpha} \tag{7}$$

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Thus (7) gives simplified rigid body dynamics of the launch vehicle. Choose  $\mu_c = 4.56$ ,  $\mu_g = 2.8$  for time period of 0.5 sec to 72 sec.

### IV. ANALYSIS OF SYSTEM WITHOUT CONTROLLER

Time domain analysis can be done on the rigid body dynamics of the launch vehicle. Rigid body parameters are selected based on time slice approach. The values corresponding to the time period of 0.5 sec to 72 sec were chosen.

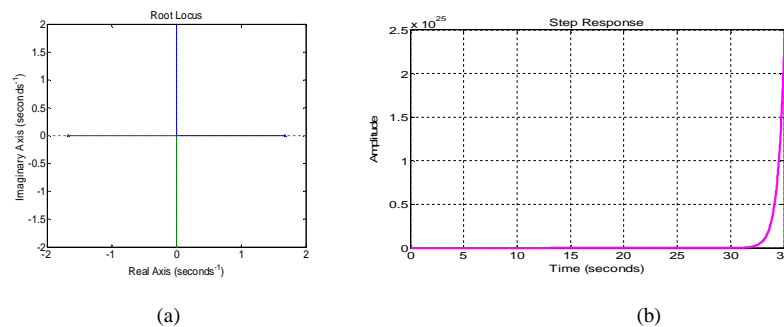


Fig 2: (a) Root locus (b) Open loop step response of Rigid body dynamics

The root locus of the rigid body dynamics in Fig. 2a indicates the presence of a right-hand side pole which makes the system unstable. The step response of the rigid body dynamics is shown in Fig.2b. In order to make stabilize the rigid body stability augmentation is needed.

Stability Augmentation Systems (SAS) are control systems that can provide artificial stability for airplanes or launch vehicles that have undesirable flying characteristics. The stability augmentation system enhances stability but does not have the ability to hold a given datum. SAS tries to place the Eigen values at desired location by using a feedback mechanism. Since the rigid body dynamics is a second order system without a damping ratio, we cannot vary only the gain of the system in order to achieve stability. So, the rate of the system should be controlled. The figure 3 shows the rigid body dynamics with stability augmentation.

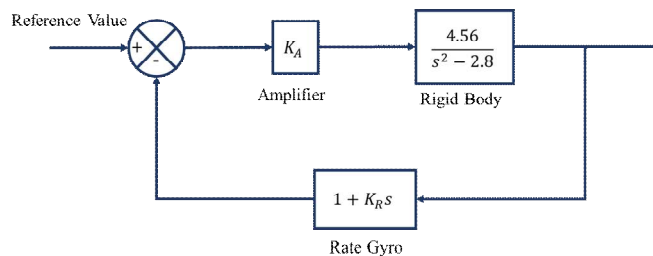


Fig 3: Closed loop system of rigid body with amplifier and rate gyro

Now the task is to design values for the amplifier gain value  $K_A$  and the gain value of rate gyro  $K_R$ . The characteristic equations of the above system and the desired characteristic equation can be compared to find these values. The desired system is a second order system with a damping ratio,  $\zeta = 0.7$  and natural frequency,  $\omega_n = 3 \text{ rad/sec}$ . Therefore, the desired characteristic equation is,

$$s^2 + 4.2s + 9 = 0 \tag{8}$$

The characteristic equation of the system shown in figure 4 is,

$$1 + K_A \frac{4.56}{s^2 - 2.8} (1 + K_R s) = 0 \tag{9}$$

Comparing equations (9) and (10) we get the values of  $K_A = 2.587$  and  $K_R = 0.356$ .

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The root locus of the rigid body in Fig. 4a indicates that for values of open loop gain greater than 0.237 the system can be made stable.

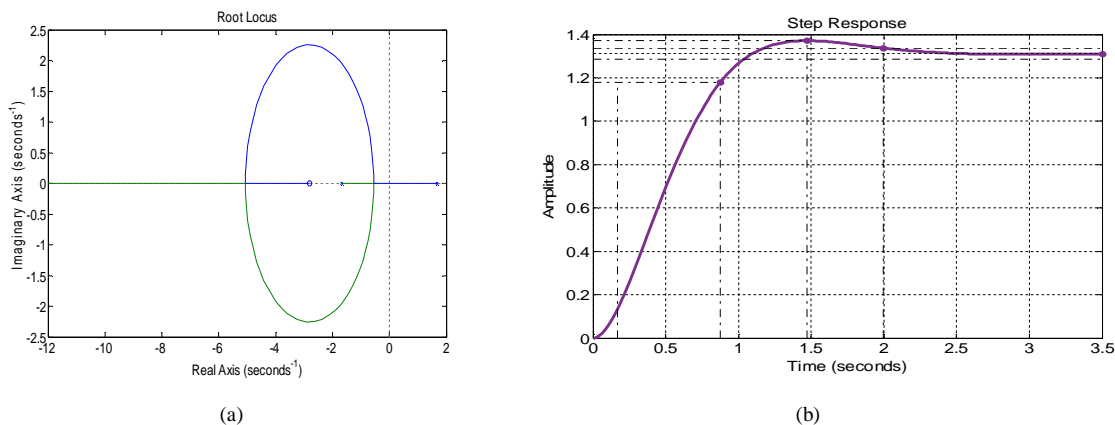


Fig 4: (a) Root locus (b) Step response of Rigid Body dynamics with stability augmentation

The step response of the system is shown in Fig 4b. In which the stability is guaranteed but the system has undesired performance. It has a peak overshoot of 4.6 % and settled at an amplitude of 1.31. These can be eliminated by giving proper control to the system.

## V. PID CONTROL

A proportional–integral–derivative controller (PID controller or three term controller) is a control loop feedback mechanism widely used in industrial control systems and a variety of other applications requiring continuously modulated control. The transfer function of PID Controller is given by equation (10)

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \tag{10}$$

The PID controller with the system is shown in Fig 5.

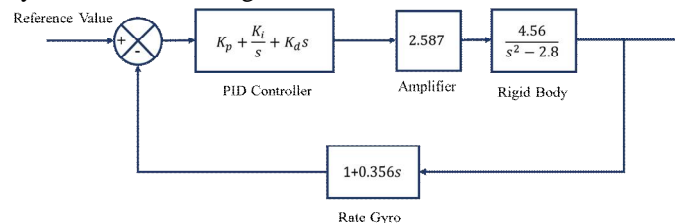


Fig 5: Block diagram of Rigid body with PID controller

Now, the PID controller have to be designed.<sup>[4]</sup> By using Root Locus Technique a controller can be designed and introduced in cascade with open loop system so that, it can have a pair of dominant closed loop poles, to satisfy specified time domain specifications  $\zeta$  and  $\omega_n$ . Here, the desired system is a second order system with a damping ratio,  $\zeta = 0.7$  and natural frequency,  $\omega_n = 3 \text{ rad/sec}$  and steady state error,  $e_{ss} \leq 1\%$ .

**Step 1:** Determine the dominant pole,  $s_d$  and calculate its magnitude and phase

$$s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = D \angle \beta \tag{11}$$

On substituting the values of  $\zeta$  and  $\omega_n$  in equation 13 we will get,  $D = 3$  and  $\beta = 134.427$

**Step 2:** Determine the magnitude and phase of system,  $G(s)$  at  $s = s_d$

Let  $G(s)$  = Open loop transfer function of the system (Rigid body dynamics with stability augmentation)

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$G_c(s)$  = Transfer function of PID controller  
 $A_d = |G(s)|$  at  $s = s_d$  (12)  
 and  $\phi_d = \angle G(s)$  at  $s = s_d$  (13)

**Step 3:** Determine transfer function of PID

Transfer function of PID given by equation (11) which can be simplified as ,

$$G_c(s) = \frac{K_p s + K_i + K_d s^2}{s} \tag{14}$$

The value of  $K_i$  can be determined from the specified error constant. From the value of  $K_i$  and using equations (11) to (14) the values of  $K_p$  and  $K_d$  can be designed.

Designed values for  $K_i = 23.7354$ ,  $K_p = 14.1143$  and  $K_d = 3.0678$

Fig. 6a shows the root locus of the system with PID controller. For values of gain greater than 0.025 the system is stable. The step response of the system with PID controller is shown in Fig 6b. It shows satisfactory results.

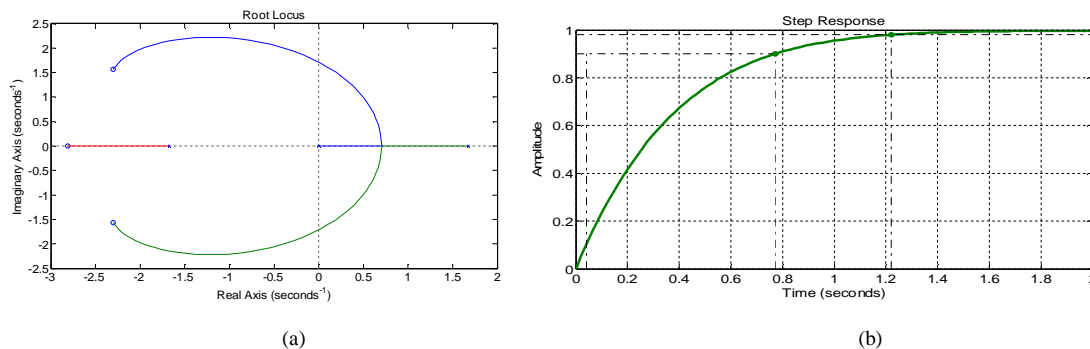


Fig 6: (a) Root locus (b) Step response of Rigid Body dynamics with stability augmentation and PID Control

## VI. SLIDING MODE CONTROL

The dynamical systems are usually affected by external matched disturbances, parametric variations, modelling uncertainties and nonlinearities (hysteresis, friction, etc.). The sliding mode control (SMC) is a well-known solution used to solve these problems. The robustness property of SMC is achieved by using a high frequency switching to steer the states of a system into the sliding surface. This leads to the appearing of an undesirable chattering phenomenon at outputs of the system and on the control input.

SMC synthesis entails two phases

PHASE I: Sliding surface design: It is designed so that the system in sliding mode evolves in the desired way.

PHASE II: Control input design: It has to be chosen in order to enforce a sliding mode.

The rigid body dynamics with stability augmentation can be represented in phase variable form as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2.8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u. \tag{15}$$

For SMC design the state space should be in regular form. i.e. the  $B$  matrix of the form,  $B = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$  Equation (15) is in regular form. The dynamics of the system with SMC is equated with the desired characteristic equation. Here the desired pole  $s_d = -2.1$ ,  $s_d$  is calculated as an equivalent first order system of previous specification in order to design SMC for reduced order dynamics.

The sliding surface design matrix,  $c = [M \quad I]$  (13)

The value for  $M$  can be calculated from the desired pole and can be used to find the sliding surface design matrix.

Now the sliding surface is  $S = cx$  (14)

Designed value of  $c = [2.1 \quad 1]$  (15)

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The total control law is the sum of equivalent control and reaching law. It can be represented as follows.

$$u_c = u_{eq} + u_{reach} \tag{16}$$

$$u_c = -(cB)^{-1}cAx - K \operatorname{sgn}(S) \tag{17}$$

The reaching law is considered as a constant rate reaching law. In order to avoid chattering Saturation function can be used instead of Signum function. Here the only unknown is the gain of the control  $K$ . It can be designed as follows, If a system  $\dot{x}_1 = x_2$  and  $\dot{x}_2 = h(x) + g(x)u$

Then  $\left| \frac{c_1 x_2 + h(x)}{g(x)} \right| \leq K_1 \leq K$  Where  $c_1$  is the element of sliding surface design matrix  $c$ . From this, the value for  $K$  can

be designed as  $K=7.75$ .

In the design of the SMC it is assumed that that initial state of  $\theta$  and  $\dot{\theta}$  is at (5,0) and the final states of  $\theta$  and  $\dot{\theta}$  is at (5,0). The phase portrait of  $\theta$  and  $\dot{\theta}$  is shown in Fig.7a.

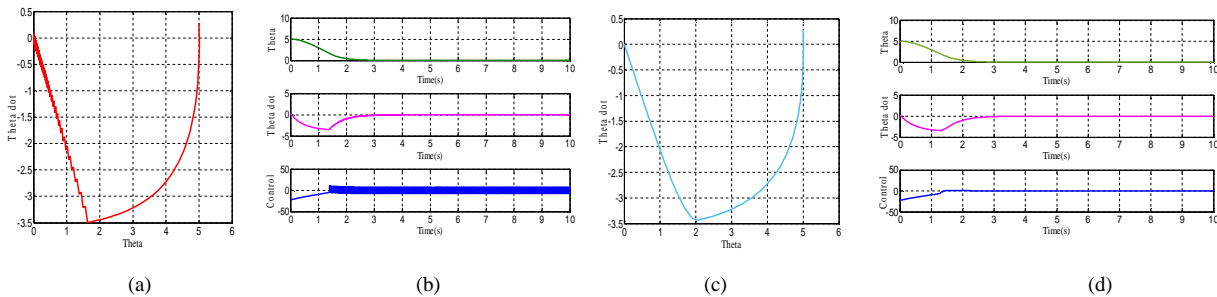


Fig 7: (a) Phase portrait of  $\theta$  and  $\dot{\theta}$  for signum function (b)Theta, theta dot and control versus time for signum function (c): Phase portrait of  $\theta$  and  $\dot{\theta}$  for saturation function (d) Theta, theta dot and control versus time for saturation function.

The ill effect of chattering can be avoided by using saturation function in control law. i.e, the control law becomes,

$$u_c = -(cB)^{-1}cAx - K \operatorname{sat}(S) \tag{19}$$

Saturation function is smooth when compared to signum or sign function. Fig. 7c shows the phase portrait of  $\theta$  and  $\dot{\theta}$  with saturation function. The variation of  $\theta$ ,  $\dot{\theta}$  and the control input versus time is plotted in Fig. 7d. From these figures it is clear that the chattering is completely avoided by using saturation function.

Two types of constant rate reaching laws are used in this work. One is based on signum function and another one based on saturation. The comparison is given in the Table I

TABLE I: COMPARISON OF EFFECT OF CONSTANT RATE REACHING LAWS BASED ON SIGNUM FUNCTION AND SATURATION FUNCTION

Signum Function	Saturation Function
Reaching law, $u_{reach} = -K \operatorname{sgn}(S)$	Reaching law, $u_{reach} = -K \operatorname{sat}(S)$
High frequency switching is needed	Switching is smooth
Chattering is present	Chattering is absent



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## VII. CONCLUSION

In an attitude control problem, the unstable launch vehicle has to track a prescribed trajectory with very small steady state error. A comparative study of classical sliding mode controller with signum and saturation reaching law is done in this paper. The controller is designed and simulated for the rigid body dynamics of the launch vehicle, even though the system is flexible. The ill effect of chattering is avoided by saturation function. A PID controller was designed initially to compare the effectiveness of the sliding mode controller with conventional method. The sliding mode controller with saturation reaching law gives satisfactory responses. In this work actuator dynamics is chosen as constant gain. The effect of controller with actual actuator dynamics can also be evaluated as a future work.

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