

# On Nano $\alpha$ g\*s-Closed Sets in Nano Topological Spaces

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**ABSTRACT:** In this paper, a new class of set called nano  $\alpha$  generalized star semi-closed sets in nano topological spaces is introduced and some of their basic properties are investigated. Further the notion of  $N\alpha g^*$ s-open sets,  $N\alpha g^*$ s-neighbourhoods,  $N\alpha g^*$ s-interior and  $N\alpha g^*$ s-closure are discussed.

**SUBJECT CLASSIFICATION :**54A05,54C10.

**KEYWORDS:**  $N\alpha g^*$ s-closed sets,  $N\alpha g^*$ s-open sets,  $N\alpha g^*$ s-neighbourhoods,  $N\alpha g^*$ s-interior,  $N\alpha g^*$ s-closure.

## I.INTRODUCTION

Levine [4] Introduced the class of g-closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology. Recently Lellis Thivagar introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and up- per approximation of X. The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets. Bhuvaneshwari introduced nano g-closed sets and obtained some of the basic interesting results..In this paper,we define a new class of sets called nano  $\alpha$  generalized star semi-closed and its open sets in nano topological spaces and study the relationships with other nano sets.

## II.PRELIMINARIES

**Definition 2.1.** [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be in discernible with one another. The pair(U,R) is said to be the approximation space. Let  $X \subseteq U$

- The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by  $L_R(X)$ . That is  $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ , where  $R(x)$  denotes the equivalence class determined by  $x \in U$ .

- The upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by  $U_R(X)$ . That is  $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$

- The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by  $B_R(X)$  That is  $B_R(X) = U_R(X) - L_R(X)$ .

**Definition 2.2.** [5] If (U,R) is an approximation space and  $X, Y \subseteq U$ , then

- $L_R(X) \subseteq X \subseteq U_R(X)$

- $L_R(\emptyset) = U_R(\emptyset) = \emptyset$

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- $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
- $L_R(X) \subseteq L_R(Y)$  and  $U_R(X) \subseteq U_R(Y)$  whenever  $X \subseteq Y$
- $U_R(X^c) = [L_R(X)]^c$  and  $L_R(X^c) = [U_R(X)]^c$
- $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
- $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

**Definition 2.3.** [5] Let  $U$  be the Universe and  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ .  $\tau_R(X)$  satisfies the following axioms:

- $U$  and  $\varphi \in \tau_R(X)$ .
- The union of elements of any subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$ .
- The intersection of the elements of any finite subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

We call  $(U, \tau_R(X))$  is a nano topological space. The elements of  $\tau_R(X)$  are called a opensets and the complement of a nano open set is called nano closed sets.

Throughout this paper  $(U, \tau_R(X))$  is a nano topological space with respect to  $X$  where  $X \subseteq U$ ,  $R$  is an equivalence relation on  $U$ ,  $U/R$  denotes the family of equivalence classes of  $U$  by  $R$ .

**Remark 2.4.** [5] If  $\tau_R(X)$  is the nano topology on  $U$  with respect to  $X$ . The set  $B = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.5.** [5] If  $(U, \tau_R(X))$  is a nano topological space with respect to  $X$ . Where  $X \subseteq G$  and if  $A \subseteq G$ , then

- The nano interior of the set  $A$  is defined as the union of all nano open subsets contained in  $A$  and is denoted by  $Nint(A)$ ,  $Nint(A)$  is the largest nano open subset of  $A$ .
- The nano closure of the set  $A$  is defined as the intersection of all nano closed sets containing  $A$  and is denoted by  $Ncl(A)$ .  $Ncl(A)$  is the smallest nano closed set containing  $A$ .

**Definition 2.6.** [10] A nano-subset of a nano topological spaces  $U$  is called nano-dense if  $Ncl(A) = U$ .

**Definition 2.7.** [12] A nano topological space  $(U, \tau_R(X))$  is said to be nano extremally disconnected, if the nano-closure of each nano-open set is nano-open.

**Definition 2.8.** [10] A space  $U$  is called nano-submaximal if each nano-dense subset of  $U$  is nano-open.

**Definition 2.9.** Let  $(U, \tau_R(X))$  be a nano topological space and  $A \subseteq G$ . Then  $A$  is said to be

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$N_r$ -closed [5] if  $A = Ncl(Nint(A))$ .

$N\alpha$ -closed [5] if  $Nint(Ncl(Nint(A))) \subseteq A$ .

$N_p$ -closed [5] if  $Nint(Ncl(A)) \subseteq A$ .

$N_s$ -closed [2] if  $Ncl(Nint(A)) \subseteq A$ .

$N\beta$ -closed [2] if  $Ncl(Nint(Ncl(A))) \subseteq A$ .

$N\pi$ -closed [1] if intersection of nano- $\alpha$ -closed.

$N_g$ -closed [2] if  $Ncl(A) \subset G$  whenever  $A \subset G$  and  $G$  is nano-open in  $U$ .

$N_g^*$ -closed [5] if  $Ncl(A) \subset G$  whenever  $A \subset G$  and  $G$  is nano  $g$ -open in  $U$ .

$N_g^*s$ -closed [4] if  $Nscl(A) \subset G$  whenever  $A \subset G$  and  $G$  is nano  $g$ -open in  $U$ .

$N\pi g$ -closed [6] if  $Ncl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is  $N\pi$ -open.

$N\alpha g$ -closed [11] if  $N\alpha cl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is nano-open.

$Nsg$ -closed [2] if  $Nscl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is  $N_s$ -open.

$N\pi g\beta$ -closed [8] if  $N\beta cl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is  $N\pi$ -open.

$N\pi gs$ -closed [7] if  $Nscl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is  $N\pi$ -open.

$Ngs$ -closed [3] if  $Nscl(A) \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is nano-open.

### III. $N\alpha g^*s$ - CLOSED SETS

**Definition 3.1.** A subset  $A$  of a nano topological space  $(U, \tau_R(X))$  is called nano  $\alpha$  generalized star semi-closed (briefly  $N\alpha g^*s$ -closed) set if  $Nscl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is  $N\alpha g$ -open in  $(U, \tau_R(X))$ .

**Theorem 3.2.** Every  $N_r$ -closed set is  $N\alpha g^*s$ -closed.

**Proof:** Let  $A$  be a  $N_r$ -closed set in  $U$ . Let  $G$  be a  $N\alpha g$ -open set such that  $A \subseteq G$ . Since  $A$  is  $N_r$ -closed we have  $Nrcl(A) = A \subseteq G$ . But,  $Nscl(A) \subseteq Nrcl(A) \subseteq G$ . Therefore  $Nscl(A) \subseteq G$ . Hence  $A$  is a  $N\alpha g^*s$ -closed set in  $X$ .

**Remark 3.3.** The converse of the above theorem need not be true as seen in the following example.

**Example 3.4.** let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{b, c, d\}\}$ . Let  $X = \{a, c\} \subseteq U$  and  $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c, d\}\}$ , nano  $r$ -closed set  $= \{\emptyset, U, \{b, c, d\}, \{a\}\}$  and nano  $\alpha g^*s$ -closed set  $= \{\emptyset, U, \{a\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$ . Let  $A = \{b, d\}$ . Thus the subset  $A$  is  $N\alpha g^*s$ -closed but not  $N_r$ -closed set.

**Theorem 3.5.** Every  $N\alpha g^*s$ -closed set is  $N\pi gs$ -closed.

**Proof:** Let  $A$  be any  $N\alpha g^*s$ -closed set in  $U$ . Let  $A \subseteq G$  and  $G$  be  $N\alpha g$ -open in  $U$ . Every nano open is  $N\pi$ -open and  $N\alpha g$ -open and since  $A$  is  $N\alpha g^*s$ -closed, we have  $Nscl(A) \subseteq G$ . Therefore  $A$  is  $N\pi gs$ -closed.

**Remark 3.6.** The converse of the above theorem need not be true as seen in the following example.

**Example 3.7.** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{b, c, d\}\}$ . Let  $X = \{b, c\} \subseteq U$  and  $\tau_R(X) = \{\emptyset, U, \{b, c, d\}\}$ .  $N\alpha g^*s$ -closed set  $= \{\emptyset, U, \{a\}, \{b\}\}$  and  $N_rwg$ -closed set  $= \{\emptyset, U, \{b, c, d\}\}$ . Let  $A = \{a\}$  and  $B = \{b, c, d\}$ . The subset  $A$  is  $N\alpha g^*s$ -closed set but

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not in  $Nr_w$ -closed set. The subset  $B$  is in  $Nr_w$ -closed set but not in  $N\alpha g^*s$ -closed set. Therefore  $Nr_w$ -closed sets and  $N\alpha g^*s$ -closed sets are independent of each other.

**Theorem 3.8.** Union of any two  $N\alpha g^*s$ -closed subset is  $N\alpha g^*s$ -closed.

**Proof:** Let  $A$  and  $B$  be any two  $N\alpha g^*s$ -closed sets in  $U$ , such that  $A \subseteq G$  and  $B \subseteq G$  where  $G$  is  $N\alpha g$ -open in  $U$  and so  $A \cup B \subseteq G$ . Since  $A$  and  $B$  are  $N\alpha g^*s$ -closed, we have  $A \subseteq Nscl(A)$  and  $B \subseteq Nscl(A)$  and hence  $A \cup B \subseteq Nscl(A) \cup Nscl(B) \subseteq Nscl(A \cup B)$ . Thus,  $A \cup B$  is  $N\alpha g^*s$ -closed set in  $(U, \tau_R(X))$ .

**Example 3.9.** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ . Let  $X = \{a, c\} \subseteq U$  and  $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, c, d\}, \{c, d\}\}$ . Then  $N\alpha g^*s$ -closed =  $\{\emptyset, U, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ . Let  $A = \{a\}$  is  $\alpha g^*s$ -closed set and  $B = \{b\}$  is  $\alpha g^*s$ -closed set, then  $A \cup B = \{a\} \cup \{b\} = \{a, b\}$  is also a  $N\alpha g^*s$ -closed set.

**Theorem 3.10.** Intersection of any two  $N\alpha g^*s$ -closed subset is  $N\alpha g^*s$ -closed

**Proof:** Let  $A$  and  $B$  be any two  $N\alpha g^*s$ -closed sets in  $U$ , such that  $A \subseteq G$  and  $B \subseteq G$  where  $G$  is  $N\alpha g$ -open in  $U$  and so  $A \cap B \subseteq G$ . Since  $A$  and  $B$  are  $N\alpha g^*s$ -closed, we have  $A \subseteq Nscl(A)$  and  $B \subseteq Nscl(A)$  and hence  $A \cap B \subseteq Nscl(A) \cap Nscl(B) \subseteq Nscl(A \cap B)$ . Thus,  $A \cap B$  is a  $N\alpha g^*s$ -closed set in  $(U, \tau_R(X))$ .

**Example 3.11.** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ . Let  $X = \{a, c\} \subseteq U$  and  $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, c, d\}, \{c, d\}\}$ . Then  $N\alpha g^*s$ -closed =  $\{\emptyset, U, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ . Let  $A = \{a, b\}$  is  $\alpha g^*s$ -closed set and  $B = \{b, c, d\}$  is a  $\alpha g^*s$ -closed set, then  $A \cap B = \{a, b\} \cap \{b, c, d\} = \{b\}$  is also a  $N\alpha g^*s$ -closed set.

**Theorem 3.12.** A subset  $A$  of  $U$  is  $N\alpha g^*s$ -closed iff  $Nscl(A) - A$  contains no non-empty nano closed set.

**Proof:** Let  $A$  be a  $N\alpha g^*s$ -closed set. Suppose  $F$  is a non-empty nano closed set such that  $F \subseteq Nscl(A) - A$ . Then  $F \subseteq Nscl(A) \cap A^c$ , since  $Nscl(A) - A = Nscl(A) \cap A^c$ . Therefore  $F \subseteq Nscl(A)$  and  $F \subseteq A^c$ . Since  $F$  is nano open, it is  $N\alpha g$ -open. Now by the definition of  $N\alpha g^*s$ -closed set,  $Nscl(A) \subseteq \overline{F}$ , that is  $F \subseteq [Nscl(A)]^c$ . Hence  $F \subseteq Nscl(A) \cap [Nscl(A)]^c = \emptyset$ . That is  $F = \emptyset$ , which is a contradiction. Thus,  $Nscl(A) - A$  contains no non-empty nano closed set in  $U$ . Conversely, assume that  $Nscl(A) - A$  contains no non-empty nano closed set. Let  $A \subseteq U$ , where  $G$  is  $\alpha g$ -open. Suppose that  $Nscl(A)$  is not contained in  $U$ , then  $Nscl(A) \cap U^c$  is a non-empty nano closed subset of  $Nscl(A) - A$ , which is a contradiction. Therefore  $Nscl(A) \subseteq U$  and hence  $A$  is  $N\alpha g^*s$ -closed.

**Theorem 3.13.** If  $A$  is a  $N\alpha g^*s$ -closed subset of  $U$  such that  $A \subseteq B \subseteq Nscl(A)$ , then  $B$  is a  $N\alpha g^*s$ -closed set in  $U$ .

**Proof:** Let  $A$  be a  $N\alpha g^*s$ -closed set of  $U$  such that  $A \subseteq B \subseteq Nscl(A)$ . Let  $G$  be a  $N\alpha g$ -open set of  $U$  such that  $B \subseteq G$ , then  $A \subseteq G$ . Since  $A$  is  $N\alpha g^*s$ -closed. We have  $Nscl(A) \subseteq G$ . Now  $Nscl(B) \subseteq Nscl(Nscl(A)) = Nscl(A) \subseteq G$ . Therefore  $B$  is a  $N\alpha g^*s$ -closed set in  $U$ .

**Theorem 3.14.** For each  $\{a\} \in U$ , either  $\{a\}$  is nano closed set (or)  $\{a\}^c$  is nano generalized closed in  $\tau_R(X)$ .

**Proof:** Suppose  $\{a\}$  is not nano closed in  $U$ . Then  $\{a\}^c$  is not nano open and the only nano open set containing  $\{a\}^c$  is  $G \subseteq U$ . That is  $\{a\}^c \subseteq U$ . Therefore  $Nscl\{a\}^c \subseteq U$  which implies  $\{a^c\}$  is nano  $N\alpha g^*s$ -closed in  $\tau_R(X)$ .

**Corollary 3.15.** Let  $A$  be a  $N\alpha g^*s$ -closed set and suppose that  $F$  is a nano closed set. Then  $A \cup F$  is a  $N\alpha g^*s$ -closed set which is given in the following example.

**Example 3.16.** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, c\}, \{b\}, \{d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X)^c = \{\emptyset, U, \{a, c, d\}, \{d\}, \{b, d\}\}$ . Then  $N\alpha g^*s$ -closed =  $\{\emptyset, U, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ . Let

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$A = \{a, c, d\}$  and  $F = \{b, c, d\}$ . Then  $A \cap F = \{c, d\}$  is a  $Nag^*$ -s-closed set.

**Definition 3.17.** A subset  $A$  of a nano topological space  $(U, \tau_R(X))$  is called nano  $\alpha$  generalized star semi-open (briefly nano  $\alpha g^*$ -s-open) if  $A^c$  is nano  $\alpha g^*$ -s-closed.

**Theorem 3.18.** A subset  $A \subseteq U$  is  $Nag^*$ -s-open iff  $F \subseteq Nint(A)$  whenever  $F$  is a  $Nag$ -closed set and  $F \subseteq A$ .

**Proof:** Let  $A$  be nano  $\alpha g^*$ -s-open suppose  $F \subseteq int(A)$  where  $F$  is  $Nag$ -closed and  $F \subseteq A$ . Let  $A \subseteq G$  where  $G = F^c$  is  $Nag$ -open. Then  $G^c \subseteq A$  and  $G^c \subseteq Nint(A)$ . Then we have  $A^c$  is  $Nag^*$ -s-closed. Hence  $A$  is  $Nag^*$ -s-open.

Conversely, If  $A$  is  $Nag^*$ -s-open,  $F \subseteq A$  and  $F$  is  $Nag$ -closed. Then  $F^c$  is  $Nag$ -open and  $A^c \subseteq F$ . Therefore  $Ncl(A^c) \subseteq (F^c)^c$ . But  $Ncl(A^c) = (Nint(A))^c$ . Hence  $F \subseteq Nint(A)$ .

**Theorem 3.19.** If  $Nint(A) \subseteq B \subseteq A$  and if  $A$  is nano  $\alpha g^*$ -s-open, then  $B$  is nano  $\alpha g^*$ -s-open.

**Proof:** Let  $Nint(A) \subseteq B \subseteq A$ , then  $A^c$  is nano  $\alpha g^*$ -s-closed and hence  $B^c$  is also nano  $\alpha g^*$ -s-closed by the above Theorem 3.8. Therefore  $B$  is nano  $\alpha g^*$ -s-open.

### IV. NANO $\alpha g^*$ -s-INTERIOR AND $\alpha g^*$ -s-CLOSURE

**Definition 4.1.** Let  $(U, \tau_R(X))$  be a nano topological space and  $A \subseteq U$  then nano  $\alpha g^*$ -s-interior is defined as  $Nag^*$ -s-int( $A$ ) =  $\cup \{B : B \text{ is nano } \alpha g^*\text{-s-open, } B \subseteq A\}$ . Clearly  $Nag^*$ -s-int( $A$ ) is the largest nano  $\alpha g^*$ -s-closed set over  $U$  which is contained in  $A$ .

**Definition 4.2.** Let  $(U, \tau_R(X))$  be a nano topological space and  $A \subseteq U$  then nano  $\alpha g^*$ -s-closure is defined as  $Nag^*$ -s-cl( $A$ ) =  $\cap \{F : F \text{ is nano } \alpha g^*\text{-s-closed, } A \subseteq F\}$ . Clearly  $Nag^*$ -s-cl( $A$ ) is the smallest nano  $\alpha g^*$ -s-closed set over  $U$  which contains  $A$ .

**Lemma 4.3.** Let  $A$  and  $B$  be any two subsets of  $U$  in a nano topological spaces  $(U, \tau_R(X))$  and the following are true

- i.  $Nag^*$ -s-int( $A$ )  $\subseteq A$
- ii.  $A \subseteq B \Rightarrow Nag^*$ -s-int( $B$ )  $\subseteq Nag^*$ -s-int( $A$ )
- iii.  $Nag^*$ -s-int( $A$ )  $\cup Nag^*$ -s-int( $B$ )  $\subseteq Nag^*$ -s-int( $A \cup B$ )
- iv.  $Nag^*$ -s-int( $A$ )  $\cap Nag^*$ -s-int( $B$ )  $\subseteq Nag^*$ -s-int( $A \cap B$ )

**Lemma 4.4** For a subset  $A$  of  $U$ .

- (i)  $Nag^*$ -s-cl( $A$ )  $\subseteq Ncl(A)$
- (ii)  $N-int(A) \subseteq Nag^*$ -s-cl( $A$ )

**Lemma 4.5.** A subset  $A$  of  $U$  is nano  $\alpha$  generalized star semi-closed if and only if  $A = Nag^*$ -s-cl( $A$ ).

**Theorem 4.6.** Let  $A$  and  $B$  be two subsets of nano topological space  $(U, \tau_R(X))$ . Then

- i.  $Nag^*$ -s-int( $U$ ) =  $U$  and  $Nag^*$ -s-int( $\emptyset$ ) =  $\emptyset$
- ii.  $Nag^*$ -s-int( $A$ )  $\subseteq A$
- iii. If  $B$  is any  $Nag^*$ -s-open set contained in  $A$ , then  $B \subseteq Nag^*$ -s-int( $A$ )
- iv. If  $A \subseteq B$ , then  $Nag^*$ -s-int( $A$ )  $\subseteq Nag^*$ -s-int( $B$ )

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$$v. \text{Nag}^*s\text{-int}(\text{Nag}^*s\text{-int}(A)) = \text{Nag}^*s\text{-int}(A).$$

**Proof:**

i. Let  $U$  and  $\phi$  are  $\text{Nag}^*s$ -open sets.

$$\text{Nag}^*s\text{-int}(U) = \cup \{B : B \text{ is a } \text{Nag}^*s\text{-open, } B \subset U\} = U \cup \text{all } \text{Nag}^*s\text{-open sets} = U.$$

$$\text{(ie) } \text{int}(U) = U.$$

Since  $\phi$  is the only  $\text{Nag}^*s$ -open set contained in  $\phi$ ,

$$\text{Nag}^*s\text{-int}(\phi) = \phi$$

ii. Let  $x \in \text{Nag}^*s\text{-int}(A) \Rightarrow x$  is a interior point of  $A$ .

$$\Rightarrow A \text{ is a nbhd of } x.$$

$$\Rightarrow x \in A$$

$$\text{Thus, (ie) } x \in \text{Nag}^*s\text{-int}(A) \Rightarrow x \in A$$

Hence  $\text{Nag}^*s\text{-int}(A) \subset A$

iii. Let  $B$  be any  $\text{Nag}^*s$ -open sets such that  $B \subset A$ . Let  $x \in B$ . Since  $B$  is a  $\text{Nag}^*s$ -open set contained in  $A$ .  $x$  is a  $\text{Nag}^*s$ -interior point of  $A$ .

$$\text{i.e } x \in \text{Nag}^*s\text{-int}(A). \text{ Hence } B \subset \text{Nag}^*s\text{-int}(A).$$

iv. Let  $A$  and  $B$  be subsets of  $(U, \tau_R(X))$  such that  $A \subset B$ . Let  $x \in \text{Nag}^*s\text{-int}(A)$ . Then  $x$  is a  $\text{Nag}^*s$ -interior point of  $A$  and so  $A$  is a  $\text{Nag}^*s$ -nbhd of  $x$ . Since  $B \supset A$ ,  $B$  is also  $\text{Nag}^*s$ -nbhd of  $x$

$$\Rightarrow x \in \text{Nag}^*s\text{-int}(B). \text{ thus we have shown that } x \in \text{Nag}^*s\text{-int}(B).$$

**Theorem 4.7.** If a subset  $A$  of space  $(U, \tau_R(X))$  is  $\text{Nag}^*s$ -open, then  $\text{Nag}^*s\text{-int}(A) = A$ .

**Proof:** Let  $A$  be  $\text{Nag}^*s$ -open subset of  $(U, \tau_R(X))$ . We know that  $\text{Nag}^*s\text{-int}(A) \subset A$ . Also,  $A$  is  $\text{Nag}^*s$ -open set contained in  $A$ . From above Theorem 4.6(iii)  $A \subset \text{Nag}^*s\text{-int}(A) = A$ .

**Theorem 4.8** If  $A$  and  $B$  are subsets of  $(U, \tau_R(X))$ , then  $\text{Nag}^*s\text{-int}(A \cap B) = \text{Nag}^*s\text{-int}(A) \cap \text{Nag}^*s\text{-int}(B)$ .

**Proof:**

We know that  $A \cap B \subset B$ . we have  $\text{Nag}^*s\text{-int}(A \cap B) \subset \text{Nag}^*s\text{-int}(A)$  and  $\text{Nag}^*s\text{-int}(A \cap B) \subset \text{Nag}^*s\text{-int}(B)$ . This implies that  $\text{Nag}^*s\text{-int}(A \cap B) \subset \text{Nag}^*s\text{-int}(A) \cap \text{Nag}^*s\text{-int}(B)$ . Again let  $x \in \text{Nag}^*s\text{-int}(B)$ . Then  $x \in \text{Nag}^*s\text{-int}(A)$  and  $x \in \text{Nag}^*s\text{-int}(B)$ . Hence  $x$  is a  $\text{Nag}^*s$ -int point of each of sets  $A$  and  $B$ . It follows that  $A$  and  $B$  is  $\text{Nag}^*s$ -nbhd of  $x$ , so that their intersection  $A \cap B$  is also a  $\text{Nag}^*s$ -nbhds of  $x$ . Hence  $x \in \text{Nag}^*s\text{-int}(A \cap B)$ . Thus  $x \in \text{Nag}^*s\text{-int}(A) \cap \text{Nag}^*s\text{-int}(B)$  implies that  $x \in \text{Nag}^*s\text{-int}(A \cap B)$ . Therefore  $\text{Nag}^*s\text{-int}(A) \cap \text{Nag}^*s\text{-int}(B) \subset \text{Nag}^*s\text{-int}(A \cap B)$ . From the above, We get  $\text{Nag}^*s\text{-int}(A \cap B) = \text{Nag}^*s\text{-int}(A) \cap \text{Nag}^*s\text{-int}(B)$ .

**Theorem 4.9.** Let  $A$  be a subset of a nano topological space  $(U, \tau_R(X))$ . Then

$$(i) (\text{Nag}^*s\text{-cl}(A))^c = \text{Nag}^*s\text{-int}(A^c)$$

$$(ii) (\text{Nag}^*s\text{-int}(A))^c = \text{Nag}^*s\text{-cl}(A^c)$$

**Proof:**

$$(i) (\text{Nag}^*s\text{-cl}(A))^c = \{F : F \text{ is } \text{Nag}^*s\text{-closed, } A \subset F\}^c$$

$$= \cup \{F^c : F^c \text{ is } \text{Nag}^*s\text{-open, } F^c \subset A^c\}.$$

$$= \text{Nag}^*s\text{-int}(A^c)$$

$$(\text{Nag}^*s\text{-int}(A))^c = (\cup \{B : B \text{ is } \text{Nag}^*s\text{-open, } B \subset A\})^c$$

$$= \cap \{B^c : B^c \text{ is } \text{Nag}^*s\text{-closed, } A^c \subset B^c\}.$$



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$$= \text{Nag}^* \text{s-cl}(A^c).$$

**Theorem 4.10.** If  $A$  is a subset of a space  $(U, \tau(X))$ , then

$\text{Nag}^* \text{s-cl}(A) \subset \text{Nag-cl}(A)$ , where  $\text{Nag-cl}(A)$  is given by  $\text{Nag-cl}(A) = \bigcap \{F \subset U : A \subset F \text{ and } F \text{ is a Nag-closed set in } U\}$ .

**Proof:**

Let  $A$  be a subset of  $U$ . By definition of  $\text{Nag-cl}(A) = \bigcap \{F \subset U : A \subset F \text{ and } F \text{ is a Nag-closed set in } U\}$ . If  $A \subset F$  and  $F$  is Nag-closed subset of  $x$ , then  $A \subset F \in \text{Nag-cl}(A)$ , because every Nag-closed subset in  $U$ . That is  $\text{Nag}^* \text{s-cl}(A) \subset F$ . Therefore  $\text{Nag}^* \text{s-cl}(A) \subset \bigcap \{F \subset U : A \subset F \text{ and } F \text{ is a Nag-closed set in } U\} = \text{Nag-cl}(A)$ .

Hence  $\text{Nag}^* \text{s-cl}(A) \subset \text{Nag-cl}(A)$ .

## V. NANO $\text{ag}^* \text{s-NEIGHBOURHOODS}$

**Definition 5.1.** A subset  $M_x \subset U$  is called a nano  $\text{ag}^* \text{s-}$ neighbourhood ( $\text{Nag}^* \text{s-nghd}$ ) of a point  $x \in U$  if and only if there exists a  $A \in \text{Nag}^* \text{sO}(U, X)$  such that  $x \in A \subset M_x$  and a point  $x$  is called  $\text{Nag}^* \text{s-nghd}$  point of the set  $A$ .

**Definition 5.2.** The family of all  $\text{Nag}^* \text{s-nghd}$  of the point  $x \in U$  is called  $\text{Nag}^* \text{s-nghd}$  system of  $U$  and is denoted by  $\text{Nag}^* \text{s-nghd}$ .

**Example 5.3.** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, c\}, \{b\}, \{d\}\}$ .

Let  $X = \{a, b\} \subseteq U$  and  $\tau R(X) =$

$\{\varphi, U, \{b\}, \{a, b, c\}, \{a, c\}\}$ ,  $\text{Nag}^* \text{sO}(U, X) = \{\varphi, U, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}\}$ . Then

$\text{Nag}^* \text{s-nghds}(a) = \{\varphi, U, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ ,

$\text{Nag}^* \text{s-nghds}(b) = \{\varphi, U, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}\}$ ,

$\text{Nag}^* \text{s-nghds}(c) = \{\varphi, U, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}\}$ ,

$\text{Nag}^* \text{s-nghds}(d) = \{\varphi, U, \{b, d\}, \{c, d\}, \{b, c\}, \{a, c, d\}\}$ .

**Lemma 5.4** An arbitrary union of  $\text{Nag}^* \text{s-nghds}$  of a point  $x$  is again a  $\text{Nag}^* \text{s-nghd}$  of  $x$ .

**Proof:** Let  $\{A_\lambda\}_{\lambda \in I}$  be an arbitrary collection of  $\text{Nag}^* \text{s-nghds}$  of a point  $x \in U$ . We have to prove that  $\bigcup A_\lambda$  for  $\lambda \in I$  (where  $I$  denote index set) also a  $\text{Nag}^* \text{s-nghd}$  of  $x$ . For all  $\lambda \in I$  there exists  $\text{Nag}^* \text{s-open}$  set  $M_x$  such that  $x \in M_x \subset A_\lambda \subset \bigcup A_\lambda$  i.e  $x \in \bigcup M_x \subset \bigcup A_\lambda$  therefore  $\bigcup A_\lambda$  for  $\lambda \in I$  is a  $\text{Nag}^* \text{s-nghd}$  of  $x$ . That is an arbitrary union of  $\text{Nag}^* \text{s-nghds}$  of  $x$  is again a  $\text{Nag}^* \text{s-nghd}$  of  $x$ . But intersection of  $\text{Nag}^* \text{s-nghds}$  of a point is not a  $\text{Nag}^* \text{s-nghd}$  of that point in general.

**Theorem 5.5.** The  $\text{Nag}^* \text{s-nghd}$  system  $\text{Nag}^* \text{s-nghd}(x)$  of a point  $x \in U$  satisfies the following properties

(a) if  $N \in \text{Nag}^* \text{s-N}(x)$  then  $x \in N$

(b) if  $N \in \text{Nag}^* \text{s-N}(x)$  and  $N \subset M$  then  $M \in \text{Nag}^* \text{s-N}(x)$

(c) if  $N \in \text{Nag}^* \text{s-N}(x)$  then there exists a  $G \in \text{Nag}^* \text{s-N}(x)$  such that  $G \subset N$  and  $G \in \text{Nag}^* \text{s-N}(y)$ , for all  $y \in G$ .

**Proof:**

(a) Let  $N \in \text{Nag}^* \text{s-N}(x)$  implies  $N$  is the  $\text{Nag}^* \text{s-nghd}$  of  $x$ . therefore  $x \in N$

(b) Let  $N \in \text{Nag}^* \text{s-N}(x)$  and  $N \subset M$ . Therefore there exists  $G \in \text{Nag}^* \text{sO}(U, X)$  such that  $x \in G \subset N \subset M$  implies  $M$  is a

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$N\alpha g^*$ -nghd of  $x$  and hence  $M \in N\alpha g^*s-N(x)$

(c) Let  $N \in N\alpha g^*s-N(x)$  implies  $G \in N\alpha g^*s-O(X)$  such that  $x \in G \subset N$ .  $G$  is  $N\alpha g^*$ -nghd of each of its points implies for all  $y \in G$ ,  $G$  is the  $N\alpha g^*$ -nghd of  $y$  and hence  $G \in N\alpha g^*s-N(y)$  for all  $y \in G$ .

**Theorem 5.6** If  $A$  be a subset of  $(U, \tau_R(X))$ . Then  $N\alpha g^*s-int(A) = \cup \{B : B \text{ is a } N\alpha g^*s\text{-open, } B \subset A\}$ .

**Proof:**

Let  $A$  be a subset of  $(U, \tau_R(X))$ .  $x \in N\alpha g^*s-int(A)$

$\Leftrightarrow x$  is a  $N\alpha g^*$ -interior point of  $A$ .

$\Leftrightarrow A$  is a  $N\alpha g^*$ -nbhd of point  $x$ .

$\Leftrightarrow$  there exists  $N\alpha g^*$ -open set  $B$  such that  $x \in B \subset A$

$\Leftrightarrow x \in \cup \{B : B \text{ is a } N\alpha g^*s\text{-open, } B \subset A\}$

Hence  $N\alpha g^*s-int(A) = \cup \{B : B \text{ is a } N\alpha g^*s\text{-open, } B \subset A\}$ .

### REFERENCES

1. Adinatha C. Upadhaya, On quasi nano  $p$ -normalspaces, International Journal of Recent Scientific Research 8(6)(2017), 17748–17751
2. K.Bhuvaneshwari and K.Mythili Gnanapriya, Nano Generalized closed sets, International Journal of Scientific and Research Publications, 14(5)(2014), 1–3.
3. K.Bhuvaneshwari and K.Ezhilarasi, On Nano Semi generalized and Nano generalized semi-closed sets, International Journal of Mathematics and Computer Applications Research, 4(3)(2014), 117–124.
4. N. Levine, Generalized closed sets in topology, Rend. Circ. math. palermo, 19(2)(1963), 89–96.
5. M. Thivagar and C. Richard, On nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1)(2012), 31–37.
6. Rajasekaran and O. Nethaji, On some new subsets of nano topological spaces, Journal of New Theory, 16(2017), 52–58.
7. Rajasekaran. I and O. Nethaji, On nano  $\pi g s$ -closed sets, Journal of New Theory, 19(2017), 56–62.
8. M.Sagan Joseph, I. Rajasekaran and O. Nethaji, On nano  $\pi g \beta$ -closed sets, Global Journal of Pure and Applied Mathematics, 14(1)(2018), 181–187.
9. P.Sathishmohan, On  $\theta g^*$  closed sets in Topological Spaces, Journal of Mathematics and Informatics, 13(2018), 65–80.
10. P. Sathishmohan, On nano semi pre neighbourhoods in nano topological spaces, Malaya Journal of Matematik, 6(1)(2018), 294–298.
11. [11] R.Thanga Nachiyar and K.Bhuvaneshwari, Nano generalized  $\alpha$ -continuous and Nano  $\alpha$ -generalized continuous functions in nano topological spaces, International Journal of Mathematics Trends and Technology, 14(2)(2014), 79–83.
12. [12] M. Thivagar and C. Richard, On nano forms of weekly open sets, International Journal of Mathematics and Statistics Invention, 1(1)(2013), 31–37.