

# Optimal Power Flow Analysis for Active Power Dispatch Considering Voltage Dependent Load Models

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**ABSTRACT:** In this paper are integrated the voltage dependent load models into the Optimal Power Flows formulation for active power dispatch in order to analyse the effect of the load models in the optimal steady-state operation point. Exponential and polynomial static load models are considered here to carry out this analysis, since these models are most common used in the open literature. The load models considered in this work are the exponential and composed models, which are integrated into OPF formulation using an existent program developed in Matlab language programming. A study case is carried out with the 5 nodes-IEEE test power system in which is determined the optimal steady-state operation point considering the above mentioned load models. The results of the case studies show that the voltage dependent load models greatly affect the optimal points of steady state operation, since there is a decrease in demand, losses and power generation, both active as reactive, thereby obtaining more economical operating points.

**KEYWORDS:** Optimal Power Flow, voltage dependent load models, optimal steady-state operation point.

## I. INTRODUCTION

Electricity supply companies must reach their maximum efficiency while minimizing the cost of generation, therefore, the profitability of the generation units is essential. This makes necessary the incorporation of the analysis of Optimal Power Flow (OPF), since by means of this analysis the demand of active power can be satisfied with the minimum cost of generation, which increases the profitability of the operation of the power system. The OPF analysis became an important research problem from 1962 [1, 2] and this analysis was formulated as a non-linear programming problem using a rigorous mathematical approach by J. Carpentier [3].

The Optimal Power Flow analysis allows to optimize an objective function subject to equality and inequality constraints, which determines the optimal steady-state operation of the electrical power system. The objective functions may consider economic, safety or environmental aspects of the electrical system. In this work, this function corresponds to the generation cost of active power [3, 4]. On the other hand, the constraints of the OPF model represent the physical laws that govern the generation of power, the power transmission capacity of the system, the nominal design limits of electrical equipment, as well as the operation of the system itself [5]. These are equality and inequality constraints, the first correspond to the equations of balance of active and reactive power in each node of the electrical system, and the second are those referring to the operational and physical limits of the system.

OPF studies have become an essential analysis tool in the planning and operation of power systems, since the OPF study allows to evaluate the steady state equilibrium point of a power system for time horizon given and for a current situation, respectively, which allows to guarantee the economic or safe operation of a power system considering its growth, evolution and operational control actions [6]. The results of the OPF analysis depend on the models used of the main electrical components of the SEP, such as transformers, generators, transmission lines and electric loads. The electric load modelling have a strong impact on the final value of the objective function and the state variables of the

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power system [7], therefore, the use of a real load model is very important to improve the quality of the studies carried out in the power systems. However, this modelling is complicated due to the different types of loads connected in the electrical system, as well as to the environmental conditions and to the several load levels [8]. Thus, the modelling of the loads in the studies of the power systems is done considering several simplifications. Load models are classified into two types: static and dynamic models; in turn the static models are exponential and composed models [9, 10]. The static models have been used in several works where the authors have integrated these models in the power systems analysis [11, 12, 13, 14, 15], but very few studies with the Optimal Power Flow have been reported in the literature [16, 17].

In this context, in this work, the static models of voltage-dependent electrical loads are integrated into the power system model to obtain the Optimal Generation Dispatch considering these load models and determine the impact of the representation of these loads in the results of the OPF studies.

Paper is structured as follows: Section II describes the mathematical formulation of the OPF analysis as a nonlinear programming problem. In Section III are described the active and reactive load models and the integration of these models in the OPF analysis are showed in Section IV. Finally, the study cases considering the voltage dependent load models in the OPF results and the conclusions of this work are presented in Section V and VI, respectively.

## II. RELATED WORK

The economic operation of the power systems has been a broad topic of study of researchers, academics and electricity supply companies around the world. It is well known that the steady-state operation point of a power system depends to a great extent on the loads connected to it, so in the last decades special attention has been paid to modelling these loads in order to represent the practical operation of the power system. It is well known that the steady-state operation point of a power system depends to a great extent on the loads connected to it, so in the last decades special attention has been paid to modelling these loads in order to represent the practical operation of the power system. In this way, several research works have been published in the open literature where the authors present the dynamic and static analysis of power systems with the modelling of electric loads, as well as, the modelling of voltage-dependent loads [18, 19, 20, 21], while many other authors have done their research in the voltage dependent load modelling [22, 23, 24, 25, 26].

Regarding the steady state operation of power systems, the power flow and the optimal power flow are the two analysis tools most used around the world to carry out the power systems analysis. For this reason, the load models have been considered and integrated in the above analysis tools, for instance, in the case of the power flow, the static load models have been used to determine the effects of these in the results of the power flows and in the steady-state operation of power systems [11, 12, 13, 14, 15].

In the Optimal Power Flow analysis, the load models have been considered too, but in smaller quantity because it is very complicated to integrate these models in a non-linear optimization model. However, in the open literature there are reported several research works, for example, in [27] it is considered the load modelling in the OPF and in [16] are studied the effects of the load models in the OPF results, while in [17] the effects of the load modelling in the security constrained OPF studies are evaluated.

## III. OPTIMAL POWER FLOW ANALYSIS

The mathematical formulation of the OPF problem can be considered as a nonlinear constrained optimization problem, in which the generation cost of active power of each generator is minimized without violating the limits of the power system. The OPF problem is formulated as follows [3, 4],

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$$\begin{aligned} &\text{Minimize } f(x) \\ &\text{Subject to } \begin{cases} h(x) = 0 \\ g(x) \leq 0 \end{cases} \\ &y^{\min} \leq y \leq y^{\max} \end{aligned} \quad (1)$$

where  $y$  is the vector of system state variables,  $f(y)$  is the objective function to be optimized, in this case the active power generation cost function,  $h(y)$  represents the equality constraints given by the equations of power balance and  $g(y)$  are the inequality constraints, which represent the limits of the control and state variables, and the functional inequality constraints.

## IV. MODELLING VOLTAGE-DEPENDENT LOADS

The operation of a power system depends on the capacity to generate power to satisfy the load demanded and the losses of the system. Consequently, the characteristics of the load have an important influence in the studies of power flows [9]. For this reason, it is very important to model loads as close to practice, but this modelling is complicated because a typical load bus is composed of a large number of devices making it difficult to estimate the exact composition of the load. In addition, the load composition changes depending on many factors, including time (hour, day, and season) and climatic conditions [8]. The static modelling of the loads is performed according to the variables that affect the active and reactive power consumed in a given node. These variables are the bus voltages and the frequency of the network [26]. The models of the voltage-dependent loads used in the analysis of power systems are of the exponential and polynomial type, which are expressed for a node  $i$ , respectively, by means of (3) and (4) as follows [9, 10],

$$P_{i,\text{exp}}(V) = P_{i,0} \left( \frac{V_i}{V_{i,0}} \right)^\alpha \quad (2)$$

$$P_{i,\text{ZIP}}(V) = P_{i,0} \left( a_0 + a_1 \left( \frac{V_i}{V_{i,0}} \right) + a_2 \left( \frac{V_i}{V_{i,0}} \right)^2 \right) \quad (3)$$

where the terms  $P_{0,i}$  and  $V_{0,i}$  represent the power and nominal voltage of the node  $i$ ,  $V_i$  is the voltage of the node  $i$  at a given moment,  $\alpha$  is the coefficient of the exponential model and  $a_{(0, 1, 2)}$  represents the coefficients of the model polynomial, whose sum of these coefficients is equal to 1. The polynomial model is also known as a ZIP or composite model. The same expressions apply for the reactive load model, only that for this case the variable  $P$ , the exponential coefficient and the coefficients of the ZIP model are denoted by  $Q$ ,  $\beta$  and  $b$ , respectively.

## V. OPF MODEL CONSIDERING VOLTAGE DEPENDENT LOAD MODELS

The OPF model used in this work considers the incorporation of voltage dependent load models in a single unified solution framework. The resulting model of OPF with the loads is expressed as,

$$\min C_T(P_{Gi}) = \sum_{i=1}^{Ng} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (4)$$

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Subject to:

$$P \sum_{i=1}^{ng} P_{Gi} - P_{Di} - \sum_{j=1}^{nl} P_{iny,i-j} = 0 \quad (5)$$

$$\sum_{i=1}^{ng} Q_{Gi} - Q_{Di} - \sum_{j=1}^{nl} Q_{iny,i-j} = 0 \quad (6)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (7)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad (8)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (9)$$

where

$$P_{Di} = \{P_{i,\text{exp}}(V_i), P_{i,\text{ZIP}}(V_i)\} \quad (10)$$

$$Q_{Di} = \{Q_{i,\text{exp}}(V_i), Q_{i,\text{ZIP}}(V_i)\}$$

Last expressions of active and reactive power correspond to voltage dependent load models described in Section III, thus the active and reactive power balance in each node of the power system is dependent of nodal voltage.

## VI. STUDY MODELS

In this paper a case study is presented with the IEEE test system of 5 nodes [28] in which the optimal steady-state operation point of this power system is determined considering the models of voltage dependent load, in order to evaluate the effect that the integration of such load models has on the dispatch of active power, as well as on the optimal steady-state equilibrium point of electrical power systems. For the study cases presented in this paper, the coefficients of the exponential model were  $\alpha = 1.5$  for the active power and  $\beta = 2.0$  for the reactive power [16], while in the case of the polynomial or ZIP model the following values were adopted for active and reactive power, respectively, ( $a_0 = 0.55, a_1 = 0.15, a_2 = 0.35$ ) and ( $b_0 = 0.50, b_1 = 0.12, b_2 = 0.38$ ). The convergence tolerance considered in the case study was  $1 \times 10^{-9}$ .

The 5 nodes power system of the IEEE has three load nodes and an additional load connected to a generation bus. In this way, the voltage dependent load models are integrated in these nodes to later calculate the optimal steady-state operation of this power system. It should be mentioned that the lower and upper limit of generation is 10 and 200 MW, respectively, while the voltage limits are  $0.9 \leq V_i \leq 1.1$  pu. A summary of the results obtained from the equilibrium point with both load models in the OPF is shown in Table 1. The results presented in this table show that the generation cost is lower with the load models, but it is important to indicate that in the case of the exponential model, the generation cost is even lower.

Table 1: Comparative results of the 5-node test power system considering load models

Parameter	Constant	Exp.	ZIP
Total cost ( \$/hr)	747.97	656.78	700.76
$P_{\text{demand}}$ (MW)	165	143.80	152.66
$P_{\text{losses}}$ (MW)	3.05	3.33	3.71
$P_{\text{generation}}$ (MW)	168.05	147.14	156.38
$Q_{\text{demand}}$ (MVar)	40	29.42	35.87
$Q_{\text{losses}}$ (MVar)	-25.29	-14.26	-13.30
$Q_{\text{generation}}$ (MVar)	14.71	15.16	22.57

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In the same way, Fig. 1 shows that the nodal voltages of the 5-node system decrease when the load models are integrated, which causes the active and reactive power flows and losses to decrease. It is important to note that all nodes of the power system the voltage decreases below the nominal value (1 pu), which is correct because with the load the voltage tends to decrease, allowing to simulate in a more practical way the operation of the power system.

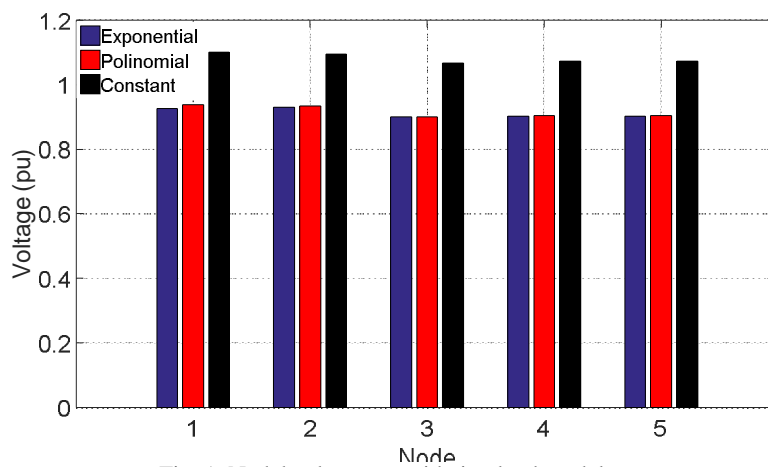


Fig. 1: Nodal voltages considering load models

Fig. 2 shows that the active power generation decreases when the load models are integrated into the OPF formulation. This effect is caused by the decrease of the nodal voltage profile, since when the voltage decreases, the power flows decrease and therefore the transmission losses, resulting in the decrease in power generation.

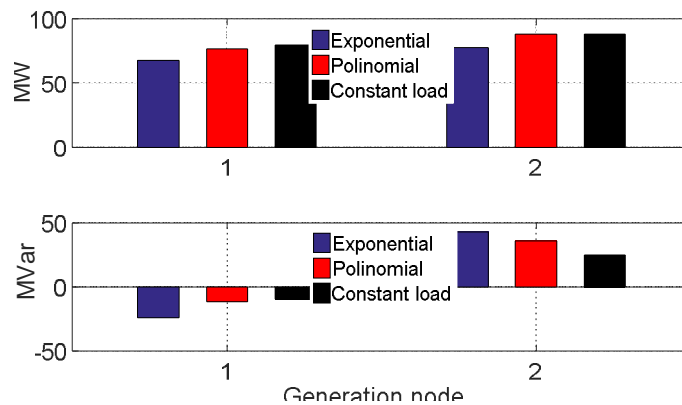


Figure 2: Active and reactive power generation considering load models

On the other hand, Fig. 3 shows that with the load models the active power demand is lower than with the constant load. This parameter decreases even more with the exponential model, but this does not mean that the exponential model is the best from a practical simulation point of view of the system. It is necessary to carry out other studies to confirm which model best represents the practical operation of power systems.

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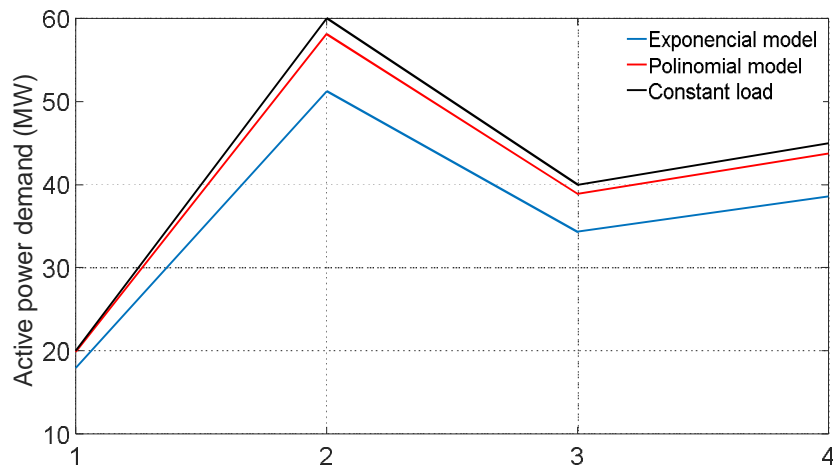


Figure 3: Active power demand considering load models

In the case of reactive power, Fig. 4 clearly shows that the demand for this power has a behavior very similar to that of the active power when the load models are integrated in the OPF analysis, that is, the reactive power demand decreases with the voltage dependent load models. In the same way with the exponential model a lower value of reactive power is obtained than when considering the polynomial model or the constant load.

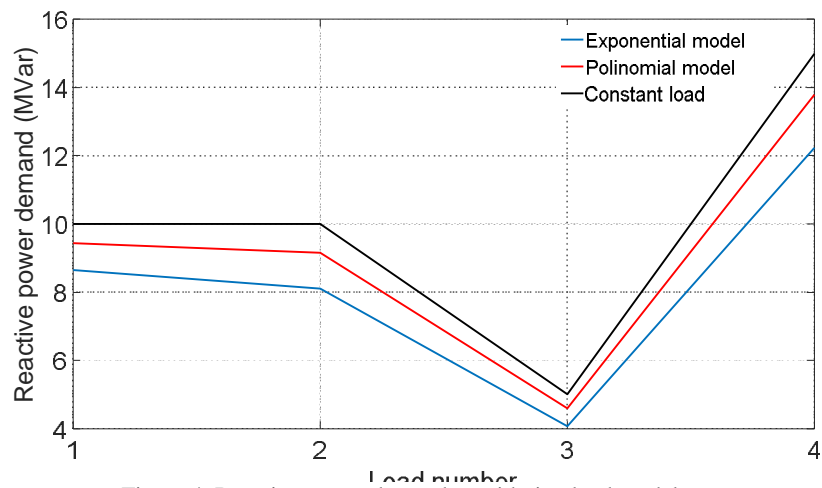


Figure 4: Reactive power demand considering load models

In this sense, it is easy to infer that the modelling of loads has a significant effect on the results of the steady state equilibrium point of power systems, which for this particular power system is a more economical operating point of the system when such load models are integrated.

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## VII. CONCLUSION

An implementation of the voltage dependent load models in the OPF formulation has been presented with the purpose of evaluating the impact that these models have on the dispatch of active power and in general on the results of the optimal equilibrium point of the steady state of power systems.

The results obtained in the study cases showed that by integrating the voltage-dependent load models, the active and reactive power losses are reduced, but also, there is a decrease in the power demanded, which makes the generation power decrease and with it the cost of generation. Thus, the generation cost is lower with the two load models, but it should be noted that in the case of the exponential model, the cost of generation is even lower.

From the aforementioned, it can be concluded that the modelling of the loads has a significant effect on the results of the optimal steady-state equilibrium point of power systems, which for this particular power system is an operation point of the system of more economical power than when load models are not integrated.

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